ON Ω_{gb}^{+} AND $\mho_{gb}^{+}$ SETS IN SIMPLE EXTENSION IDEAL TOPOLOGICAL SPACES

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Abstract: This paper serves as a platform to discuss and bring out the basic properties of newly defined Ω_{gb}^{+} and $\mho_{gb}^{+}$ -sets, under the light of simple extension topological spaces.

Introduction: A new class of generalized open sets called b-open sets in topological spaces was defined by Andrijevic [2]. The class of all b open sets generates the same topology as the class of all pre-open sets. In 1986, Maki [9] introduced the concept of generalized Λ sets and defined the associated closure operators by using the work of Levine [7] and Dunhem [4]. Caldas and Dontchev [3] introduced Λ_s-sets, V_s-sets, $g\Lambda_s$-sets and gV_s-sets. Ganster and et al. [5] introduced the notion of pre Λ-sets and pre V-sets and obtained new topologies via these sets. M.E. Abd El-Monsef et al. [1] defined bΛ-sets and bV-sets on a topological space and proved that it forms a topology. In 1963 Levine [8] introduced the concept of a simple extension topology τ as $\tau(B)$ ={(B $\cap$ O) $\cup$ O$'$ /O,O$'$ $\in \tau$ and B $\notin \tau$ }. Sr. I. Arockiarani and F. Nirmala Irudayam [10] introduced the concept of b^{+}-open sets in extended topological spaces. S. Reena and F. Nirmala Irudayam [12] devised a new form of continuity and T. Noiri, Sr. I. Arockiarani and F. Nirmala Irudayam [11] coined the idea of Ω_{gb}^{+*}, $\mho_{gb}^{+*}$ sets in simple extended topological spaces.

Preliminaries: All through the paper the space X is a SEITS in which no separation axioms are assumed unless and otherwise stated.

Definition 2.1: A subset A of a topological space (X, τ) is said to be, (i) b-open set[2], if A $\subseteq$ cl(int(A)) $\cup$ int(cl(A)) and b-closed set cl(int(A)) $\cup$ int(cl(A)) $\subseteq$ A. (ii) a generalized closed (briefly g-closed) [6] if cl(A) $\subset$ U whenever A $\subset$ U and U is open.

Definition 2.2[13]: A subset A of (X, τ) is called πgb - closed if $bcl(A) \subset U$ whenever $A \subset U$ and U is π - open in (X,τ). By $\pi GBO(X,\tau)$ we mean the family of all πgb - closed subsets of the space (X,τ).

Definition 2.2[10]: A subset A of a topological space (X, τ^{+}) is said to be, (i) a b^{+}-open set If A $\subseteq$ cl^{+}(int(A)) $\cup$ int(cl^{+} (A)). (ii) a generalized b^{+}-closed (briefly gb^{+}-closed) if b^{+} cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open.

Definition 2.3[12]: A subset A of (X, τ^{+}) is called πgb^{+} -closed if $b^{+}cl(A) \subset U$ whenever $A \subset U$ and U is π^{+} -open in (X,τ^{+}). By $\pi GBO^{+}(X,\tau^{+})$ we mean the family of all πgb^{+} closed subsets of the space (X,τ^{+}).

Definition 2.3[11]: Let (X,τ^{+},I) be a simple extension ideal topological space (SEITS) and a subset of X. We defined Ω_{gb}^{+*} (A) and $\mho_{gb}^{+*}$ (A) as follows, Ω_{gb}^{+*}(A) $=\cap\{G : A \subseteq G, G \in BI^{+}O(X,\tau^{+},I)\}$

$\mho_{gb}^{+*}$(A) $=\cup\{F : F \subseteq A, F \in BI^{+}C(X,\tau^{+},I)\}$.

Ω_{gb}^{+}-sets and $\mho_{gb}^{+}$ -sets

Definition 3.1: Let S be a subset of a topological space (X, τ^{+}) we define the sets Ω_{gb}^{+} (S) and $\mho_{gb}^{+}$(S) as follows,

$\Omega_{gb}^{+}(S) = \cap\{G \mid G \in \pi GB^{+}O(X,\tau^{+})\, and\, S \subseteq G\}$

$\mho_{gb}^{+}(S) = \cup\{F \mid F \in \pi GB^{+}C(X,\tau^{+})\, and\, S \supseteq F\}$

Lemma 3.2: For subsets S, Q and $S_i, i \in I$ of a topological space (X,τ^{+}) the following properties hold

(1) $S \subseteq \Omega_{gb}^{+}(S)$

(2) $Q \subseteq S \Rightarrow \Omega_{gb}^{+}(Q) \subseteq \Omega_{gb}^{+}(S)$ (3) $\Omega_{gb}^{+}(\Omega_{gb}^{+}(S)) = \Omega_{gb}^{+}(S)$ (4) *If* $S \in \pi GB^{+}O(X,\tau^{+})$, then $S = \Omega_{gb}^{+}(S)$ (5) $\Omega_{gb}^{+}(\cup\{S_i : i \in I\}) = \cup\{\Omega_{gb}^{+}(S_i) : i \in I\}$ (6) $\Omega_{gb}^{+}(\cap\{S_i : i \in I\}) \subseteq \cap\{\Omega_{gb}^{+}(S_i) : i \in I\}$ (7) $\Omega_{gb}^{+}(S^{c}) = (\mho_{gb}^{+}(S))^{c}$ (8) $\Omega_{gb}^{+}(X-S) = X - \mho_{gb}^{+}(S)$

Proof: (1) Let $x \notin \Omega_{gb}^{+}(S)$,then there exists a πgb^{+} -open set G such that $S \subseteq G$ and $x \notin G$.

Hence $x \notin S$ and so $S \subseteq \Omega_{gb}^{+}(S)$

(2) Let $x \notin \Omega_{gb}^{+}(S)$,then there exists a πgb^{+} -open set G such that $S \subseteq G$ and $x \notin G$. By our assumption $Q \subseteq S$. Hence $Q \subseteq G$ and hence $x \notin \Omega_{gb}^{+}(Q)$

Hence $Q \subseteq S \Rightarrow \Omega_{gb}^{+}(Q) \subseteq \Omega_{gb}^{+}(S)$.

(3) From (1) and (2) using (1), $S \subseteq \Omega_{gb}^{+}(S) \Rightarrow \Omega_{gb}^{+}(S) \subseteq \Omega_{gb}^{+}(\Omega_{gb}^{+}(S))$..........(1)

If $x \notin \Omega_{gb}^{+}(S)$, then there exists a πgb^{+} -open set G such that S $\subseteq$ G and x $\notin$ G from the definition of $\Omega_{gb}^{+}(S)$, $\Omega_{gb}^{+}(S) \subseteq G$ and hence $x \notin \Omega_{gb}^{+}(\Omega_{gb}^{+}(S))$

Therefore $\Omega_{gb}^{+}(\Omega_{gb}^{+}(S)) \subseteq \Omega_{gb}^{+}(S)$...........(2) From (1) and (2) we get. $\Omega_{gb}^{+}(\Omega_{gb}^{+}(S)) = \Omega_{gb}^{+}(S)$

(4) From definition if $S \in \pi GB^+O(X, \tau^+)$, then $S \in \Omega_{gb}^+(S) \Rightarrow S \in S$.
Therefore $\Omega_{gb}^+(S) \subseteq S$ ....(i).
From (1) $S \subseteq \Omega_{gb}^+(S)$ .... (ii).
Hence from (i) and (ii), $S = \Omega_{gb}^+(S)$

(5) Let $S = \cup\{S_i : i \in I\}$.
By (2) we have, $\cup\{\Omega_{gb}^+(S_i) : i \in I\} \subseteq \Omega_{gb}^+(S)$.
If $x \notin \cup\{\Omega_{gb}^+(S_i) : i \in I\}$, then, for each $i \in I$, there exists $G_i \in \pi GB^+O(X, \tau^+)$, such that $S_i \subseteq G_i$ and $x \notin G_i$. If $G = \cup\{G_i : i \in I\}$ then $G \in \pi GB^+O(X, \tau^+)$ with $S \subseteq G$ and $x \notin G$.
Hence $x \notin \Omega_{gb}^+(S)$ and hence (5) holds.

(6) Follows from definition (3.1)

(7) Let $x \in \Omega_{gb}^+(S^c)$. Then for every πgb^+-open set G containing S^c, $x \in G$.
Hence $x \notin G^c$ for every πgb^+-closed set $G^c \subseteq S$.
Hence $x \notin \mho_{gb}^+(S)$
Hence $x \in (\mho_{gb}^+(S^c))$
Therefore $\Omega_{gb}^+(S^c) \subseteq (\mho_{gb}^+(S))^c$............(1).
Let $x \in (\mho_{gb}^+(S))^c \Rightarrow x \notin \mho_{gb}^+(S)$.
Then for every πgb^+-closed set $G^c \subseteq S$, $x \notin G^c$
$\Rightarrow x \in G$ for every πgb^+-open set G containing S^c.
$S^c \subseteq G$.
Hence $x \in \Omega_{gb}^+(S^c)$.
Therefore $\Omega_{gb}^+(S^c) \supseteq (\mho_{gb}^+(S))^c$ .....(2).
From (1) and (2), $\Omega_{gb}^+(S^c) = (\mho_{gb}^+(S))^c$

(8) Follows from definition (3.1)

Lemma 3.3: For subsets S, Q and $S_i, i \in I$ of a topological space (X, τ^+) the following properties hold

1. $\mho_{gb}^+(S) \subseteq S$
2. $Q \subseteq S \Rightarrow \mho_{gb}^+(Q) \subseteq \mho_{gb}^+(S)$
3. $\mho_{gb}^+(\mho_{gb}^+(S)) = \mho_{gb}^+(S)$

$If\ S \in \pi GB^+C(X, \tau^+)$, then $S = \mho_{gb}^+(S)$

4. $\mho_{gb}^+(\cap\{S_i : i \in I\}) = \cap\{\mho_{gb}^+(S_i) : i \in I\}$

$\cup\{\mho_{gb}^+(S_i) : i \in I\}) \subseteq \mho_{gb}^+(\cup\{(S_i) : i \in I\})$

Definition 3.4: A subset S of a space (X, τ^+) is called a (1) gb^+-Ω-set briefly Ω_{gb}^+-set if $S = \Omega_{gb}^+(S)$ (2) gb^+-$\mho$-set briefly $\mho_{gb}^+$-set if $S = \mho_{gb}^+(S)$ The set of all Ω_{gb}^+-sets (respectively $\mho_{gb}^+$-sets) is denoted by $\Omega_{gb}^+(X, \tau^+)$ (resp. $\mho_{gb}^+(X, \tau^+)$).

Remark 3.5: Clearly Ω- sets are gb^+-Ω sets and $\mho$-sets are gb^+-$\mho$-sets. Observe that a subset S is a gb^+-Ω- set if S^c is a gb^+-$\mho$-set. Also every gb^+-Ω-set is a πgb^+-open set.

Theorem 3.6: For a space (X, τ^+), the following statements hold (1) ϕ and X are Ω_{gb}^+-sets and $\mho_{gb}^+$-sets
(2) Every union of $\mho_{gb}^+$-sets(resp. Ω_{gb}^+-sets) is a $\mho_{gb}^+$-set(resp. Ω_{gb}^+-sets) (3) Every intersection of Ω_{gb}^+-sets (resp. $\mho_{gb}^+$ sets) is a Ω_{gb}^+-set(resp. $\mho_{gb}^+$ sets)

Proof:(1) It is obvious.
(2) Let $\{S_i \mid i \in I\}$ be a family of $\mho_{gb}^+$-sets in (X, τ^+).
Then $S_i = \mho_{gb}^+(S_i)$ for each $i \in I$.
Let $S = \bigcup_{i \in I} S_i$. Then
$\mho_{gb}^+(S) = \mho_{gb}^+(\bigcup_{i \in I} S_i) \supseteq \bigcup_{i \in I} \mho_{gb}^+(S_i) = \bigcup_{i \in I} S_i = S$ Also $\mho_{gb}^+(S) \subseteq S$.
Hence S is a $\mho_{gb}^+$-set.
(3) By using
$\Omega_{gb}^+(\bigcap_{i \in I} S_i) \subseteq \bigcap_{i \in I} \Omega_{gb}^+(S_i) \subseteq \bigcap_{i \in I} S_i = S$.
Also $S \subseteq \Omega_{gb}^+(S)$.
Hence S is a Ω_{gb}^+-set.

Definition3.7: Let (X, τ^+) be a topological space then the πgb^+-closure of A denoted by $\pi gb^+ cl(A)$ is defined by
$\pi gb^+\text{-}cl(A) = \cap\{F \mid F \in \pi GB^+C(X, \tau^+) \ \&F \supset A\}$

Lemma 3.8: Let (X, τ^+) be a topological space and $x \in X$. Then $y \in \Omega_{gb}^+(\{x\})$ iff $x \in \pi gb^+\text{-}cl(\{y\})$.

Proof: Suppose $y \in \Omega_{gb}^+(\{x\})$. Then for every πgb^+-open set $G \supseteq \{x\}$, $y \in G$. If $x \notin \pi gb^+\text{-}cl(\{y\})$,
then $\exists$ $H \in \pi GB^+C(X, \tau^+) \ni \{y\} \subset H$ and $x \notin H$. This implies $x \in X-H$,
$X-H \in \pi GB^+O(X, \tau^+)$ and $y \notin X-H$.
Take X-H =G. Then $G \in \pi GB^+O(X, \tau^+)$, $\{x\} \subseteq G$ and $y \notin G$ which is a contradiction. Hence $x \in \pi gb^+ cl(\{y\})$.
Conversely, suppose $x \in \pi gb^+\text{-}cl(\{y\})$ then for every πgb^+-closed set $G \supset \{y\}$, $x \in G$.
If $y \notin \Omega_{gb}^+(\{x\})$ then there exists $H \in \pi GB^+O(X, \tau^+)$ such that $\{x\} \subseteq H$ and $y \notin H$.
Take X-H =G.
Then $G \in \pi GB^+C(X, \tau^+)$, $y \in G$ and $x \notin G$.
So $\exists$ a πgb^+-closed set $G \supset \{y\} \ni x \notin G$.
By this contradiction, we get $y \in \Omega_{gb}^+(\{x\})$.

Theorem 3.9: The following statements are equivalent for any points x and y in a topological space (X, τ^+).

(1) $\Omega_{gb}^+(\{x\}) \neq \Omega_{gb}^+(\{y\})$

(2) πgb^+-cl $(\{x\}) \neq \pi gb^+$-cl$(\{y\})$

Proof: (1) ⇒ (2)

Suppose $\Omega_{gb}^+(\{x\}) \neq \Omega_{gb}^+(\{y\})$.

Then $\exists$ $z \in X \ni z \in (\{x\})$ and $z \in \Omega_{gb}^+(\{y\})$. Therefore $x \in \pi gb^+$-cl $(\{z\})$ and $y \in \pi gb^+$-cl$(\{z\}$. Hence $\{x\} \cap \pi gb^+$-cl $(\{z\}) \neq \phi$ and $\{y\} \cap \pi gb^+$-cl$(\{z\}) \neq \phi$.

Since $x \in \pi gb^+$-cl$(\{z\})$, πgb^+-cl$\{x\} \subset \pi gb^+$-cl$(\{z\})$ and hence $\{y\} \cap \pi gb^+$-cl $(\{x\}) \neq \phi$.

Thus πgb^+-cl $(\{x\}) \neq \pi gb^+$- cl$(\{y\})$.

(2)⇒ (1). Suppose $\Omega_{gb}^+(\{x\}) \neq \Omega_{gb}^+(\{y\})$,then $\exists$ $z \in \pi gb^+$-cl $(\{x\})$ and $z \in \pi gb^+$cl$(\{y\})$. Therefore $x \in \Omega_{gb}^+(\{x\})$ and $y \notin \Omega_{gb}^+(\{z\})$.

So $\exists$ a πgb^+-open set $G \supset \{z\}$ such that $x \in G$ and $y \notin G$.Hence $y \notin \Omega_{gb}^+(\{x\})$.

Hence $\Omega_{gb}^+(\{x\}) \neq \Omega_{gb}^+(\{y\})$. **Lemma 3.10:** Let (X,τ^+) be a topological space and $A \in \pi GB^+O(X,\tau^+)$. Then $\Omega_{gb}^+(A)=\{x \in X \mid \pi gb^+\text{-cl}(\{x\}) \cap A \neq \phi\}$.

Proof: Let $x \in \pi GB^+O(X,\tau^+)$, $A= \Omega_{gb}^+(A)$.

Also $x \in \pi gb^+$-cl$(\{x\})$.

Hence πgb^+-cl$\{x\} \cap A \neq \phi$. Conversely,

let $x \in X$ such that πgb^+-cl$(\{x\}) \cap A \neq \phi$.

If $x \notin \Omega_{gb}^+(A)$,then $\exists$ $V \in \pi GB^+O(X,\tau^+)$ such that $A \subseteq V$ and $x \notin V$. Let $y \in \pi gb^+$-cl$(\{x\}) \cap A$. Since $y \in \Omega_{gb}^+(\{y\})$.

Therefore for every πgb^+-open set $G \supseteq \{y\}$ in (X,τ^+), $x \in G$. Since $y \in A$ and $A \subseteq V$, $y \in V$ where $V \in \pi GB^+O(X,\tau^+)$.Hence $x \in V$.

By this contradiction, we get $x \in \Omega_{gb}^+(A)$.

Ω_{gb}^+ - Closed Sets And Its Properties

Definition 4.1: (1) Let A be a subset of a space (X, τ^+). Then A is called a Ω_{gb}^+-closed set if $A = S \cap C$ where S is Ω_{gb}^+ -set and C is a closed set. (2) The complement of a Ω_{gb}^+ -closed set is called a Ω_{gb}^+-open set. (3) The collection of all Ω_{gb}^+-open sets in (X,τ^+) is denoted by $\Omega_{gb}^+ O(X, \tau^+)$.The collection of all Ω_{gb}^+-closed sets in (x, τ^+) is denoted by $\Omega_{gb}^+ C(X, \tau^+)$. (4) A point $x \in X$ is called Ω_{gb}^+-cluster point of A if for every Ω_{gb}^+-open set U containing x, $A \cap U \neq \phi$. (5) The set of all Ω_{gb}^+-cluster points of A is called the Ω_{gb}^+-closure of A and is denoted by Ω_{gb}^+-cl(A). Let (X, τ^+) be a topological space and A, B and A_k where $K \in I$, subsets of X. Then we have the following properties.

Proposition4.2: $A \subset \Omega_{gb}^+$-cl(A).

Proof: Let $x \notin \Omega_{gb}^+$-cl(A) .Then x is not a Ω_{gb}^+-cluster point of A. So there exists a Ω_{gb}^+ -open set U containing x such that $A \cap U = \phi$ and hence $x \notin A$.

Proposition4.3: Ω_{gb}^+-cl(A) = $\cap\{F/A \subset F$ and F is Ω_{gb}^+-closed$\}$

Proof: Let $x \notin \Omega_{gb}^+$- cl(A).

Then there exists a Ω_{gb}^+-open set U containing x such that $A \cap U = \phi$

Take $F=U^c$.

Then F is Ω_{gb}^+ -closed, $A \subset F$ and $x \notin F$ and hence $x \notin \cap\{F/A \subset F$ and F is Ω_{gb}^+-closed$\}$.Similarly Ω_{gb}^+ - cl(A) $\subset$ $\{F/A \subset F$ and F is Ω_{gb}^+ -closed$\}$.

Proposition4.4: If $A \subset B$, then Ω_{gb}^+-cl(A) $\subset \Omega_{gb}^+$-cl(B)

Proof: Let $x \notin \Omega_{gb}^+$-cl(B).

Then there exists Ω_{gb}^+a-open set U containing x such that $B \cap U =\phi$.

Since $A \subset B$, $A \cap U =\phi$.

Hence x is not a Ω_{gb}^+ - cluster point of A.

Therefore $x \notin \Omega_{gb}^+$-cl(A)

Proposition4.5: A is Ω_{gb}^+ -closed iff $A= \Omega_{gb}^+$-cl(A).

Proof: Suppose A is Ω_{gb}^+-closed.

Let $x \notin A$, then $x \in A^c$ and A^c is Ω_{gb}^+-open. Take $A^c = U$, Then U is a Ω_{gb}^+-open set containing x and $A \cap U=\phi$.

Hence $x \notin \Omega_{gb}^+$-cl(A).

Hence Ω_{gb}^+-cl(A) $\subset$ A .By using Proposition 4.2, we get $A \subset \Omega_{gb}^+$-cl(A). Hence A = Ω_{gb}^+-cl(A).

Conversely, Suppose $A= \Omega_{gb}^+$-cl(A) . Since $A= \cap\{F/A \subset F$ and F is Ω_{gb}^+ -closed$\}$,by Proposition 4.3, A is Ω_{gb}^+ -closed. **Proposition4.6:** Ω_{gb}^+ -cl(A) is Ω_{gb}^+ closed.

Proof: By using proposition 4.2 and 4.4, we have Ω_{gb}^+-cl(A) $\subset \Omega_{gb}^+$-cl (Ω_{gb}^+-cl(A)). Let $x \in \Omega_{gb}^+$-cl(Ω_{gb}^+-cl(A)) ⇒ x is a Ω_{gb}^+-cluster point of Ω_{gb}^+ -cl(A).That implies for every Ω_{gb}^+-open set U containing x, (Ω_{gb}^+-cl(A)) $\cap$ U$\neq \phi$. Let $y \in \Omega_{gb}^+$-cl(A)$\cap$U. Then y is a Ω_{gb}^+-cluster point of A. Therefore for every Ω_{gb}^+-open set G containing y, $A \cap G \neq \phi$. Since U is Ω_{gb}^+-open and $y \in U$, $A \cap U \neq \phi$.

Hence $x \in \Omega_{gb}^+$-cl(A).

Hence Ω_{gb}^+-cl(A) = Ω_{gb}^+-cl(Ω_{gb}^+-cl(A)). By Proposition4.5, Ω_{gb}^+-cl(A) is Ω_{gb}^+-closed.

Remark 4.7: (1) X and ϕ are both Ω_{gb}^+-open and Ω_{gb}^+-closed. (2) By using properties 4.3 and 4.6, Ω_{gb}^+-cl(A) is the smallest Ω_{gb}^+-closed set containing A. **Proposition4.8:** If A_k is Ω_{gb}^+ -closed for each $K \in I$, then $\bigcap_{k \in I} A_k$ is Ω_{gb}^+ -closed. **Proof**: Let $A = \bigcap_{k \in I} A_k$ and $x \in \Omega_{gb}^+$-cl(A).

Then x is Ω_{gb}^+-cluster point of A.

Hence for every Ω_{gb}^+-open set U containing x, $A \cap U \neq \phi \Rightarrow (\bigcap_{k \in I} A_k) \cap U \neq \phi$.

That implies $A_k \cap U \neq \phi$ for each $K \in I$.

If $x \notin A$, then for some $k \in I$, $x \notin A_k$. Since A_k is Ω_{gb}^+-closed, $A_k = \Omega_{gb}^+$-cl(A_k) Hence $x \notin \Omega_{gb}^+$-cl(A_k).

Therefore x is not a Ω_{gb}^+-cluster point of A_k. So $\exists$ a Ω_{gb}^+-open set V containing x such that $A_k \cap V = \phi$.

By this contradiction, $x \in A$. Therefore Ω_{gb}^+-cl(A) $\subset$ A.

By Proposition 4.2, $A \subset \Omega_{gb}^+$-cl(A). Hence A= Ω_{gb}^+ -cl(A).

By Proposition 4.5, A is Ω_{gb}^+-closed.

Hence $\bigcap_{k \in I} A_k$ is Ω_{gb}^+-closed

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