# OPERATING CHARACTERISTICS OF M/M (alb)/1/MWV/BD QUEUEING SYSTEMS

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**Abstract:** In this paper, steady state solution of M/M(a,b)/1 queueing system along with server breakdowns and multiple working vacation is analyzed under exponential distribution. Numerical illustration is given to justify the validity of the model. Further particular cases are also evaluated.

**Keywords:**Bulk service, multiple working vacations, breakdowns, mean queue length.

**Introduction:** In queueing models, there are situations particularly in transportation system, where the service is provided such that a group of customers can be served simultaneously. Examples include shuttle-bus service, freight trains, express elevators, tour guides and batch servicing in manufacturing process. The theory of batch queues originated with the work of Bailey [3].He considered a queue with Poisson arrival and fixed size service. Numerous authors (Jaishwal [5]&Madil and Choudhury [12]) have investigated a variety of extensions of the basic model. Neuts[13] has introduced the most general bulk service rule in considering a queueing system with Poisson arrival and general service time distribution.

Most of the general bulk service queueing models with servers vacation have been analysed by many authors using matrix geometric method. Some of the notable works for matrix geometric method can be seen in Neuts [15]. Analytic solution of bulk service queueing models can be found in (Madhil and choudhury [12] and Medhi [13].But these works do not include vacations. In general it is difficult to obtain a closed form solution for bulk service queueing models with servers vacation. Afthab Begum [1] has obtained analytic solutions for M/M(a,b)/1 queue, $E_k$/M(a,b)/1 queue and Gl/M(a,b)/1 queue with servers single and multiple vacation in her Ph.D thesis and presented the steady state results.

In 2002, Servi and Finn introduced a class of semi vacation policies, in which servers work at a lower rate rather than completely stopping primary service during vacation. Such a vacation is called **Working vacation (WV).**In working vacation queues, the server works at a lower service rather than completely stopping service during the vacation period. At the vacation termination epochs, if there are customers in the system, the server will start a new regular busy period, otherwise, he takes another working vacation or joins the system and stays idle according as, he follows multiple or single working vacation policy. The M/M/1 queueing system with working vacation has been analysed by Servi and Finn [18] and Tian. N, Zhao .X and Wang .K [19]. Most of the general bulk service queueing models with servers vacation have been analysed by many authors using matrix geometric method. Moreover, Julia Rose Mary and Afthab Begum [7] have analysed the Markovian M/M(a,b)/1 queueing model under multiple working vacations and derived the steady state probability distribution and the mean queue length for the model.

Baba [2], Li et al., [10] and Banik [4]studied the GI/M/1 type working vacation queues. Using different methods, Kim et al., [9] and Li et al.,[11] discussed several M/G/1 type working vacation queues. Recently Tian et al., [20] provided a survey of the results of working vacation queues and demonstrated that the matrix analytic methods developed by Neuts([15] & [16]) are powerful tools for analyzing the working vacation queues and can be considered as a unified approach to this class of queueing models.

Wang first proposed Markovian queueing system under the N-policy and server break downs. Ke, J.C and Pearn, W. L [8] discussed the optimal management policy of heterogenous arrival M/M/1 queue with server's break down and vacations. Later Ke, J.C[6] studied the operating system characteristics of $M^X$/G/1 queueing system under vacation policies with start up and break down times in which the server may breakdown according to a Poisson process while working and his repair time has general distribution. With the help of available literature in this paper we are analysing M/M(a,b)/1/MWV queueing model, for an unreliable server.

**Model Description:** In this model, it is assumed that the arrival process is Poisson with parameter $\lambda$ . The server processes the customers in batches according to the general bulk service rule (GBSR) introduced by Neuts [14].

According to this rule the server starts service only when a minimum of '**a**' customers are present in the system. If the server after a service completion finds **a** (or) more but at most **b** customers present in the system, then he takes them all in a batch, and if he finds more than **b**, then he takes in the batch the first **b**- customers for service, while others wait. Thus each batch for service contains a minimum of '**a**' units and a maximum of '**b**' units. This rule is called general bulk service rule(GBSR).The service time of batches of size S (a ≤s ≤b) is assumed to be independent identically distributed random variable with exponential distribution of parameter μ.

The server subject to break downs at any time while working, with Poisson rate α. Whenever the system fails, the server is sent immediately for repair facility where the repair time is an independent and identically distributed random variable Br following an exponential distribution $(1-e^{-\beta t})$ the customer, who is just being served when the server breaksdown, joins the head of waiting line and resumes the service as soon as the server returns from the repair facility. This type of service continues until the system becomes empty again.

Whenever the server completes a service and finds less than '**a**' customers in the queue he begins a vacation which is an exponentially distributed random variable V with parameter η. After completing a vacation, if the system length is still less than '**a**' he takes another vacation and the vacations are continued until the server finds at least '**a**' customers in the queue (i.e) multiple vacations is adopted. Suppose during vacation if the queue size becomes at least '**a**'the server starts his service under the GBSR with service rate $\mu_v$ which is different from the regular service rate μ. When the vacation ends he switches his service rate from $\mu_v$ to μ when the server is working, the size of the batch in service is x with a≤x ≤b and the service rates are independent of the size of the batch in service, The server may breakdown at any time, and it is sent for repair and thus it is completly repaired. Then the server continues the service. The above queueing model is denoted by **M/M(a,b)/1/MWV/Br.** The steady state results including the probability distribution of the queue size and the expected number of units in the queue are obtained for the model.

**Steady state solution:** R.Rajalakshmiet al.,[17] discussed the steady state equations and performance measures for M/M(a,b)/1/MWV for an unreliable server. According to that, the steady state solution for the number of customers in vacation and in busy state are given by,

$$Q_n = r_v^n Q_0; \quad (n \geq 0) \tag{1}$$

$$P_n = (Ar^n + Br_v^n)Q_0; \quad (n \geq 0) \tag{2}$$

$$A = \left[\frac{\mu(1-r^a)}{(1-r)} - \frac{\alpha\beta r}{\lambda(r-1)+\beta r}\right]^{-1}\left[\frac{\eta}{(1-r_v)} - \frac{B\mu(1-r_v^a)}{(1-r_v)} + \frac{\alpha\beta B r_v}{\lambda(r_v-1)+\beta r_v}\right]$$

$$B = \frac{\eta r_v\left(\lambda(r_v-1)+\beta r_v\right)}{\left[\lambda(r_v-1)+\mu r_v(1-r_v^b)+\alpha r_v\right]\left[\lambda(r_v-1)+\beta r_v\right]-\alpha\beta r_v^2}$$

$$R_n = \left[\frac{\mu}{\lambda}\left(\frac{A(1-r^{n+1})}{(1-r)} + \frac{B(1-r_v^{n+1})}{(1-r_v)}\right) + \frac{\mu_v}{\lambda}\left(\frac{1-r_v^{n+1}}{1-r_v}\right)\right]Q; \quad (0 \leq n \leq a-1) \tag{3}$$

$$Q_0^{-1} = F(r_v,\mu_v) + AF(r,\mu) + BF(r_v,\mu) + AS(r,\alpha) + BS(r_v,\alpha)$$

$$\text{where} \quad F(x,y) = \frac{1}{(1-x)}\left[1+\frac{y}{\lambda}\left(a - \frac{x(1-x^a)}{(1-x)}\right)\right]$$

$$\text{and} \quad S(x,y) = \frac{1}{(1-x)}\left[\frac{xy}{\lambda(x-1)+\beta x}\right]$$

$$L_q = AH(r,\mu) + BH(r_v,\mu) + H(r_v,\mu_v) + AJ(r,\alpha) + BJ(r_v,\alpha) \tag{4}$$

$$\text{where} \quad H(x,y) = \frac{x}{(1-x)^2} + \frac{y}{\lambda(1-x)}\left\{\frac{a(a-1)}{2} + \frac{ax^{a+1}(1-x) - x^2(1-x^a)}{(1-x)^2}\right\}$$

$$J(x,y) = \frac{x}{(1-x)^2}\left[\frac{xy}{\lambda(x-1)+\beta x}\right]$$

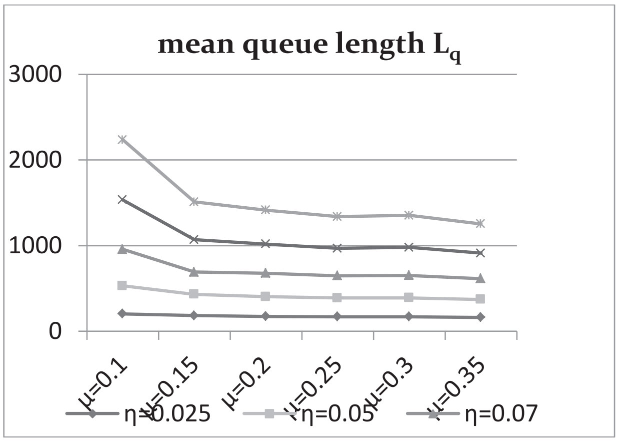
**Numerical example:** For M/M(a,b)/1/MWV with break down queueing system, we consider the parameters for arrival rate, service rate for busy, service rate for vacation, vacation parameter, break down and repair rate as,

$\mu_V = 0.05, \quad \lambda = 0.1, \quad \alpha = 0.05, \quad \beta = 0.25, \quad r = 0.8, \quad r_v = 0.9,$ the $a = 6 \quad and \quad b = 15,$

mean queue length ($L_q$) is calculated, by using equation(4) and the results are tabulated. The tabulated value is represented by graph.

**Table 1: mean queue length for M/M(a,b)/1/MWV/Br model**

| η / μ | 0.025 | 0.05 | 0.07 | 0.1 | 0.125 |
|---|---|---|---|---|---|
| 0.1 | 202.75 | 327.41 | 427.11 | 576.69 | 701.32 |
| 0.15 | 183.22 | 247.75 | 261.02 | 376.81 | 441.34 |
| 0.2 | 174.23 | 229.76 | 274.21 | 340.86 | 396.41 |
| 0.25 | 169.07 | 219.44 | 259.74 | 320.19 | 370.56 |
| 0.3 | 169.63 | 220.54 | 261.27 | 328.43 | 373.13 |
| 0.35 | 163.36 | 208.03 | 243.76 | 297.36 | 342.03 |




From the table and graph, we conclude that the mean queue length Lq increases when the vacation parameter increases. Also we find that the mean queue length Lq decreases when the service rate increases.

**Particular cases:** In this section Particular cases are analysed.

**Case(i):** M/M(a,b)/1 Multiple working vacation model.

In equations (2) and (3) by letting α=β=0 then the results of the Multiple working vacations (Julia rose mary (2011)) queueing model is deduced.
We get,

(i.e) $Q_n = r_v^n Q_0; \quad (n \geq 0)$ $P_n = (Ar^n + Br_v^n)Q_o; \quad (n \geq 0)$

where $A = \frac{(1-r)}{\mu(1-r^a)}\left[\frac{\eta}{(1-r_v)} - \frac{B\mu(1-r_v^a)}{(1-r_v)}\right]$

with $B = \frac{\eta r_v}{\mu r_v(1-r_v^b) + \lambda(r_v - 1)}$

$$R_n = \left[\frac{\mu}{\lambda}\left(\frac{A(1-r^{n+1})}{(1-r)} + \frac{B(1-r_v^{n+1})}{(1-r_v)}\right) + \frac{\mu_v}{\lambda}\left(\frac{1-r_v^{n+1}}{1-r_v}\right)\right]Q_0$$

**Case(ii):** M/M/1 Muliple Working vacation model.
The steady state queue size probabilities of M/M/1 Multiple working vacation (Liu et al.,(2007)) queueing model are deduced by letting a=b=1 with α=β=0 in the Queueing model M/M/(a,b)/1/MWV/Br.

$(i.e) \quad Q_n = r_v^n Q_0; \quad (n \geq 0)$ ,

$$P_n = \frac{B}{r_v}\left(r_v^{n+1} - r^{n+1}\right)Q_0; \quad (n \geq 0) \quad and \quad R_0 = \left(\frac{Q_0}{r_v}\right)$$

Where $r = \frac{\lambda}{\mu} = \rho; \; A = -\frac{B\rho}{r_v}$ and $B = \frac{\eta r_v}{\mu(1-r_v)(r_v - \rho)}$

Hence the queue size probabilities of M/M/1 Multiple working vacation queueing model of (Liu et al.,2007) is deduced.

**Case(iii):** M/M(a,b)/1 classical multiple vacations model.
The results of the classical multiple vacations (Afthab Begum 1996) Queueing model M/M(a,b)/1 can be derived by letting $\mu_v = 0,$ in the Multiple working vacations model. When $\mu_v$=0, $r_v = \frac{\lambda}{\lambda + \eta}$ and α=β=0, we get,

$Q_n = r_v^n Q_0; \quad (n \geq 0),$

$P_n = (Ar_n + Br_v^n)Q_0; \quad (n \geq 0)$

And $R_n = \frac{\mu}{\lambda}\left(Ah_n(r) + Bh_n(r_v)\right)Q_0$

*Where* $B = \frac{\eta}{\mu(1-r_v^b) - \eta}$ *and*

$$\frac{A(1-r^a)}{(1-r)} = \frac{\lambda+\eta}{\mu} - \frac{B(1-r_v^a)}{(1-r_v)}$$

Further $Q_0^{-1} = AF(r,\mu) + BF(r_v,\mu) + \frac{1}{(1-r_v)}$

Where $F(x,y) = \frac{1}{(1-x)}\left(1 + \frac{y}{\lambda}\left(a - \frac{x(1-x^a)}{(1-x)}\right)\right)$

And the mean queue length is given by,

$$Lq = AH(r,\mu) + BH(r_v,\mu) + \frac{r_v}{(1-r_v)^2}$$

*Where* $H(x,y) = \frac{x}{(1-x)^2} + \frac{y}{\lambda(1-x)}\left\{\frac{a(a-1)}{2} + \frac{ax^{a+1}(1-x) - x^2(1-x^a)}{(1-x)^2}\right\}$

Thus the classical M/M(a,b)/1 multiple vacations model is deduced.

**Conclusion:** In the present paper we approach the M/M(a,b)/1/MWV queueing model for unreliable server, by considering Chapman Kolmogrov equations. With the help of those equations and Steady state solutions, numerical example is given, to check the validity of the model. Further particular cases are also evaluated.

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