A SIMULATION STUDY ON M/M/1 AND M/M/C QUEUEING MODELS IN A MEDICAL CENTRE

P. UMA RANI, S. SHANMUGASUNDARAM

Abstract: In this paper, the application of Simulation in M/M/1 and M/M/C Queueing models in a Medical centre is analyzed. Chi-Square test is used for fitting the arrival and service time distribution. In the Simulation table,the queue time, system time, queue length and system length are studied. Also this paper focuses on studying both single and multi-server queues. Numerical examples illustrate the coincidence of Monte-Carlo Simulation with Analytical method.

Keywords: Inter-arrival time, Service time, Waiting time, Queue time, Length of system, Length of queue andIdle time.

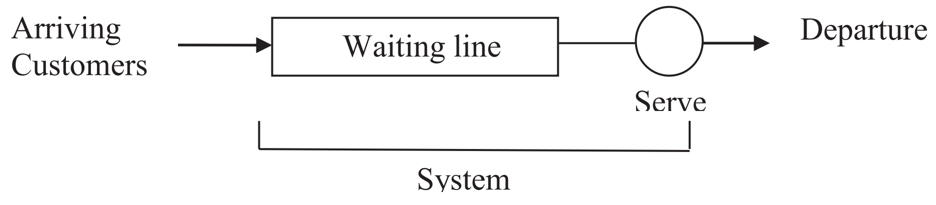
Introduction: Queueing Theory is the mathematical study of waiting position or queues. The theory enables mathematical analysis of several related processes waiting in the queues which is a common phenomenon of life. A group of items waiting to receive service is known as queue. Queues are in every daily life. These are formed to get service at that time when demand for a service is more than the capacity of service facility. In queues, customers arrive at a greater rate than service facility.A.K.Erlang (1878-1929) Danish Engineer is called the Father of Queuing Theory. He published his papers relating to the study of overcrowding in telephone traffic[1].

An analysis of a given queueing system involves the study of its different operating qualities. Some of the more commonly considered characteristics are discussed here.Length of queue:The average number of customer in the queue waiting to get service. Large queues may indicate pitiable server performance while small queues may imply too much server capacity.Length of system:The average number of customers in the system, those waiting to be and those being serviced. Large values of this statistic imply congestion and possible customer dissatisfaction and a potential need for greater service capacity.Waiting time in the queue:The average time that a customer has to wait in the queue to get service. Long waiting times are directly related to customer's dissatisfaction and potential loss of future revenues while very small waiting times may indicate too much service capacity.Total time in the system:The average time that a customer spends in the system from entry in the queue to completion of service.Server idle time :The relative frequency with which the service system is idle. However, reducing idle time may have adverse effects on the other characteristics mentioned above.

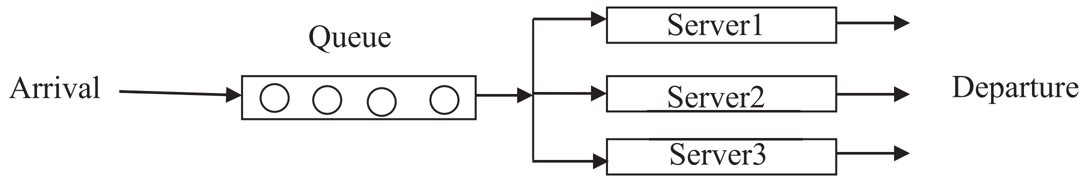
Elements of the queueing system may also be termed as the elements of the queueing theory.Input process : The arrivals can be from population that are infinite or finite.For example, the number of people coming to a medical centre is an example from infinite populations.The crew responsible for maintaining the ten machines of a factory is an example of finite population as here the number of machines to be maintained is limited.Each machine is regarded as a customer here. The customers may arrive for service individually or in groups. The customers may arrive in the system at known times, or they might arrive in a random way. A substantial majority of the queueing models are based on the premise that the customers enter the system stochastically, at random points in time.

Service system:

(i)A single server facility:A single server model involves a single server and single line of customers[4].



(ii)Multiple parallel facilities with single queue: There is more than one server with single queue.



Queue Discipline: This is the method by which customers are selected for service when a queue has created. The most common discipline is FCFS – First Come, First Served. The other disciplines are LCFS – Last Come First Served and SIRO –Service In Random Order and including Priority.

Customer behavior in queue:(i)Balking: A customer unlike to wait in a queue due to lack of time or space or otherwise.(ii)Reneging: A customer may leave the queue due to impatience.(iii)Collusion: Some customer may collaborate and only one of them join the queue as at the cinema ticket window one person may join the queue and purchase tickets for his friends.(iv)Jockeying: If there are excess of queues then one customer may leave one queue and move the other. This occurs generally in the supermarket[6].

There are number of operations research tools and techniques for solving various types of managerial decision making problems. Techniques like linear programming, dynamic programming, queueing theory, network models etc., are not sufficient to tackle all the important managerial problems requiring data analysis. Each technique has its own limitations. When the characteristics such as uncertainty, complexity, dynamic iteration between the discussion and subsequent event, and the need to develop detailed procedures and finely divided times intervals combine together in one situation, it becomes too complex to be solved by any of the techniques of mathematical programming and probabilistic models. It must be analysed by some other kind of quantitative technique which may give quite accurate and reliable results. Many new techniques are coming up, but, so far, the best available is Simulation. Simulation is used in the analysis of queueing models[5].

Simulation is a method of solving decision-making problems by designing, constructing and manipulating a model of the real system. It is defined to be the action of performing experiments on a model of a given system. It duplicates the essence of a system or activity without actually obtaining the reality. The basic steps in Simulation are step 1: Formulate the variables that influence the situation and an exact or probabilistic description of their possible values or states. step 2: Obtain a consistent set of values (or states) for the variables. In the case of deterministic variables, this is simple and in the case of probabilistic variables, random sampling technique may be used. Step 3: Use the sample obtained in step 2 to calculate the value of the decision criterion, by actually following the relationships among the variables for each of the alternative decisions. Step 4: Repeat steps 2 and 3 until a sufficient number of samples are available. Step 5: Tabulate the various values of the decision criterion and choose the best policy.

The Simulation models can be classified into following four categories: (i) Simulation of Deterministic Models. In the case of these models, the input and output variables are not permitted to be random variables and models are described by exact functional relationships. (ii) Simulation of Probabilistic Models. In such cases, method of random sampling is used. The technique used for solving these models is termed as “Monte – Carlo Technique”. (iii) Simulation of Static Models. These models do not take variable time into consideration. (iv) Simulation of Dynamic Models. These models deal with time varying interaction.

The Monte Carlo method of simulation was developed by the two mathematicians John von Neumann and Stainslaw Ulam, during world travel through different materials. The technique provided an approximate but quite workable solution to the problem. With the remarkable success of this technique on neutron problem, it soon became popular and found many applications in business and industry and at present forms a very important tool of operation researcher’s tool kit. The technique employs random numbers and is used to solve problems that involve probability and where in physical experimentation is impracticable and formulation of mathematical model is impossible. It is a method of simulation by sampling technique. Though in any real problem to be solved by simulation, the variables in the problem are probabilistic in nature. Simulation has also been defined as “the use of a system model that has the designed characteristics of reality in order to produce the essence of actual operation”[2].

Goodness-of-fit tests provide helpful guidance for evaluating the suitability of a potential input model. It is especially important to understand the effect of sample size. One procedure for testing the hypothesis that a random sample of size n of the random variable X follows a specific distributional form is the

Chi-square Goodness-of-fit test. This test formalizes the intuitive idea of comparing the histogram of the data to the shape of the candidate density or mass function. This test is valid for large sample sizes and for both discrete and continuous distributional assumptions when parameters are estimated by maximum likelihood. The test procedure begins by assuming the n observations into a set of $k$class intervals or cells. The test statistic is given by $\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$ where O_i is the observed frequency in the i^{th} class interval and E_i is the expected frequency in that class interval. The expected frequency for each class interval is computed as $E_i = np_i$ where p_i is the theoretical, hypothesized probability associated with the i^{th} class interval. It can be shown that χ^2 approximately follows the Chi-square distribution with k-s-1 degrees of freedom, where s represents the number of parameters of the hypothesized distribution estimated by the sample statistics. The hypotheses are H_o: The random variable X, conforms to the distributional assumption with the parameter given by the parameter estimates and H_1 :The random variable Xdoes not conform[9].

Queueing theory has been applied in many sectors like supermarkets, banks, post offices, library, ATM centres, cinema theatres, railway stations etc.,This paper discusses the applications of queueing theory at a medical centre.It describes the patients arrival as a random phenomenon and the time between the arrivals varying from 6 a.m to 12 p.m and the service time varying from four minutes to thirty two minutes. The frequency distributions are given below.

Table 1: Arrival Distribution:

S.No	Time	No of Patients	Probability
1	0-6	0	0
2	6-7	4	0.02
3	7-8	5	0.02
4	8-9	10	0.04
5	9-10	12	0.05
6	10-11	16	0.07
7	11-12	18	0.07
8	12-13	14	0.06
9	13-14	29	0.12
10	14-15	34	0.14
11	15-16	13	0.05
12	16-17	16	0.07
13	17-18	12	0.05
14	18-19	18	0.07
15	19-20	17	0.07
16	20-21	13	0.05
17	21-22	7	0.03
18	22-23	5	0.02
19	23-24	0	0
Total	-	243	

Table 2: Tag Number Table For Arrival Distribution

S.No	Time	No of Patients	Probability	Cummulative Probability	Tag numbers
1	0-6	0	0	0	0
2	6-7	4	0.02	0.02	0-1
3	7-8	5	0.02	0.04	2-3
4	8-9	10	0.04	0.08	4-7
5	9-10	12	0.05	0.13	8-12
6	10-11	16	0.07	0.20	13-19
7	11-12	18	0.07	0.27	20-26
8	12-13	14	0.06	0.33	27-32
9	13-14	29	0.12	0.45	33-44
10	14-15	34	0.14	0.59	45-58
11	15-16	13	0.05	0.64	59-63
12	16-17	16	0.07	0.71	64-70
13	17-18	12	0.05	0.76	71-75
14	18-19	18	0.07	0.83	76-82
15	19-20	17	0.07	0.90	83-89
16	20-21	13	0.05	0.95	90-94
17	21-22	7	0.03	0.98	95-97
18	22-23	5	0.02	1	98-100
19	23-24	0	0	0	0
Total	-	243			

Table 3: Service Distribution:

S.No	Time	No of Patients	Probability
1	0 - 4	5	0.08
2	4 - 8	7	0.11
3	8 - 12	15	0.23
4	12 - 16	13	0.20
5	16 - 20	12	0.19
6	20 - 24	7	0.11
7	24 - 28	3	0.05
8	28 - 32	2	0.03
Total	-	64	1

Table 4: Tag Number Table for Service Distribution:

S.No	Time	No of Patients	Probability	Cummulative Probability	Tag numbers
1	0 - 4	5	0.08	0.08	0 -7
2	4 - 8	7	0.11	0.19	8-18
3	8 - 12	15	0.23	0.42	19-41
4	12 - 16	13	0.20	0.62	42-61
5	16 - 20	12	0.19	0.81	62-80
6	20 - 24	7	0.11	0.92	81-91
7	24 - 28	3	0.05	0.97	92-96
8	28 - 32	2	0.03	1	97-100
Total	-	64	1		-

Table 5 : Chi-Square Test For Arrival Distribution:

X	No of Patients f	fX	P(x)	E_i	χ^2
0	0	0	0	0	0
1	4	4	0.0009	0	0
2	5	10	0.004	0	0
3	10	30	0.21	6	1.5
4	12	48	0.0319	8	2
5	16	80	0.0561	13	0.6
6	18	108	0.0858	21	0.4
7	14	98	0.11	26	505
8	29	232	0.129	32	0.2
9	34	306	0.131	32	0.1
10	13	130	0.12	29	8.8
11	16	176	0.1	24	2.6
12	12	144	0.076	18	2.0
13	18	234	0.054	13	1.9
14	17	238	0.04	10	1.6
15	13	195	0.027	7	5.1
16	7	112	0.01	4	2.2
17	5	85	0.004	0	0
18	0	0	0.002	0	0
Total	243	2230	-	243	34.5

Table 6: Chi-Square Test for Service Distribution

X	No of Patients f	fX	P(x)	E_i	χ^2
1	5	5	0.2	13	4.923
2	7	14	0.154	10	0.9
3	15	45	0.2	13	0.307
4	13	52	0.10	7	5.143
5	12	60	0.10	7	3.571
6	7	42	0.10	7	0
7	3	21	0.05	4	0.25
8	2	16	0.0324	3	0.33
Total	64	255	-	64	15.424

Null Hypothesis H_0: The poisson distribution fits well into thearrival distribution at 1% level of significance.
Alternative hypothesis H_1 : The poisson distribution does not fit well into the arrival distribution.$\lambda = 9.176$; P(x) = $\frac{e^{-9.176} \quad (9.176)^x}{x!}$. Tabulated value of χ^2 for 18 degrees of freedom at 1% level of significance is 34.8.Since calculated χ^2<tabulated $\chi\ 2_{0.01}$, we accept H_0 and conclude that the poisson distribution is a good fit to the given data.
Null HypothesisH_0: The Exponential distribution fits well into the service distribution at 1% level of significance. Alternative hypothesis H_1 : The Exponential distribution does not fit well into the service distribution.$\bar{X} = 3.9$; λ = $\frac{1}{mean}$ = 0.26 ; P(x) = 0.26 $e^{-0.26x}$. Tabulated value of χ^2 for 7 degrees of freedom at 1% level of significance is 18.475. Since calculated χ^2< tabulated $\chi\ 2_{0.01}$, we accept H_0 and conclude that the exponential distribution is a good fit to the given data.

Table 7: Simulation Table for Single Server Model

S. No	Random number	Inter arrival time (min)	Arrival time	Random number	Service begins	Service Time (min)	Service ends	Waiting time in Queue (min)	Waiting time in Medical Centre (min)	Idle Time of Doctor (min)	Length of queue
1	81	-	6.00	13	6.00	4	6.04	0	4	0	-
2	28	7	6.07	16	6.07	4	6.11	0	4	3	-

3	12	4	6.11	12	6.11	8	6.19	0	8	0	-
4	57	9	6.19	10	6.19	4	6.23	0	4	0	-
5	99	17	6.36	88	6.36	20	6.56	0	20	13	-
6	45	9	6.45	64	6.56	16	7.12	11	27	0	1
7	49	9	6.54	9	7.12	4	7.16	18	22	0	1
8	3	2	6.56	20	7.16	8	7.24	20	28	0	1
9	64	11	7.07	11	7.24	4	7.28	17	21	0	1
10	93	15	7.22	15	7.28	4	7.32	6	10	0	1
11	92	15	7.37	72	7.37	16	7.53	0	16	5	-
12	97	16	7.53	35	7.53	8	8.01	0	8	0	-
13	7	3	7.56	24	8.01	8	8.09	5	13	0	1
14	84	14	8.10	35	8.10	8	8.18	0	8	1	-
15	40	8	8.18	8	8.18	4	8.22	0	4	0	-
16	90	15	8.33	59	8.33	12	8.45	0	12	11	-
17	66	11	8.44	65	8.45	16	9.01	1	17	0	1
18	95	15	8.59	55	9.01	12	9.13	2	14	0	1
19	8	4	9.03	25	9.13	8	9.21	10	18	0	1
20	70	11	9.14	46	9.21	12	9.33	7	19	0	1
21	99	17	9.31	65	9.33	16	9.49	2	18	0	1
22	64	11	9.42	41	9.49	8	9.57	7	15	0	1
23	65	11	9.53	36	9.57	8	10.05	4	12	0	1
24	54	9	10.02	35	10.05	8	10.13	3	11	0	1
25	80	13	10.15	16	10.15	4	10.19	0	4	2	-
26	26	6	10.21	33	10.21	8	10.29	0	8	2	-
27	85	14	10.35	26	10.35	8	10.43	0	8	6	-
28	27	7	10.42	8	10.43	4	10.47	1	5	0	1
29	76	13	10.55	34	10.55	8	11.03	0	8	9	-
30	27	7	11.02	44	11.03	12	11.15	1	13	0	1
31	82	13	11.15	9	11.15	4	11.19	0	4	0	-
32	99	17	11.32	46	11.32	12	11.44	0	12	13	-
33	24	6	11.38	9	11.44	4	11.48	6	10	0	1
34	41	8	11.46	18	11.48	4	11.52	2	6	0	1
35	55	9	11.55	23	11.55	8	12.03	0	8	3	-
36	17	5	12.00	96	12.03	24	12.27	3	27	0	1
37	98	17	12.17	62	12.27	16	12.43	10	26	0	1
38	74	12	12.29	76	12.43	16	12.59	14	30	0	1
39	91	15	12.44	82	12.59	20	1.19	15	35	0	1
40	38	8	12.52	70	1.19	16	1.35	27	43	0	1
41	34	8	1.00	39	1.35	8	1.43	35	43	0	1
42	84	14	1.14	16	1.43	4	1.47	29	33	0	1
43	91	15	1.29	17	1.47	4	1.51	18	22	0	1
44	68	11	1.40	8	1.51	8	1.59	11	19	0	1
45	95	16	1.56	63	1.59	16	2.15	3	19	0	1
46	40	8	2.04	85	2.15	20	2.35	11	31	0	1
47	35	8	2.12	9	2.35	4	2.39	23	27	0	1
48	90	15	2.27	31	2.39	8	2.47	12	20	0	1
49	75	12	2.39	17	2.47	4	2.51	8	12	0	1
50	92	15	2.44	59	2.51	12	3.03	7	19	0	1
-	-	535	-	-	-	476	-	349	817	78	33

Simulation Calculation:

Average Length of queue = 0.66

Number of customers in the medical centre = $L_s = 1$

Average Waiting Time of a Patient in queue = 6.98min

Average Waiting Time of a Patient in a Medical centre = 16.34min

Idle Time of a Doctor = 1.5min

Analytical Calculation :

Length of queue = $L_q = 1$

Number of customers in the medical centre = $L_s = 2$

Average Waiting Time of a Patient in queue = 24 min

Average Waiting Time of a Patient in a Medical Centre = 38 min

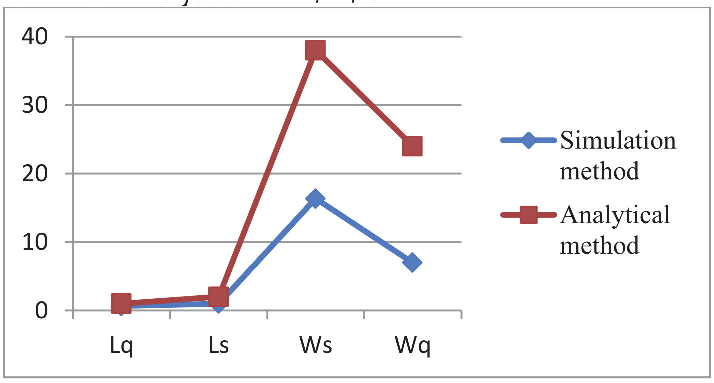
Idle Time of a Doctor = 0.366min

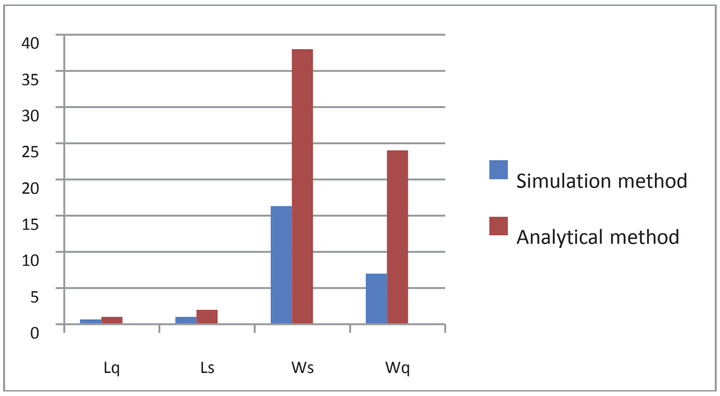
Table 8: Simulation Table for Multi - Server Model

S. No	Random number	Inter arrival time (min)	Random number	Service Time (min)	Arrival time (min)	SERVER 1		SERVER 2		Patient waiting time in queue (min)	Idle time		Length of queue
						Service begins	Service ends	Service begins	Service ends		Server 1 (min)	Server 2 (min)	
1	81	-	13	4	6.00	6.00	6.04	-	-	-	-	-	-
2	28	7	16	4	6.07	6.07	6.11	-	-	-	3	-	-
3	12	4	12	8	6.11	6.11	6.19	-	-	-	-	-	-
4	57	9	10	4	6.19	6.19	6.23	-	-	-	-	-	-
5	99	17	88	20	6.36	6.36	6.56	-	-	-	13	-	-
6	45	9	64	16	6.45	-	-	6.45	7.01	-	-	-	-
7	49	9	9	4	6.54	6.56	7.00	-	-	-	-	-	-
8	3	2	20	8	6.56	7.00	7.08	-	-	-	-	-	-
9	64	11	11	4	7.07	-	-	7.07	7.11	-	-	6	-
10	93	15	15	4	7.22	7.22	7.26	-	-	-	14	-	-
11	92	15	72	16	7.37	7.37	7.53	-	-	-	11	-	-
12	97	16	35	8	7.53	7.53	8.01	-	-	-	-	-	-
13	7	3	24	8	7.56	-	-	7.56	8.04	-	-	45	-
14	84	14	35	8	8.10	8.10	8.18	-	-	-	9	-	-
15	40	8	8	4	8.18	-	-	8.18	8.22	-	-	14	-
16	90	15	59	12	8.33	8.33	8.45	-	-	-	15	-	-
17	66	11	65	16	8.44	-	-	8.44	9.00	-	-	22	-
18	95	15	55	12	8.59	8.59	9.11	-	-	-	14	-	-
19	8	4	25	8	9.03	-	-	9.03	9.11	-	-	3	-
20	70	11	46	12	9.14	9.14	9.26	-	-	-	3	-	-
21	99	17	65	16	9.31	9.31	9.47	-	-	-	5	-	-
22	64	11	41	8	9.42	-	-	9.42	9.50	-	-	31	-
23	65	11	36	8	9.53	9.53	10.01	-	-	-	6	-	-
24	54	9	35	8	10.02	10.02	10.10	-	-	-	1	-	-
25	80	13	16	4	10.15	10.15	10.19	-	-	-	5	-	-
26	26	6	33	8	10.21	10.21	10.29	-	-	-	2	-	-
27	85	14	26	8	10.35	10.35	10.43	-	-	-	6	-	-
28	27	7	8	4	10.42	-	-	10.42	10.46	-	-	52	-
29	76	13	34	8	10.55	10.55	11.03	-	-	-	12	-	-
30	27	7	44	12	11.02	-	-	11.02	11.14	-	-	16	-
31	82	13	9	4	11.15	11.15	11.19	-	-	-	12	-	-
32	99	17	46	12	11.32	11.32	11.44	-	-	-	13	-	-
33	24	6	9	4	11.38	-	-	11.38	11.42	-	-	24	-
34	41	8	18	4	11.46	11.46	11.50	-	-	-	2	-	-
35	55	9	23	8	11.55	11.55	12.03	-	-	-	5	-	-
36	17	5	96	24	12.00	-	-	12.00	12.24	-	-	18	-
37	98	17	62	16	12.17	12.17	12.33	-	-	-	14	-	-
38	74	12	76	16	12.29	-	-	12.29	12.45	-	-	5	-
39	91	15	82	20	12.44	12.44	1.04	-	-	-	11	-	-
40	38	8	70	16	12.52	12.52	1.08	-	-	-	-	7	-
41	34	8	39	8	1.00	1.04	1.12	-	-	-	-	-	-
42	84	14	16	4	1.14	1.14	1.18	-	-	-	2	-	-
43	91	15	17	4	1.29	1.29	1.33	-	-	-	11	-	-
44	68	11	8	8	1.40	1.40	1.48	-	-	-	7	-	-
45	95	16	63	16	1.56	1.56	2.12	-	-	-	8	-	-
46	40	8	85	20	2.04	-	-	2.04	2.24	-	-	56	-
47	35	8	9	4	2.12	2.12	2.16	-	-	-	-	-	-
48	90	15	31	8	2.27	2.27	2.35	-	-	-	11	-	-
49	75	12	17	4	2.39	2.39	2.43	-	-	-	4	-	-
50	92	15	59	12	2.44	2.44	2.56	-	-	-	1	-	-
-	-	535	-	476	-	-	-	-	-	-	223	289	-

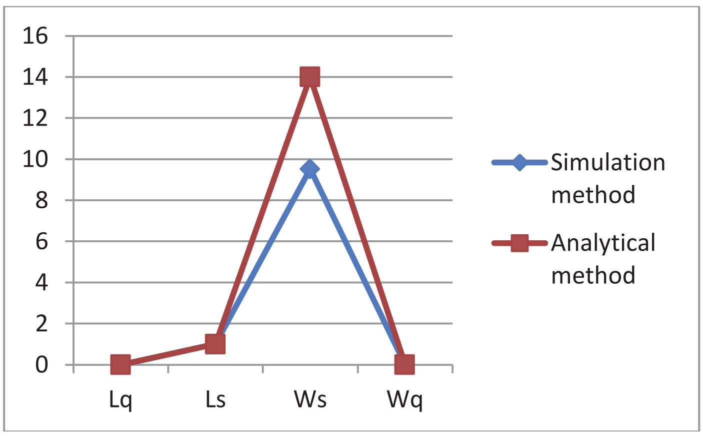
Simulation Calculation:
Average Waiting Time of a Patient in queue = 0min
Average Waiting Time of a Patient in a Medical centre = 9.52 min
Number of customers in the medical centre = L_s = 1
Average Length of queue = L_q= 0
Idle Time of a Doctor 1 = 4.46min
Idle Time of a Doctor 2 = 5.78min
Analytical Calculation :
Average Waiting Time of a Patient in queue = 0min
Average Waiting Time of a Patient in a medical centre = 13.9 min
Number of customers in the medical centre = L_s = 1
Length of queue = L_q= 0
Idle Time of Doctors = 0.366min

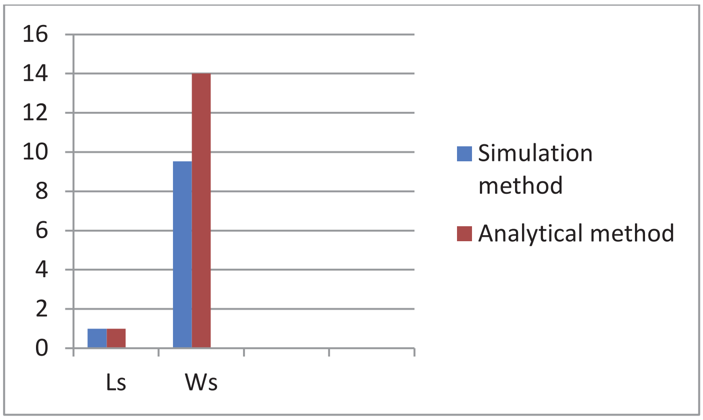
Numerical Study
Comparison of Simulation With Analytical In M/M/1:





Comparison Of Simulation With Analytical In M/M/C:





Conclusion: The Arrival followingPoisson distribution and Service followingExponential distribution have been verifiedby using Chi-Square test. The length of queue, arrival time, service time, idle time of a doctor, patient waiting time in the queue as well as in a medical centre have been discussed in both Simulation and Analytical methods in single and multi-server queueing models.The numerical examples show the feasibility of the system. The main objective of this paper is to develop an efficient procedure to reduce the waiting time of the patients in the Medical Centre in due course.

References:

1. Erlang,A.K (1909), "*The Theory of Probabilities of Telephone Conversations*", Nyt Jindsskriff Mathematic, B20, 33-39.
2. Saraswathi.A , A.Praveenprakash, Mary Binu Ashwini, A Study on Problems of Transgender in Tamil Nadu; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 3 Issue 1 (2014), Pg 365-370
3. Shanmugasundaram,S. and Punitha,S (2014), "*A Study on Multi Server Queueing Simulation*", IJSR – International Journal of Science and Research, ISSN: 2319– 7064,Volume 3, Issue 7.
4. Wijewickrama,A.K.A (2006), "*Simulation Analysis for Reducing Queues in Mixed Patients' Outpatient Department*", International Journal of Simulation Modelling, 5, 56 - 68.
5. Shanmugasundaram,S. and Banumathi.P (2015), "*A Study on Single Server Queue in Southern Railway using Monte Carlo Simulation*", Global Journal of Pure and Applied Mathematics, ISSN 0973 – 1768,Volume 11.
6. *Parvinder Singh,* integral Quaternions, Four-Square theorem and Number of Representations of An INTEGER As Sum of Four Squares; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Spl Issue (2014), Pg 931-940
7. Abdelaziz Nasroallah(2004),"*Monte Carlo Simulation of Markov Chain Steady-State Distribution*",Extracta Mathematicae, Volume 19,Num.2, p279-288.
8. Sokolowski,J.A., Banks,C.M(2009), "*Principals of Modelling and Simulation*", Hoboken, NJ: Wiley.p.6, ISBN 978–0-470-28943-3.
9. Kim,B.J(2011), "*Conceptualization of Traffic Flow for Designing, Toll Plaza Configuration: A Case Study using Simulation with Estimated Traffic*", Volume. 18(1): 51 – 57.
10. Sundarapandian,V(2009), "*7.Queueing Theory*", Probability, Statistics and Queueing Theory, PHI Learning, ISBN 8120338448.
11. C. Balasubramanyam, Ajay M.S, Amogh B. Shetty, K R Spandana, K N Seetharamu, Curve Fitting for Coarse and Fine Data Using Artificial; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Issue 2 (2014), Pg 519-537
12. Jerry Banks, John S.Carson, Barry L.Nelson and David M.Nicol(2007),"*Discrete-Event System Simulation*",Fourth Edition, Pearson Education.
13. Laguna,M and Marklund,J(2005), "*Business Process Modelling, Simulation and Design*", ISBN 0-13-091519-X, Pearson Prentice Hall.
14. Shanmugasundaram,S. and Umarani.P(2016), "*A Simulation Study on M/M/1 Queueing Model in a Medical Centre*", IJSR – International Journal of Science and Research, ISSN: 2319– 7064,Volume 5, Issue 3.

P. Umarani, Department of Mathematics, AVS Engineering College, Salem – 3, Tamil Nadu,
S. Shanmugasundaram , Department of Mathematics, Government Arts College, Salem – 7, Tamil Nadu.