ANALYSING AN INTUITIONISTIC FUZZY LINEAR PROGRAMMING PROBLEM USING (α, β)-CUTS

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Abstract: Obtaining the solution of Intuitionistic Fuzzy Linear Programming Problem (IFLPP) without affecting its originality is an interesting task. In this paper we propose a new ranking to solve an IFLPP in its original version. We have considered IFLPP with Hexagonal Intuitionistic Fuzzy Numbers (HIFN) in the analysis. Numerical examples are studied to show the efficiency of the proposed study.

Keywords: Fuzzy Linear Programming , Hexagonal Intuitionistic Fuzzy Numbers (HIFN), (α, β)-Cuts.

Introduction: The traditional optimization models dealt only with crisp and exact information. Fuzzy optimization models reflect real life uncertainty. IFLPP is a useful tool for understanding complex problems. Atanassov [1] introduced the concept of Intuitionistic Fuzzy Sets (IFS), which is a generalization of the concept of fuzzy set [1]. An IFS has been applied in many areas including pattern recognition and image processing. The first method for solving Fuzzy Linear Programming was proposed by Zimmermann [7]. Numerous ranking methods have been proposed in literature to rank Intuitionistic Fuzzy numbers [2, 3, 4, 5]. In this paper we have adopted the ranking method and have developed an algorithm to solve an Intuitionistic Fuzzy Linear Programming problem.

Preliminaries :

Intuitionistic fuzzy set [6] : Let X be a nonempty set. An intuitionistic fuzzy set $\widetilde{A}^I$ of X is defined as $\widetilde{A}^I = \{\langle x, \mu_{\widetilde{A}^I}, \vartheta_{\widetilde{A}^I} \rangle / x \in X\}$ where $\mu_{\widetilde{A}^I}(x)$ *and* $\vartheta_{\widetilde{A}^I}(x)$ are membership and nonmember ship functions such that $\mu_{\widetilde{A}^I}(x), \vartheta_{\widetilde{A}^I}(x) : X \to [0,1]$ and $0 \le \mu_{\widetilde{A}^I}(x) + \vartheta_{\widetilde{A}^I}(x) \le 1$ for all x ε X

Intutionistic fuzzy number [6] :

An Intutionistic fuzzy number $\widetilde{A}^I$ is

i) an intuitionistic fuzzy subset of the real line,

ii) normal, that is, there is some $x_0 \in R$ such that $\mu_{\widetilde{A}}^I(x_0) = 1, \vartheta_{\widetilde{A}}^I(x_0) = 0$

iii)convex for the membership function $\mu_{\widetilde{A}^I}(x)$, that is,

$$\mu_{\widetilde{A}}^I(\lambda x_1 + (1-\lambda)x_2) \ge \min(\mu_{\widetilde{A}}^I(x_1), \mu_{\widetilde{A}}^I(x_2)),$$

for every $x_1, x_2 \in R$, $\lambda \in [0,1]$,

iv) concave for the membership function

$$\vartheta_{\widetilde{A}}^I(\lambda x_1 + (1-\lambda)x_2) \ge \min(\vartheta_{\widetilde{A}}^I(x_1), \vartheta_{\widetilde{A}}^I(x_2)),$$

for every $x_1, x_2 \in R$, $\lambda \in [0,1]$,

Hexagonal Intutionistic fuzzy number[6] : A hexagonal Intutionistic fuzzy number is specified by $\widetilde{A}_H^I = (a_1, a_2, a_3, a_4, a_5, a_6), (a_1', a_2', a_3, a_4, a_5', a_6')$ where $a_1, a_2, a_3, a_4, a_5, a_6, a_1', a_2', a_5', a_6'$ are real numbers such that $a_1' \le a_1 \le a_2' \le a_2 \le a_3 \le a_4 \le a_5 \le a_5' \le a_6 \le a_6'$ and its membership and non membership are given by

$$\mu_{\widetilde{A}^I}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & \text{for } a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right), & \text{for } a_2 \le x \le a_3 \\ 1, & \text{for } a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), & \text{for } a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right), & \text{for } a_5 \le x \le a_6 \\ 0, & \text{otherwise} \end{cases}$$

and

$$\vartheta_{\widetilde{A}^I}(x) = \begin{cases} 1 - \frac{1}{2}\left(\frac{x-a_1'}{a_2'-a_1'}\right), & \text{for } a_1' \le x \le a_2' \\ \frac{1}{2}\left(\frac{a_3-x}{a_3-a_2'}\right), & \text{for } a_2' \le x \le a_3 \\ 0, & \text{for } a_3 \le x \le a_4 \\ \frac{1}{2}\left(\frac{x-a_4}{a_5'-a_4}\right), & \text{for } a_4 \le x \le a_5' \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-a_5'}{a_6'-a_5'}\right), & \text{for } a_5' \le x \le a_6' \\ 1, & \text{otherwise} \end{cases}$$

Intutionistic fuzzy optimum solution[8]: Let S be the set of all intutionistic fuzzy feasible solutions of $\sum_{j=1}^{n} \widetilde{a}_{ij} x_j \le \widetilde{b}^I$. Any vector $x_0 \in S$ is said to be an intutionistic fuzzy optimum solution to $\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i^I$ if $Cx_0 \geq Cx$ for all $x \in S$ where C = ($c_1, c_2, \ldots, c_n$) and $Cx = c_1x_1+c_2x_2+ \ldots +c_nx_n$.

Intutionistic fuzzy feasible solution [8] : Any vector $x \in R^n$ which satisfies the constraints and non negative restrictions of $\sum_{j=1}^{n} \tilde{a}_{ij}^{\ I} x_j \leq \tilde{b}^I$ is said to be an intutionistic fuzzy feasible solution.

Ranking of hexagonal intutionistic fuzzy numbers : Let $\tilde{A}^I = ((a_1, a_2, a_3, a_4, a_5, a_6), (b_1, b_2, b_3, b_4, b_5, b_6))$ be an intutionistic hexagonal fuzzy number. Then the accuracy function of L of $\tilde{A}^I$ is defined as

$$L(\tilde{A}^I) = \frac{a_1 + b_1 + a_2 + b_2 + a_3 + b_3 + a_4 + b_4 + a_5 + b_5 + a_6 + b_6}{6}$$

Intutionistic fuzzy linear programming problem [8] : Linear programming is one of the most frequently applied operations research technique. The parameters of linear programming models must be well defined and precise. But in the real world situation the available information in the system under consideration are not exact, therefore the parameters of linear programming may be represented as fuzzy or intutionistic fuzzy numbers. In the case of intutionistic fuzzy numbers an additional non membership degree is added, which may express more abundant and flexible information as compared with the fuzzy numbers.

An intutionistic fuzzy linear programming problem with intutionistic fuzzy technological coefficients is defined as

$$\text{Maximize } Z = \sum_{j=1}^{n} \tilde{c}_i^{\ I} x_j$$

$$\text{Subject to } \sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i^{\ I}; \qquad 1 \leq i \leq m$$

$$x_j \geq 0; \qquad 1 \leq j \leq n$$

Where atleast one x_j>o and $\tilde{c}_i^{\ I}, b_i^{\ I}$ are the interval arithmetic intutionistic fuzzy numbers.

Proposed method :

Step 1: A hexagonal intutionistic fuzzy number is denoted by $\tilde{A}_H^{\ I} = (a_1, a_2, a_3, a_4, a_5, a_6), (a_1', a_2', a_3, a_4, a_5', a_6')$ where $a_1' \leq a_1 \leq a_2' \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_5' \leq a_6 \leq a_6'$ then their alpha cuts are,

For membership function:

$$\begin{pmatrix} 2\alpha(a_2 - a_1) + a_1,\ 2\left(\alpha - \frac{1}{2}\right)(a_3 - a_2) + a_2, \\ 2(1-\alpha)(a_5 - a_4) + a_4,\ a_6 - 2\alpha(a_6 - a_5) \end{pmatrix}$$

For nonmember ship function:

$$\begin{pmatrix} 2(1-\alpha)(a_2' - a_1') + a_1',\ a_3 - 2\alpha(a_3 - a_2'), \\ 2\alpha(a_5' - a_4) + a_4,\ 2\left(\alpha - \frac{1}{2}\right)(a_6' - a_5') + a_5' \end{pmatrix}$$

Step 2: Using the ranking function the IHFLPP is formulated as

$$\max \tilde{z} \approx \sum_{j=1}^{n} R[c_1, c_2, c_3, c_4, c_5, c_6][d_1, d_2, d_3, d_4, d_5, d_6]\tilde{x}_j$$

Subject to

$$\sum_{j=1}^{n} a_{ij}\tilde{x}_j \leq, \geq, = R[b_1, b_2, b_3, b_4, b_5, b_6][e_1, e_2, e_3, e_4, e_5, e_6]$$

and $\tilde{x}_j \geq \tilde{0}$

Step 3 : The optimum solution is obtained by solving the crisp LPP obtained in step 2 using simplex method.

Numerical example:

Consider the following IFLPP:

$$\max \tilde{z} = [3,4,5,6,7,8][4,5,6,7,8,9]\tilde{x}_1 + [2,3,4,5,6,7][3,4,5,6,7,8]\tilde{x}_2$$

Subject to

$$15\tilde{x}_1 + 10\tilde{x}_2 \leq [11,12,13,14,15,16][10,11,12,13,14,15]$$

$$16\tilde{x}_1 + 18\tilde{x}_2 \leq [12,13,14,15,16,17][13,114,15,16,17,18]$$

and $\tilde{x}_1, \tilde{x}_2 \geq 0$

Solution :

Step 1: The formulated IHFLPP is now converted to alpha cut intutionistic hexagonal fuzzy number

$$\max \tilde{Z} = \left(2\alpha + 3,\ 2\left(\alpha - \frac{1}{2}\right) + 4,\ 2(1-\alpha) + 6,\ 8 - 2\alpha\right)$$

$$\left(2(1-\alpha) + 4,\ 6 - 2\alpha,\ 2\alpha + 7,\ 2\left(\alpha - \frac{1}{2}\right) + 8\right)\tilde{x}_1 +$$

$$\left(2\alpha + 2,\ 2\left(\alpha - \frac{1}{2}\right) + 3,\ 2(1-\alpha) + 5,\ 7 - 2\alpha\right)$$

$$\left(2(1-\alpha) + 3,\ 5 - 2\alpha,\ 2\alpha + 6,\ 2\left(\alpha - \frac{1}{2}\right) + 7\right)\tilde{x}_2$$

Subject to

$$15\tilde{x}_1 + 10\tilde{x}_2 \leq \begin{bmatrix} 2\alpha + 11, 2\left(\alpha - \frac{1}{2}\right) + 12, \\ 2(1-\alpha) + 14, 16 - 2\alpha \end{bmatrix}$$

$$\begin{bmatrix} 2(1-\alpha) + 10, 12 - 2\alpha, \\ 2\alpha + 13, 2\left(\alpha - \frac{1}{2}\right) + 14 \end{bmatrix}$$

$$16\tilde{x}_1 + 18\tilde{x}_2 \leq \begin{bmatrix} 2\alpha + 12, 2\left(\alpha - \frac{1}{2}\right) + 13, \\ 2(1-\alpha) + 15, 17 - 2\alpha \end{bmatrix}$$

$$\begin{bmatrix} 2(1-\alpha) + 13, 15 - 2\alpha, \\ 2\alpha + 16, 2\left(\alpha - \frac{1}{2}\right) + 17 \end{bmatrix}$$

Step 2 :

$$\max \tilde{z} = R[3,4,5,6,7,8][4,5,6,7,8,9]\tilde{x}_1 + R[2,3,4,5,6,7][3,4,5,6,7,8]\tilde{x}_2$$

Subject to

$$15\tilde{x}_1 + 10\tilde{x}_2 \leq R[11,12,13,14,15,16][10,11,12,13,14,15]$$

$$16\tilde{x}_1 + 18\tilde{x}_2 \leq R[12,13,14,15,16,17][13,114,15,16,17,18]$$

and $\tilde{x}_1, \tilde{x}_2 \geq 0$

Step 3: By Solving the crisp linear programming problem, obtained in Step 2 using simplex method The optimum solution obtained is max $\tilde{Z}$ = 21.42, $\tilde{x}_1 = 1.53$, $\tilde{x}_2 = 0.31$

Conclusion : In this paper we have obtained an optimal solution for intutionistic fuzzy linear programming problem with hexagonal fuzzy numbers by a ranking method using alpha. beta cut operations. The decision maker is involved in all the steps of the decision process which makes our work very useful to apply in a real-time applications and problems where the information is uncertain, like sales forecast, Project investment etc,.

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