A NOTE ON VARIOUS TYPES OF CONTINUITY VIA J_p^{μ} – CLOSED SETS AND $J_p^{\mu *}$ -CLOSED SETS

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Abstract: In this paper we define the new closed sets J_p^{μ}- closed set and $J_p^{\mu *}$- closed set and their properties are analysed.Then we define J_p^{μ}- totally continuous maps and Its characterizations are investigated.

Keywords: $g^{\mu}b$ -closed set, $g^{\mu}br$ closed set, J_p^{μ}- closed set and $J_p^{\mu *}$- closed set.

Introduction: In 1983, Mashhour et al[3] investigated the notion of supra topological spaces and studied S-Scontinuous functions and S*-continuous functions. In 2010, O.R. Sayed and Takashi Noiri[4] introduced supra b-open sets and supra b-continuity. M. Trinita Pricilla and I.Arockiarani[2] derived a new class of generalised b-closed set called $g^{\mu}b$-closed set and $g^{\mu}br$-closed set. In 2013 M. Trinita Pricilla and I.Arockiarani[7] introduced and studied strong continuity called $g^{\mu}b$-totally continuity .

Preliminaries:

Definition 2.1:[2] Let (X ,μ) be a supra topological space.A subset A of X is called supra regular open if A $=int^{\mu}(cl^{\mu}$(A)) . The complement of supra regular open is supra regular closed.

Definition 2.2:[3] Let (X ,μ) be a supra topological space.A set A of (X , μ) is calledSupra generalized closed set if cl^{μ}(A) ⊆ U whenever A ⊆ U and U is supra open.

Definition 2.3:[3] A subset A of a supra topological space (X ,μ) is called supra b – open ifA $\subseteq int^{\mu}$ $(cl^{\mu}$(A)) $\cup cl^{\mu}(int^{\mu}$ (A)) and the complement of supra b – open set is supra b-closed.

Definition 2.4:[2] A set A of (X ,μ) is called supra generalized b–closed set (simply $g^{\mu}b$-closed) if bcl^{μ} (A) ⊆ U wheneverA⊆U and U is supra open. The complement of supra generalized b-closed set is supra generalized b -open set (simply $g^{\mu}b$ -open).

Definition 2.5:[9] A set A of (X ,μ) is called supra generalized b –regular closed set (simply $g^{\mu}br$ -closed) if bcl^{μ} (A) ⊆ U whenever A ⊆ U and U is supra regular open.The complement of supra generalized b- regular closed set is supra generalized b - regular open set (simply $g^{\mu}br$ -open).

J_p^{μ} - closed sets and $J_p^{\mu *}$ -closed sets:

Definition:3.1 A set A of a supra topological space (X,μ) is said to be

(i) J_p^{μ} -closed set if rcl^{μ} (A)⊆U whenever A⊆U and U is $g^{\mu}b$-open in X. (ii) $J_p^{\mu *}$-closed set if rcl^{μ} (A) ⊆U whenever A⊆U and U is $g^{\mu}br$ -open in X.

Theorem:3.2 Every $J_p^{\mu *}$ -closed set is J_p^{μ} -closed set.

Proof: Let A be $J_p^{\mu *}$ -closed set.

To prove: A is J_p^{μ} -closed set.Let A⊆U and U is $g^{\mu}b$ -open in X. But every $g^{\mu}b$ -open set is $g^{\mu}br$ -open set.Since A is $J_p^{\mu *}$ -closed set, rcl^{μ} (A)⊆U. Therefore rcl^{μ} (A)⊆U whenever A⊆U and U is $g^{\mu}b$ -open set.Therefore A is J_p^{μ} –closed.

Theorem:3.3 Every J_p^{μ} -closed set is

1) $g^{\mu}b$ -closed set 2) $g^{\mu}br$ closed set 3) T^{μ}-closed set 4) bT^{μ} -closed set 5) αg^{μ} -closed set 6) gr^{μ}- closed set 7) rg^{μ}- closed set.

Theorem:3.4 Every $J_p^{\mu *}$ -closed set is

1) $g^{\mu}b$ -closed set 2) $g^{\mu}br$ -closed set

3) T^{μ} closed set 4) bT^{μ} –closed set 5) αg^{μ} -closed set 6) gr^{μ} -closed set 7) rg^{μ}- closed set.

Remark:3.5 The converse of the above theorems are not true as shown by the following example.

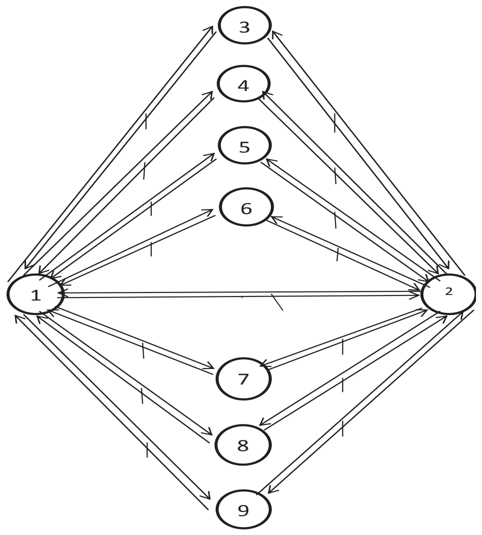
Let X={ a,b,c,d}; μ ={ϕ,X,{a},{b},{a,b},{c,d},{b,c,d},{a,b,d},{a,c,d}}.

Here $J_p^{\mu *}$-closed set ={ϕ,X,{a},{b},{a,b},{c,d},{b,c,d},{a,c,d}} and rg^{μ}-closed set={ϕ,X,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,d},{c,d},{a,b,c},{b,c,d},{a,c,d},{a,b,d}} . Thus {b,d} is rg^{μ} -closed set but not $J_p^{\mu *}$-closed.

Remark:3.6 From the above theorems and examples we have the following diagrammatic representation of J_p^{μ}-closed set and $J_p^{\mu *}$-closed set.The numbers 1 to 9 in the following diagram are represented by 1. J_p^{μ}-closed set 2. $J_p^{\mu *}$-closed set 3. $g^{\mu}b$-closed set 4. $g^{\mu}br$-closed set 5. αg^{μ}-closed set 6. rg^{μ}-closed set 7. gr^{μ}-closed set 8. bT^{μ}-closed set 9. T^{μ}closed set

Theorem :3.7 If A is J_p^{μ} - closed set and B is any set such that A ⊆ B $\subseteq rcl^{\mu}$(A) then B is J_p^{μ} –closed set.

Proof: Let B⊆U where U is $g^{\mu}b$- open.Then A⊆B ,implies A⊆U. Since A is J_p^{μ} - closed and A⊆U then rcl^{μ}(A)⊆U. B$\subseteq rcl^{\mu}$(A) implies rcl^{μ}(B)$\subseteq rcl^{\mu}$(A).Thus rcl^{μ}(B)⊆U. Hence B is J_p^{μ} – closed set.



Theorem :3.8 Arbitrary union of J_p^μ - closed set is always J_p^μ -closed set.

Proof: Let A and B be J_p^μ closed sets. Thenrcl^μ(A)∪rcl^μ(B)⊆G. Thisimplies thatrcl^μ(A∪B)⊆G .Therefore A∪B isJ_p^μ- closed.

Theorem:3.9 Let A be J_p^μ -closed subset of (X,μ) then $rcl^\mu(A)$ −A does not contain any non empty $g^\mu b$ -closed set.

Proof: Let F be $g^\mu b$ -closed set such that F⊆$rcl^\mu(A) - A$. Then F⊆rcl^μ(A) and F⊆X – A. This implies that A⊆X – F.Since A isJ_p^μ -closed and X–F is $g^\mu b$ -open. Therefore$rcl^\mu(A)$⊆X–F.That is F⊆X−rcl^μ(A).

Hence,F⊆ rcl^μ(A) ∩ (X−rcl^μ(A))=ϕ . This shows F = ϕ.

Theorem:3.10 Let A beJ_p^μ - closed set in (X,μ).Then A is supra regular closed iffrcl^μ(A)–A is $g^\mu b$-closed.

Proof : Let A be J_p^μ -closed. If A is supra regular closed. We havercl^μ(A) – A = ϕ which is $g^\mu b$ -closed set.

Conversely, letrcl^μ(A) – A be $g^\mu b$- closed. Then by theorem 3.9 rcl^μ (A) – A does not contain any non empty $g^\mu b$ - closed subset and so $rcl^\mu(A) - A$ is $g^\mu b$ - closed subset of itself then $rcl^\mu(A)$ −A = ϕ. This implies that A = rcl^μ(A).Therefore A is supra regular closed set.

On J_p^μ– totally continuous functions in supra topology

Definition:4.1 A function f : X → Y is said to be J_p^μ -continuous function if $f^{-1}(V)$ is J_p^μ -closed in X for every supra closed set V of Y.**Definition:4.2** A function f : X → Y is said to be J_p^μ -irresolute function if $f^{-1}(V)$ is J_p^μ -closed in X for every J_p^μ -closed set V of Y

Definition: 4.3 A function f : X → Y is said to be J_p^μ -totally continuous function if the inverse image of every J_p^μ -open subset of Y is $Cl^\mu open^\mu$ in X.**Theorem: 4.4** A bijective function f:X→ Y is J_p^μ -totally continuous if and only if the inverse image of every J_p^μ -closed subset of Y is $Cl^\mu open^\mu$ in X.

Proof: Let F be any J_p^μ -closed set in Y.

Then Y – F isJ_p^μ -open set in Y.By definition f^{-1} $(Y - F)$ is $Cl^\mu open^\mu$ in X. That is $X - f^{-1}(F)$ is $Cl^\mu open^\mu$ in X. This implies $f^{-1}(F)$ is $Cl^\mu open^\mu$ in X.

Conversely, if V is J_p^μ -open in Y, then Y – V is J_p^μ -closed in Y. By assumption, f^{-1} $(Y$ – V) = $X - f^{-1}(V)$ is $Cl^\mu open^\mu$ in X, Which implies $f^{-1}(V)$ is $Cl^\mu open^\mu$ in X. Therefore$f$ is J_p^μ -totally continuous function.

Theorem: 4.5(i) Every J_p^μ -totally continuous function is $totally^\mu$ continuous function.

(ii) Every J_p^μ -totally continuous function isJ_p^μ– continuous.

Proof: It is obvious.

Theorem: 4.6 (i)If f: X→Y is J_p^μ– totally continuous and g: Y→Z is J_p^μ -irresolute, then $g∘f$: X→ Z is J_p^μ -totally continuous.(ii) If : X→Y is J_p^μ -totally continuous and$g$:Y→Z isJ_p^μ-continuous, then$g∘f$: X→Z is $totally^\mu$continuous.

Proof: (i) Let V be a J_p^μ -open set in Z. Since g is J_p^μ–irresolute $g^{-1}(V)$ is J_p^μ -open set in Y.Since f is J_p^μ -totally continuous $f^{-1}(g^{-1}(V))$ is $Cl^\mu open^\mu$ in X. Therefore g ∘ f is J_p^μ -totally continuous.

(ii) Proof is Similar to (i).

Theorem: 4.7 The composition of twoJ_p^μ -totally continuous function is J_p^μ –totally continuous.

Proof: It is obvious

Theorem: 4.8 Let f:X→Y be J_p^μ -closed map and g:Y→Z be any function. If $g∘f$: X → Z is J_p^μ -totally continuous then g is J_p^μ -irresolute

Proof: It is obvious.

Definition: 4.9 A supra topological space (X,τ) is said to be J_p^μ -space if every J_p^μ -closed set of X is supra closed in X. **Definition: 4.10**A supra topological space X is said to

(i) J_p^μ–T_0 if for each pair of distinct points in X, there exists a J_p^μ -open set containing one point but not the other. (ii) J_p^μ– T_1 if for each pair of distinct points x and y of X , there exist J_p^μ-open sets U and V containing x and y respectively such that x∈U , y∉U and x∉ V, y∈V.

(iii) J_p^μ– T_2 if every two distinct points of X can be separated by disjoint J_p^μ -open sets. (iv) *J_p^μ-normal if each pair of non-empty disjoint supra closed sets can be separated by disjoint J_p^μ -open sets. (v) *J_p^μ -regular if for each supra closed set F of X and each $x \notin F$, there exist disjoint J_p^μ -open sets U and V such that $F \subset U$ and $x \in V$. (vi) J_p^μ -normal if for each pair of disjoint J_p^μ -closed sets U and V of X, there exist J_p^μ -open sets U and V such that $F \subset U$ and $x \in V$. (vii) J_p^μ -connected if X is not the union of two non-empty disjoint J_p^μ -open subsets of X.

Theorem: 4.11(i) Every J_p^μ -regular is *J_p^μ -regular space

(ii) Every J_p^μ -normal is *J_p^μ -normal space

Proof: (i) Let X be J_p^μ -regular space by definition, if for each J_p^μ -closed set F.There exist disjoint J_p^μ -open sets U and V such that F ⊂ U and x ∈ V.Since every J_p^μ -closed set is supra closed set.Therefore X is *J_p^μ -regular space.

(ii) Proof is similar to (i).

Theorem: 4.12 If $f: X \to Y$ is J_p^μ -totally continuous injection and Y is J_p^μ–T_1, then X is $Cl^\mu open^\mu - T_1$.**Proof:** Let x and y be any two distinct points in X. Since f is injective we have $f(x)$ and $f(y)$ $\in Y$ such that $f(x) \neq f(y)$. Since Y is J_p^μ–T_1, there exist J_p^μ–open sets U and V in Y such that $f(x) \in U$, $f(y) \notin U$, $f(y) \in V$ and $f(x) \notin V$. Therefore we have $x \in f^{-1}(U)$, $y \notin f^{-1}(U)$, $y \in f^{-1}(V)$ and $x \notin f^{-1}(V)$, where $f^{-1}(U)$ and $f^{-1}(V)$ are $Cl^\mu open^\mu$ subsets of X because f is J_p^μ -totally continuous. This shows that X is $Cl^\mu open^\mu - T_1$.

Theorem: 4.13 (i) If $f: X \to Y$ is J_p^μ -totally continuous injection and Y is $J_p^\mu - T_0$, then X is $ultra^\mu$ Hausdorff.

(ii) If $f: X \to Y$ is J_p^μ -totally continuous injection and Y is $J_p^\mu - T_2$, then X is $ultra^\mu$ Hausdorff.

(iii)If $f: X \to Y$ is J_p^μ -totally continuous, supra closed injection and Y is J_p^μ - normal, then X is $ultra^\mu$ normal.

Proof:

(i)Let a and b be any pair of distinct points of X and f be injective.

Then $f(a) \neq f(b)$ in Y. Since Y is $J_p^\mu - T_0$, there exists a J_p^μ -open set containing $f(a)$ but not $f(b)$. Then we have $a \in f^{-1}(U)$ and $b \notin f^{-1}(U)$.

Since f is J_p^μ–totally continuous $f^{-1}(U)$ is $Cl^\mu open^\mu$ in X.

Also $a \in f^{-1}(U)$ and $b \in X - f^{-1}(U)$.

This implies every pair of distinct points of X can be separated by disjoint $Cl^\mu open^\mu$ sets in X.

Therefore X is $ultra^\mu$ Hausdorff.

$ultra^\mu$ Hausdorff.

(ii)Let $x_1, x_2 \in X$ and $x_1, \neq x_2$. Since f is injective, $f(x_1) \neq f(x_2)$ in Y.

Further, since Y is $J_p^\mu - T_2$ there exist V_1 and $V_2 \in J_p^\mu O(Y)$ such that $f(x_1) \in V_1$, $f(x_2) \in V_2$, and $V_1 \cap V_2 = \phi$.

This implies $x_1 \in f^{-1}(V_1)$ and $x_2 \in f^{-1}(V_2)$.

Since f is J_p^μ–totally continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are $Cl^\mu open^\mu$ sets in X.

Also $f^{-1}(V_1) \cap f^{-1}(V_2) = f^{-1}(V_1 \cap V_2) = \phi$.

Thus every two distinct points of X can be separated by disjoint $Cl^\mu open^\mu$ sets.

Therefore X is $ultra^\mu$ Hausdorff.

(iii)Let F_1 and F_2 be disjoint supra closed subsets of X. Since f is supra closed and injective, $f(F_1)$ and $f(F_2)$ are disjoint supra closed subsets of Y. Since Y is J_p^μ -normal, $f(F_1)$ and $f(F_2)$ are separated by disjoint J_p^μ -open sets V_1 and V_2 separately. Therefore we obtain,

$F_1 \subset f^{-1}(V_1)$ and $F_2 \subset f^{-1}(V_2)$. Since f is J_p^μ–totally continuous, $f^{-1}(V_1)$ and $f^{-1}(V_1)$ are $Cl^\mu open^\mu$ sets in X.

Also $f^{-1}(V_1) \cap f^{-1}(V_2) = (V_1 \cap V_2) = \phi$.

Thus each pair of non-empty disjoint supra closed sets in X can be separated by disjoint $Cl^\mu open^\mu$ sets in X.

Therefore X is $ultra^\mu$ normal.

Theorem: 4.14 If $f: X \to Y$ is J_p^μ -totally continuous, surjection and X is supra connected then Y is J_p^μ -connected.

Proof: Suppose Y is not J_p^μ -connected. Let A and B form disconnection of Y. Then A and B are J_p^μ -open sets in Y and $Y = A \cup B$ where $A \cap B = \phi$. Also $X = f^{-1}(Y) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are non-empty $Cl^\mu open^\mu$ sets in X, because f is J_p^μ -totally continuous. Further $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = \phi$.

This implies X is not supra connected, which is a contradiction. Hence Y is J_p^μ -connected.

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