APPLICATION OF SYMMETRIC HEXAGONAL INTUITIONIST FUZZY NUMBERS IN A TRANSPORTATION PROBLEM

A.SAHAYA SUDHA, K.R. VIJAYALAKSHMI

Abstract: The main objective of this paper is to introduce a fuzzy transportation problem with symmetric hexagonal Intuitionist fuzzy number (SHIIFN) based on supply and demand and obtains an initial basic feasible solution. Hence it reduces the computational complexity of deriving the solutions Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness This is illustrated with a suitable example.

Keywords: Fuzzy Transportation problem, Symmetric Intuitionist Hexagonal fuzzy number Intuitionistic fuzzy number, Initial Basic Feasible Solution ,

1. Introduction: Intuitionistic fuzzy set (IFS) is one of the generalizations of fuzzy sets theory [9].Out of several higher-order fuzzy sets. IFS were first introduced by Atanassov [1].He found that IFS is very much compatible to deal with vagueness or uncertainty. The main advantage of IFSs is that include both the degree of Membership and non membership of each element in the set. As an advantage intuitionistic fuzzy set (IFS) theory is introduced to deal the transportation problems with two functions. The past year research works on IFS was done by the great people like Atanassov and Gargov [3], Buhaescu [5], Ban [4], Deschrijver and Kerre [6]. Mahapatra and Roy [7] defined the triangular intuitionistic fuzzy number (TrIFN) and trapezoidal intuitionistic fuzzy number (TrIFN) and their arithmetic operations based on intuitionistic fuzzy extension principle using alpha cuts method .

2. Preliminaries: 2.1 Intuitionistic Fuzzy set [IFS] [2]: An Intuitionistic Fuzzy Set (IFS) $\widetilde{A}^I$ in X is defined as an object of the form $\widetilde{A}^I = \left\{ \left\langle x, \mu_{\widetilde{A}^I}(x), v_{\widetilde{A}^I}(x) \right\rangle : x \in X \right\}$ where the functions $\mu_{\widetilde{A}^I} : X \to [0,1]$ and $v_{\widetilde{A}^I} : X \to [0,1]$ define the degree of the membership and the degree of non membership of the element $x \in X$ in $\widetilde{A}^I$

$$0 \le \mu_{\widetilde{A}^I}(x) + v_{\widetilde{A}^I}(x) \le 1$$

$$\mu_{\widetilde{A}_H^I}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-(a_L-s-t)}{t}\right) & for\ a_L - s - t \le x \le a_L - s \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-(a_L-s)}{s}\right) & for\ a_L - s \le x \le a_L \\ 1, & for\ a_L \le x \le a_U \\ 1 - \frac{1}{2}\left(\frac{x-a_U}{s}\right), & for\ a_U \le x \le a_U + s \\ \frac{1}{2}\left(\frac{(a_U+s+t)-x}{t}\right), & for\ a_U + s \le x \le a_U + s + t \\ 0, & otherwise \end{cases}$$

2.2 Intuitionistic Fuzzy number [8]:
An Intuitionistic Fuzzy Number (IFN) $\widetilde{A}^I$ is an intuitionistic fuzzy subset of the real line,

i) convex for the membership function $\mu_{\widetilde{A}^I}(x)$,
(i.e.) $\mu_{\widetilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \ge \min\left(\mu_{\widetilde{A}^I}(x_1), \mu_{\widetilde{A}^I}(x_2)\right)$, for every $x_1, x_2 \in R, \lambda \in [0,1]$.

ii) concave for the membership function $v_{\widetilde{A}^I}(x)$, that is, $v_{\widetilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \le \max\left(v_{\widetilde{A}^I}(x_1), v_{\widetilde{A}^I}(x_2)\right)$ for every $x_1, x_2 \in R, \lambda \in [0,1]$.

iii) normal, that is, there is some $x_0 \in R$ such that $\mu_{\widetilde{A}^I}(x_0) = 1, v_{\widetilde{A}^I}(x_0) = 0$.

2.3 Symmetric Hexagonal intuitionistic fuzzy numbers: A symmetric hexagonal intuitionistic fuzzy numbers

$$\widetilde{A}_H^I = (a_L - s - t,\ a_L - s, a_L, a_U, a_U + s, a_U + s + t),$$
$$(a'_L - s' - t', a'_L - s', a_L, a_U, a'_U + s', a'_U + s' + t')$$

where $a_L, a_U, a'_L, a'_U, s, t, s', and\ t'$ are real numbers such that

$$(a'_L - s' - t' \le a_L - s - t \le a'_L - s' \le a_L - s \le a_L \le, a_U \le a_U + s \le a'_U + s' \le a_U + s + t \le a'_U + s' + t')$$

and its membership and non-membership functions are given below

$$v_{\widetilde{A}_H^I}(x) = \begin{cases} 1 - \frac{1}{2}\left(\frac{x-(a'_L-s'-t')}{t'}\right) & for\ a'_L - s' - t' \le x \le a'_L - s' \\ \frac{1}{2}\left(\frac{a_L - x}{a_L - (a'_L - s')}\right) & for\ a'_L - s' \le x \le a_L \\ 0, & for\ a_L \le x \le a_U \\ \frac{1}{2}\left(\frac{x - a_U}{a'_U + s' - a_U}\right), & for\ a_U \le x \le a'_U + s' \\ \frac{1}{2} + \frac{1}{2}\left(\frac{(x-(a'_U+s'))}{t'}\right), & for\ a'_U + s' \le x \le a'_U + s' + t' \\ 1, & otherwise \end{cases}$$

3. Mathematical formulation: Consider a transportation problem with 'm' sources and 'n' destinations. The mathematical formulation of the SHIFTP whose parameters are SHIFNs under the case that the total supply is equivalent to the total demand is given by

Minimize $Z' = \sum_{i=0}^{m} \sum_{j=0}^{n} \tilde{x}_{ij}' \tilde{c}_{ij}'$ Subject to

$\sum_{i=0}^{m} \tilde{x}_{ij}'' = \tilde{a}_i'$,i=1, 2, 3...m, $\sum_{j=0}^{n} \tilde{x}_{ij}'' = \tilde{b}_j'$,j=1, 2, 3....n

$x_{ij} \geq 0 \quad \forall_{ij}$ In the above model the transportation costs $\tilde{c}_{ij}'$ supply $\tilde{a}_i'$ demand $\tilde{b}_j'$ are in SHIFNs

3.1 Intuitionistic Fuzzy Initial Basic Feasible Solution (IFIBFS) – Proposed Method: A feasible solution to a 'm' sources and 'n' destinations transportation problem is said to be basic feasible solution if the number of positive allocations are 'm+n-1'.Here the IFIBFS is based on intuitionistic fuzzy version of vogel's approximation method (IFVAM). The method proceeds as follows.

I. Fuzzy transportation table

Step 1: Calculate the magnitude of difference between the minimum and next to minimum transportation cost in each row and column and write it as "Diff." along the side of the table against the corresponding row/column.

Step 2:In the row /column corresponding to maximum "Diff.", make the maximum allotment at the box having minimum transportation cost in that row/ column.

Step 3:If the maximum "Diff." corresponding to two or more rows or columns are equal, select the top most row and the extreme left column. Repeat the above procedure until all the SHIF supplies are fully used and IF demands are fully received.

3.2 Application of IFIBFS in a Transportation Problem: A textile company has three metro outlets .Each outlet has to deliver six types of textile materials to four production centers at different destinations. The Fuzzy supply and demand are given below. Here in this problem supply and demand are considered as symmetric intuitionistic Hexagonal fuzzy numbers.

Origin	Destination				
	D_1	D_2	D_3	D_4	SUPPLY
SB_1	6	7	13	10	(7,9,11,13,15,17), (5,8,11,13,16,19)
SB_2	4	3	9	5	(8,10,12,14,16,18), (6,9,12,14,17,20)
SB_3	8	12	21	10	(8,11,14,15,18,21), (6,,10,14,15,19,23)
DEMAND	(9,11,13,15,17,19), (7,10,13,15,18,21)	(3,5,7,8,10,12) (1,4,7,8,11,14)	(10,12,14,16,18,20 8,11,14,16,19,22)	(11,13,15,17,19,21) (9,12,15,17,20,23)	

Solution: Step:1 The Intuitionistic Fuzzy IBFS of the above SHIFTP can be obtained by IFVAM as follows: Now using Step 1 of the VAM calculate the value "Diff" for each row and column as mentioned in the last row and column the following table.

Step - 2 Using step 2 identify the row/column corresponding to the highest value of "Diff". In this case it occurs at column 4. In this column minimum cost cell is (2,4) and the corresponding demand and supply are(11,13,15,17,19,21),(9,12,15,17,20,23) and(7,9,11,13,15,17),(5, 8, 11, 13, 16, 19).

Now allocate the maximum possible units to the minimum cost position (2, 4), write the remaining in column 4. After removing the second row repeats the step 1, till all the demand and supply are exhausted. Now allocate the remaining demands and supplies, we get the following complete allocation table.

II. Allocation table:

ORIGIN	DESTINATION				SUPPLY	
	D_1	D_2	D_3	D_4		
SB_1	6	7	13	10	(7,9,11,13,15,17), (5,8,11,13,16,19)	1
SB_2	4	3	9	**5**	(8,10,12,14,16,18), (6,9,12,14,17,20),	1
SB_3	8	12	21	10	(18,22,26,29,33,37), 14,20,26,29,35,41	2
Demand	(9,11,13,15,17,19), (7,10,13,15,18,21),	(3,5,7,8,10,12), (1,4,7,8,11,14)	(10,12,14,16,18,20), (8,11,14,16,19,22)	(11,13,15,17,19,21), (9,12,15,17,20,23)		
	2	4	4	5		

III. Optimal solution table:

ORIGIN	DESTINATION					
	D_1	D_2	D_3	D_4		
SB_1	-	-	(7,9,11,13,15,17), (5,8,11,13,16,19)	-	-	
SB_2	-	-	-	(8,10,12,14,16,18), (6,9,12,14,17,20)	-	-
SB_3	(9,11,13,15,17,19), (7,10,13,15,18,21)	(3,5,7,8,10,12), (1,4,7,8,11,14)	(-7,-3,1,5,9,13), (-11,-5,1,5,11,17)	(-7,-3,1,5,9,13), (-11,-5,1,5,11,17)	-	
DEMAND	-	-	-	-		

Therefore, the intuitionistic fuzzy optimal solution in terms of SHIFNs for the given IFTP is, by applying the ranking $R(\tilde{A}) = 2\,[(a_L + a_U)\,,(a'_L + a'_u)]$

$$x_{13} = (7,9,11,13,15,17),(5,8,11,13,16,19)$$

$$x_{31} = (9,11,13,15,17,19),(7,10,13,15,18,21)$$

$$x_{24}(8,10,12,14,16,18),(6,9,12,14,17,20)$$

$$x_{32} = (3,5,7,8,10,12),(1,4,7,8,11,14)$$

$$x_{33} = (-7,-3,1,5,9,13),(-11,-5,1,5,11,17)$$

$$x_{34} = (-7,-3,1,5,9,13),(-11,-5,1,5,11,17)$$

The total minimum fuzzy transportation cost is given by,

Minimize $Z' =$ (22,222,422,610,810,1010), (-178,122,422,610,910,1210)

By Symmetric hexagonal intuitionistic fuzzy ranking the minimum transportation cost is $Z' =$ **Rs.2064**

Conclusion: Generally the transportation problems will be solved by using triangular fuzzy numbers trapezoidal fuzzy numbers, hexagonal fuzzy number or various other methods. In the present paper symmetric hexagonal Intuitionistic Fuzzy number has been newly introduced to deal IFTP. The arithmetic operations on symmetric hexagonal intuitionistic fuzzy numbers is used to find the solutions of decision problems where all the six parameters are fuzzy numbers Also it can be applied in many other applications such as production process, when the requirement are uncertain.

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A. Sahaya Sudha
Department of Mathematics, Nirmala College for women, Coimbatore-641018
K.R. Vijayalakshmi
Ph.D. Research Scholar, Nirmala College for women Coimbatore- 641018.