NANO STAR PRE GENERALIZED CLOSED SETS IN NANO TOPOLOGICAL SPACES.

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Abstract : The purpose of this paper is to introduce, nano star pre generalized closed sets in Nano Topological Spaces.

Keywords: Nano topological space, Nano*pg closed sets.

1. **Introduction:** In 1970, Levine[1] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. In [2] Maki et al introduced the concepts of generalized pre closed sets and pre generalized closed sets in an analogous manner. The notion of Nano topology was introduced by Lellis Thivagar[3] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined Nano closed sets, Nano interior and Nano closure of a set and Nano continuous functions. In [4,5] Bhuvaneshwari et al Introduced and studied some properties of Nano generalized closed sets, Nano generalized pre closed sets and Nano pre generalized closed set.

This paper mainly deals with the study of a new type of set in Nano topological space called Nano*pre generalized closed setsand study the concept of Nano*pre generalized continuous function..

Throughout this paper, a space $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We recall the following
definition, notions and characterizations.
said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$

1. The lower approximation of X with respect to R is the set of all objects, which can be for certainly classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = \cup_{x \in U}\{ R(x): R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by X∈U.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = \cup_{x \in U}\{R(x): R(x) \cap X \neq \emptyset\}$
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) - L_R(X)$.

II. Preliminaries:

Definition:2.1[7]A subset A of a topological space (X,τ) is called a preopen set if A ⊆Int(Cl(A)). The complement of a preopen set of a space X is called preclosed set in X.

Definition:2.2 [6]A pre-closure of a subset A of X is the intersection of all preclosed sets that contains A and it isdenoted by pcl(A).

Definition:2.3[6]The union of all preopen subsets of X contained in A is called preinterior of A and it is denoted by pInt(A).

Definition:2.4 [1] A subset A of (X,τ) is called a generalised closed set (briefly g-closed) if cl(A) ⊆U whenever A ⊆ U and U is open in X.

Definition:2.5 [6]A subset A of (X,τ) is called a generalisedpre closed set (briefly gp-closed) ifpcl(A) ⊆ Uwhenever A⊆ U and U is open in X.

Definition:2.6 [6] A subset A of (X,τ) is called a pre generalised closed set (briefly pg-closed) if pcl(A) ⊆U whenever A⊆ U and U is preopen in X.

Definition:2.7[9] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are
Then $\tau_R(X)$ is a topology on U called the nano topology on with respect to X. The elements of $\tau_R(X)$are called as nano-open sets.

Definition:2.8[9] If (U,R) is an approximation space and X,Y⊆U, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$.
(ii) $L_R(\emptyset) = U_R(\emptyset) = \emptyset$ and $L_R(U) = U_R(U) = U$.
(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$.
(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$.
(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$.
(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$.
(vii) $L_R(X) \subseteq L_R(Y)$and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$.
(viii) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$.
(ix) $U_R U_R(X) = L_R U_R(X) = U_R(X)$.
(x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition:2.9[9] Let U be non-empty, finite universe of objects and R be an equivalence relation on U. Let X⊆U. Let $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$. If $\tau_R(X)$ satisfies the following axioms:

(i) $U, \phi \epsilon \tau_R(X)$.
(ii) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Definition:2.10[9] If $\tau_R(X)$ is the nano topology on U with respect to X, then the set B={U, $L_R(X)$, $U_R(X)$,

$B_R(X)$} is the basis for $\tau_R(X)$.

Definition:2.11[9] If $(U,\tau_R(X))$ is a nano topological space with respect to X where X⊆U and if A⊆U , then

i)Thenano interior of A is defined as the union of all nano-open subsets of A and it is denoted by Nint(A). That is Nint(A), is the largest nano open subset of A.

ii)Thenano closure of A is defined as the intersection of all nano closed sets containing A and is denoted by Ncl(A). That is ,Ncl(A) is the smallest nano closed set containing A.

Definition:2.12[9] Let $(U,\tau_R(X))$ be a nano topological space and A⊆U. Then A is said to be

Nano semi open if A⊆Ncl(Nint(A))

Nano pre-open if A⊆Nint(Ncl(A))

Nano α-open if A⊆Nint(Ncl(Nint(A)))

Nano regular-open if A=Nint(Ncl(A))

NSO(U,X), NPO(U, X), NαO(U,X) and NRO(U,X) respectively denote the families of all nano semi-open, nano pre-open, nano α-open and nano r-open subsets of U. Let $(U,\tau_R(X))$ be a nano topological space and A⊆U. A is said to be nano semi closed, nano pre-closed and nano α-closed, nano regular-closed if its compliment is respectively nano semi-open, nano pre-open, nano α-open, nano regular-open.

Definition:2.13[9]:A subset of $(U,\tau_R(X))$ is called Nanogeneralized closed set (briefly Ng-closed) if Ncl(A)⊆V whenever A⊆V and V is Nano open set in $(U,\tau_R(X))$.

Definition:2.14:[8] A subset A of $(U,\tau_R(X))$ is called Nano pregeneralized closed set (briefly Npg-closed) if Npcl(A)⊆V whenever A⊆V and V is Nanopre open in $(U,\tau_R(X))$.

III Nano*Pre Generalized Closed Set: Throughout this paper, (U, $\tau_R(X)$) is a Nano topological space with respect to X where X ⊆U, R is an equivalence relation on U. Then U/R denotes the family of equivalence classes of U by R.

open in U. Therefore A is nano*pre generalized closed set.

In this section ,we define and study the forms of Nano*pre generalized pre closed sets.

Definition:3.1A subset A of $(U,\tau_R(X))$ is called Nano*pre generalized closed set (briefly N*pg closed) if Nint(Npcl(A))⊆V whenever A⊆V and V is Nano pre open in $(U,\tau_R(X))$.

Example:3.2 Let U = {a,b,c,d} with U/R = { {a}, {c}, {b,d}} and X = {a,b}.

Then $\tau_R(X)$={ U,Φ, {a}, {a,b,d}, {b,d}} which are open sets.

The Nano closed sets = { U,Φ, {b,c,d}, {c}, {a,c}} .

The Nano pre closed sets = {Φ,U, {b} , {c}, {d}, {a,c}, {b,c}, {c,d}, {a,b,c}, {b,c,d}, {a,c,d}} .

The Nanogeneralised closed sets are { Φ,U,{c}, {a,c}, {b,c} , {c,d} , {a,b,c} , {b,c,d} ,{a,c,d}}

The Nanopregeneralised closed sets are{Φ,U,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{c,d},{a,b,c},{a,c,d},{b,c,d}}

The Nano*pre generalised closed sets are {Φ,U,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{c,d},{b,d},{a,b,d},{a,b,c},{a,c,d},{b,c,d}}

Theorem:3.3[4] Every Nano closed set is a Nanopre closed set .

Remark:3.4 The converse of the above theorem is not true.In the example 3.2 , the sets {a,b,c},{a,c,d} are Nanopreclosed sets but not Nano closed sets.

Theorem:3.5 Every nano closed set is nano*pre generalized closed set.

Proof: Let A be a nano closed set of U and

Here the sets {b,c} and {d} are nano*pre generalized closed sets but not nano α-generalized closed sets.

Remark:3.6 The converse of the above theorem need not be true and it is shown in example 3.2

Theorem:3.7 Every nano pre closed set in nano*pre generalized closed set.

Proof: The proof of the theorem is obvious. The converse of the theorem need not be true a

nd it is shown in example 3.2

Theorem:3.8 Every nano generalized closed set is nano*pre generalized closed set.

Proof: Let A be a nano generalized closed set. Then Ncl(A)⊆V whenever A⊆V and V is nano open in U.But every nano open set is nanopre open set (by theorem 2.1.2) which implies V is nanopre open in U. Also Nint(Npcl(A))⊆Ncl(A)⊆V, A⊆V, V is nano pre

{c,d},{a,c} {a,b,c},{a,b,d},{a,c,d}}.The nano* pre generalized closed sets are:

generalized b-closed sets.

Remark:3.9 The converse of the above theorem need not be true and it is shown in example 3.2

Theorem:3.10 Every nano αg-closed set isnano*pre generalized closed set.

Proof: Let A be nano αg-closed in U and V is nano open in U such that A⊆V. But every nano open set is nanopre open which implies V is nanopre open. Also Nint(Npcl(A))⊆Nαcl(A)⊆V, A⊆V and V is nano per open in U.Hence A is nano*pre generalized closed set.

Remark:3.11 The converse of the above theorem need not be true and it is shown by the following example.

Example: 3.12 Let U={a,b,c,d} with U/R={ {a}, {b,d}, {c}} and X={a,b}.

Then the nano topology is defined as $\tau_R(X)$={U,∅, {a},{a,b,d},{b,d}}.

The nano α-generalised closed sets are:{U, ∅, {a},{c},{b,c},{c,d},{a,c}, {b,c,,d},{a,c,d},{a,b,c}} .The nano* pre generalized closed sets are: {U,∅,{a},{b},{c},{d}, {a,b},{b,c},{c,d},{b,c},{a,c},{b,d},{a,b,c},{a,b,d},{b,c,d}{a,c,d}}.

Here the sets {b} and {d} are nano*pre generalized closed sets but not nano α-generalized closed sets.

Theorem:3.13 Every nano generalized b-closed set is nano*pre generalized closed set.

Proof: Let A be nano generalized b-closed in Uand V is nano open in U such that A⊆V.But everynano open set is nanopre open which implies V is nanopre open. Also Nint(Npcl(A))⊆Nbcl(A)⊆V, A⊆V and V is nano per open in U.Hence A is nano*pre generalized closed set.

Remark:3.14 The converse of the above theorem need not be true and it is shown by the following example.

Example: 3.15 Let U={a,b,c,d} with U/R={ {a}, {b,d}, {c}} and X={b,d}.

Then the nano topology is defined as $\tau_R(X)$={U,∅, {a},{a,b,d},{b,d}}.

The nanogeneralised b-closed sets are : {U,∅,{c},{b,c},{c,d},{a,c}{a,b,c},{a,b,d},{a,c,d}}.Here the sets {b} and {a,d} are nano*pre generalized closed sets but not nano

closed sets but not nano generalized regular closed sets.

Theorem:3.16 Every nano weakly generalized set is nano*pre generalized closed set.

Proof: Let A be nano weakly generalized closed in U and V is nano open in U such that A⊆V. But every nano open set is nanopre open which implies V is nanopre open. Also Nint(Npcl(A))⊆Ncl(Nint(A))⊆V, A⊆V and V is nano per open in U. Hence A is nano*pre generalized closed set.

Remark:3.17 The converse of the above

Nint(Npcl(A))⊆Nscl(A)⊆V, A⊆V and V is nano per open in U. Hence A is nano*pre generalized closed set.

Example: 3.18 Let U={a,b,c,d} with U/R={ {a}, {b}, {c,d}} and X={a,c}.

Then the nano topology is defined as $\tau_R(X)$={U,∅,{a},{c,d},{a,c,d}}.

The nano weakly generalised closed sets are :{U,∅,{b},{c},{d},{a,b},{b,c}, {b,d},{a,b,c},{b,c,d},{a,b,d} }.The nano* pre generalized closed sets are: {U,∅,{a},{b},{c},{d},{a,b},{b,c},{a,d},{b,c},{a,c},{a,b,c},{a, b,d},{b,c,d}{a,c,d}}.Here the sets {a} and {a,c,d} are nano*pre generalized closed sets but not nano weakly generalized closed sets.

Theorem:3.19 Every nano generalized regular closed set is nano*pre generalized closed set.

Proof: Let A be nano generalized regular closed in U and V is nano open in U such that A⊆V. But every nano open set is nanopre open which implies V is nanopre open. Also Nint(Npcl(A))⊆Nrcl(A)⊆V, A⊆V and V is nano per open in U. Hence A is nano*pre generalized closed set.

Remark:3.20 The converse of the above theorem need not be true and it is shown by the following example.

Example: 3.21 Let U={a,b,c,d} with U/R={ {a}, {b,d}, {c}} and X={a,b}.

Then the nano topology is defined as $\tau_R(X)$={U,∅,{a},{a,b,d},{b,d}}.

The nanogeneralised regular closed sets are :{U,∅,{c},{b,c},{c,d},{a,c}, {a,b,c},{b,c,d},{a,c,d}}.The nano* pre generalized closed sets are: {U,∅,{a},{b},{c},{d},{a,b},{b,c},{c,d}{a,d} ,{b,c},{a,c},{a,b,c},{a,b,d},{b,c,d}{a,c,d}}.Here the sets {b} and {d} are nano*pre generalized

Now Nint(Npcl(AUB))= [Nint(Npcl(A))UNint(Npcl(B))]⊆V. Thus we have Nint(Npcl(AUB))⊆V, whenever AUB⊆V, V is nanopre open set in (U,$\tau_R(X)$). This implies AUB is a nano*pre generalized closed set in (U,$\tau_R(X)$).

Theorem:3.22: Everynano semi generalized closed is nano*pre generalized closed set.

Proof: Let A be nano semi generalized closed in U and V is nano open in U such that A⊆V.But every nano open set is nanopre open which implies V is nanopre open. Also Also Nint(Npcl(A))⊆Nscl(A)⊆V, A⊆V and V is nano per open in U.Hence A is nano*pre

nano*pre generalized closed set in (U,$\tau_R(X)$)

Remark:3.23 The converse of the above theorem need not be true and it is shown by the following example.

Example: 3.24 Let U={a,b,c,d} with U/R={ {a}, {b,d}, {c}} and X={a,b}.

Then the nano topology is defined as $\tau_R(X)$={U,∅, {b,d},{a,b,d}}.

The nano semi generalised closed sets are :{U,∅,{a},{b},{c},{d},{b,c},{c,d} ,{b,d},{a,b,c},{b,c,d},{a,c,d} }The nano* pre generalized closed sets are: {U,∅,{a},{b},{c},{d},{a,b},{b,c},{c,d}{a,d},{b,c},{a,c},{a,b, c},{a,b,d},{b,c,d}{a,c,d}}. Here the sets {a,b,d} and {a,b} are nano*pre generalized closed sets but not nano semi generalized closed sets.

Theorem:3.25 Every nano pre generalized closed set are nano*pre generalized closed set.

Proof: Let A be nano pre generalized closed in U and V is nanopre open in U such that A⊆V. Also Nint(Npcl(A))⊆Npcl(A)⊆V, A⊆V and V is nanopre open in U. Hence A is nano*pre generalized closed set.

Remark:3.26 The converse of the above theorem need not be true and it is shown example 3.2 .Here {a} is nano*pre generalized closed set but not nano pre generalized closed set.

Theorem:3.27 The union of two nano*pre generalized closed sets in (U,$\tau_R(X)$) is also a nano*pre generalized closed set in (U,$\tau_R(X)$).

Proof: Let A and B be two nano*pre generalized closed sets in (U,$\tau_R(X)$).Let V be ab nanopre open set in U such that A⊆V and B⊆V. Then we have AUB⊆V.As A and B are nano*pre generalized closed set in (U,$\tau_R(X)$) .Nint(Npcl(A))⊆V and Nint(Npcl(B))⊆V.

Remark:3.28 The intersection of two nano*pre generalized closed set in (U,$\tau_R(X)$) is also a nano*pre generalized closed set as seen from the following example

Example:3.29 Let U={a,b,c,d} with U/R={ {a}, {d}, {b,c}} and X={a.b.c}.

Then the nano topology is defined as $\tau_R(X)$={U,∅,{a},{b,c},{a,b,c}}.

The nano* pre generalized closed sets are: {U,∅,{a},{b},{c},{d},{a,b},{b,c},{c,d}, {a,d},{b,c},{a,c},{a,b,c},{a,b,d},{b,c,d}{a,c,d}}.Here {a,b,c}∩{a,c,d}={a,c} which is again a nano*pre generalized closed set.

Theorem:3.30 If A is nano*pre generalized closed set iffNint(Npcl(A))-A contains no non-empty nano pre closed set.

Proof:

Necessity: Let F be nano pre closed set in (U,$\tau_R(X)$) such that F⊆Nint(Npcl(A))-A. Then A⊆X-F. Since A is nano*pre generalized closed set and X-F is nano pre closed set. Then Nint(Npcl(A))⊆X-F. That is F⊆X-Nint(Npcl(A)).So F⊆ [X-Nint(Npcl(A))]∩[Nint(Npcl(A))-A]

Therefore F=∅.

Sufficiency: Let us assume that Nint(Npcl(A))-A contains no non-empty nano pre closed set.

Let A⊆V, V is nanopre open. Suppose that Nint(Npcl(A)) is not contained in V, Nint(Npcl(A))∩V^C is non empty nano pre closed set of Nint(Npcl(A))-A, which is a contradiction. Therefore Nint(Npcl(A))⊆V. Hence A is Nano*pre generalized closed set.

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