ON SOFT JP CLOSED SETS IN SOFT TOPOLOGICAL SPACES

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Abstract: This paper is devoted to soft JP Interior, soft JP Closure, Soft JP border, Soft JP exterior, Soft JP Frontier and Soft JP Kernal respectively.

Keywords: Soft JP closed set, soft closed set, Soft open, Soft JP open set, soft JP closure

1.Introduction: Molodtsov[2] (1999) initiated the theory of soft sets as a new mathematical tool for dealing uncertainty, which is completely a new approach for modeling vagueness and uncertainties. Shabir and Naz[5](2011) introduce the notion of soft topological spaces. In 2016,The authors of this paper introduced soft JP closed sets in soft topological spaces. In this paper, we introduce soft JP Interior, soft JP Closure, Soft JP border, Soft JP exterior, Soft JP Frontier and Soft JP Kernal respectively.

2.Preliminaries:

Throughoutthiswork,Xreferstoaninitialuniverse,$E$is asetofparameters,$P(X)$isthepowersetof$X$,and$A\subseteq E$.

Definition 2.1[4]: A soft set (F,A) over X is said to be **Null Soft Set** denoted by $F_\phi$or $\widetilde{\Phi}$if for all e ∈A, F(e) = ϕ. A soft set (F,E) over X is said to be an **Absolute Soft Set** denoted by $F_X$or $\widetilde{X}$if for all e∈A,F(e) = X.

Definition 2.2[5]: The **Union** of two soft sets(F, A) and (G, B)over X is the soft set (H, C), where C = A ∪B, and for all e ∈C,H(e) = F(e), if e ∈A\B, H(e) = G(e) if e ∈B\A and H(e) = F(e) ∪G(e)if e ∈A ∩B and is denoted as(F, A) $\widetilde{\cup}$(G, B) = (H, C).

Definition 2.3[5]: The **Intersection** of two soft sets (F, A) and (G, B)over X is the soft set (H,C), where C = A ∩B and H(e) = F(e) ∩G(e)for all e ∈C and is denoted as

(F, A) $\widetilde{\cap}$(G,B) = (H, C).

Definition 2.6[5]: Let (F,A) and (G, B) be soft sets over X, we say that (F,A) is a **Soft Subset** of (G,B) if A ⊆B and for all e ∈A,F(e)and G(e) are identical approximations. We write (F, A) $\widetilde{\subseteq}$(G, B).

Definition 2.7[5]: Let τ be a collection of soft sets over X with the fixed set E of parameters. Then τ is called a **Soft Topology** on X if **i**. $\widetilde{\Phi}$,$\widetilde{X}$belongs to τ. **ii.** The union of any number of soft sets in τ belongs to τ.**iii.** The intersection of any two soft sets in τ belongs to τ.The triplet (X, τ, E) is called **Soft Topological Spaces** over X.The members of τ are called **Soft Open** sets in X and complements of them are called **Soft Closed** sets in X.

Definition 2.8[5]: Let (X, τ, E) be a Soft Topological Spaces over X. The **Soft Interior** of (F, E) denoted by Int(F, E) is the union of all soft open subsets of (F,E). Clearly Int(F, E) is the largest soft open set over X which is contained in (F, E).

The **Soft Closure** of (F, E) denoted by Cl (F,E) is the intersection of soft closed sets containing (F, E). Clearly (F, E) is the smallest soft closed set containing (F,E).

i)Int (F, E) =$\widetilde{\cup}${(O.E): (O, E) is soft open and (O,E) $\widetilde{\subseteq}$(F,E)}.

ii)Cl (F,E) =$\widetilde{\cap}${(O. E): (O, E) is soft closed and (F, E) $\widetilde{\subseteq}$(O,E)}.

Definition 2.9:[5] Let (X, τ, E) be a Soft Topological Spaces over X.

1. The **Soft Border** of (F,E) denoted by Bd(F,E),defined by Bd(F,E)=(F,E)\Int(F,E).
2. The **Soft Frontier** of (F,E) denoted by Fr(F,E),defined by Fr(F,E)=Cl(F,E)\Int(F,E).
3. The **Soft Exterior** of (F,E) denoted by Ext(F,E),defined by Ext(F,E)= Int((F,E)^C)..

Definition 2.11:[4]: A Subset of a soft topological space (X, τ, E) is said to be**Soft JP closed set** if Scl(F,E)$\widetilde{\subseteq}$Int(U,E) whenever (F,E) $\widetilde{\subseteq}$(U,E) and (U,E) is soft $\hat{g}$ open. The complement of soft JP Closed set is called soft JP Open set .

3. On Soft Jp Closed Sets: Defnition3.1: Let (X, τ, E) be a Soft Topological Spaces over X. The **Soft JP Interior** of (F, E) denoted by JPInt(F, E) is the union of all soft JP open subsets of (F,E). Clearly JPInt(F, E) is the largest soft JP open set over X which is contained in (F, E).

JPInt (F, E) =$\widetilde{\cup}${(O.E): (O, E) is soft JP open and (O,E) $\widetilde{\subseteq}$(F,E)}.:

Defnition3.2: Let (X, τ, E) be a Soft Topological Spaces over X. The **Soft**JP **Closure** of (F, E) denoted by JP Cl (F,E) is the intersection of soft JP closed sets containing (F, E). Clearly JPCl (F,E) is the smallest soft JP closed set containing (F,E).

JPCl (F,E) =$\widetilde{\cap}${(O. E): (O, E) is soft JP closed and (F, E) $\widetilde{\subseteq}$(O,E)}.

Defnition3.3: Let (X, τ, E) be a Soft Topological Spaces over X. The **Soft JP Border** of (F,E) denoted by JP Bd(F,E),defined by **JP Bd(F,E)=(F,E)\JPInt(F,E).**

Defnition3.4: Let (X, τ, E) be a Soft Topological Spaces over X. The **Soft JPFrontier** of (F,E) denoted by JPFr(F,E),defined by **JP Fr(F,E)=JPCl(F,E)\JPInt(F,E).**

Defnition3.5: Let (X, τ, E) be a Soft Topological Spaces over X.The **Soft JP Exterior** of (F,E) denoted by JP Ext(F,E),defined by **JP Ext(F,E)= JPInt((F,E)^C).**

Theorem 3.6: Let (X, τ, E) be a Soft Topological Spaces over X. For any two subsets (A,E) and (B,E)of (X, τ, E),

i. JPCl($\tilde{\Phi}$)= $\tilde{\Phi}$.
ii. JPCl($\tilde{X}$)= $\tilde{X}$.
iii. (A,E) $\tilde{\subseteq}$JPCl(A,E)$\tilde{\subseteq}$Cl(A,E).
iv. If (A,E)$\tilde{\subseteq}$ (B,E) then JPCl(A,E)$\tilde{\subseteq}$ JPCl(B,E).
v. If (A,E) is soft JP Closed then JPCl(A,E)=(A,E).
vi. JPCl(JPCl(A,E))= JPCl(A,E).

Proof:(i)&(ii) are obivious.(iii).Since every soft closed set is softJP Closed .The proof follows.(iv)&(v)follows from the Definition 3.2.(vi). follows from the Definition 3.2 and (v).

Theorem 3.7: Let (X, τ, E) be a Soft Topological Spaces over X. For any two subsets (A,E) and (B,E)of (X, τ, E),

1. JPInt($\tilde{\Phi}$)=$\tilde{\Phi}$.
2. JPInt($\tilde{X}$)=$\tilde{X}$.
3. Int(A,E)$\tilde{\subseteq}$JPInt(A,E).
4. (A,E) is soft JP Open then (A,E)=JPInt(A,E).
5. If (A,E)$\tilde{\subseteq}$ (B,E) then JPInt(A,E)$\tilde{\subseteq}$ JPInt(B,E).
6. If (B,E) is any soft JP Open set contained in (A,E) then (B,E)$\tilde{\subseteq}$JPInt(A,E).
7. JPInt(A,E)$\tilde{\cup}$JPInt(B,E)$\tilde{\subseteq}$ JPInt[(A,E) $\tilde{\cup}$(B,E)].
8. JPInt(JPInt(A,E))= JPInt(A,E).

Proof:(i)&(ii) are obivious.(iii)Since Every soft Open set is soft JP Open set.Then the proof follows.(iv)&(v)Follows from the Defnition 3.1.(vi)Let (x,E)$\tilde{\in}$(B,E).By (v)&(v) (x,E)$\tilde{\in}$JPInt(A,E).Hence the proof.(vii)It follows from (v).(viii)It follows from definition 3.1 and (iv).

Theorem3.8: Let (X, τ, E) be a Soft Topological Spaces over X and (A,E) be a soft subset of (X, τ, E).Then for any (x,E)$\tilde{\in}$ $\tilde{X}$, (x,E)$\tilde{\in}$JPCl[(A,E) if and only if (U,E) $\tilde{\cap}$ (A,E) ≠ $\tilde{\Phi}$ for Every soft JP Open set (U,E) containing (x,E).

Proof: Necessity: Suppose that (x,E)$\tilde{\in}$JPCl[(A,E),Let (U,E) be a soft JP Open set containing (x,E) such that (U,E) $\tilde{\cap}$(A,E)= $\tilde{\Phi}$.Then,(A,E)$\tilde{\subseteq}$(U,E)^C.But (U,E)^C is soft JP closed and hence JPCl(A,E)$\tilde{\subseteq}$(U,E)^C.Since (x,E)$\tilde{\notin}$(U,E)^C,Then (x,E)$\tilde{\notin}$ JPCl[(A,E)],which is a contrary to the hypothesis.

Sufficiency: Suppose that Every soft JP Open set (U,E) containing (x,E) intersects (A,E).If (x,E)$\tilde{\notin}$ JPCl[(A,E)],there exists a soft JP closed set (F,E) in (X, τ, E) such that (A,E)$\tilde{\subseteq}$ (F,E) and (x,E)$\tilde{\notin}$ (F,E).Then (x,E)$\tilde{\in}$(F,E)^C and (F,E)^C is a soft JP open set containing (x,E).But (F,E)^C$\tilde{\cap}$ (A,E)= $\tilde{\Phi}$, which is a contrary to the hypothesis.

Theorem 3.9: For any subset (A,E) of the soft topological space (X, τ, E),

1. [JPInt(A,E)]^C = JPCl[(A,E)^C].
2. JPInt(A,E)]= [JPCl((A,E)^C)]^C.
3. JPCl(A,E)]= [JPInt((A,E)^C)]^C.

Proof:(i)Let (x,E)$\tilde{\in}$[JPInt(A,E)]^C .Then (x,E)$\tilde{\notin}$[JPInt(A,E)],i.e) Every soft JP Open set (U,E) containing (x,E) suchthat (U,E)$\tilde{\subseteq}$ (A,E).Then (U,E) $\tilde{\cap}$ (A,E) ≠ $\tilde{\Phi}$.By Theorem 3.6 (iii), (x,E)$\tilde{\in}$JPCl((A,E)^C).And therefore, [JPInt(A,E)]^C $\tilde{\subseteq}$ JPCl[(A,E)^C].Conversely suppose, (x,E)$\tilde{\in}$JPCl[(A,E)^C]. By Theorem 3.6 (iii), Every soft JP Open set (U,E) containing (x,E) such that (U,E) $\tilde{\cap}$ (A,E) ≠ $\tilde{\Phi}$.Then By theorem 3.7(iv), (x,E)$\tilde{\notin}$[JPInt(A,E)].Therefore, (x,E)$\tilde{\in}$[JPInt(A,E)]^C .(ii)It follows by taking complements in (i).(iii)It follows by replacing (A,E) by (A,E)^C in (i).

Theorem 3.10: For any subset (A,E) of the soft topological space (X, τ, E),

1. JPBd($\tilde{\Phi}$)= $\tilde{\Phi}$.
2. JPBd($\tilde{X}$)= $\tilde{X}$.
3. JPBd(A,E)$\tilde{\subseteq}$(A,E).
4. JPInt(A,E) $\tilde{\cup}$JPBd(A,E)=(A,E).
5. JPInt(A,E) $\tilde{\cap}$JPBd(A,E)=$\tilde{\Phi}$.
6. JPBd(A,E)$\tilde{\subseteq}$ Bd(A,E).
7. JPBd[JPInt(A,E)] =$\tilde{\Phi}$.
8. JPBd(JPBd(A,E))= JPBd(A,E).
9. JPBd(A,E)=(A,E)$\tilde{\cap}$JPCl((A,E)^C)

Proof:(i),(ii),(iii),(iv),(v) are obivious from the Definition 3.3.(vi)follows from 3.7(iii).
(vii) Let (x,E)$\tilde{\in}$JPBd[JPInt(A,E)].Then (x,E)$\tilde{\in}$JPBd(A,E).Since JPBd(A,E)$\tilde{\subseteq}$ (A,E).Then , (x,E)$\tilde{\in}$JPInt[JPBd(A,E)]$\tilde{\subseteq}$ JPInt(A,E).Then (x,E)$\tilde{\in}$ JPInt(A,E) $\tilde{\cap}$JPBd(A,E), which contradicts.(v).Then JPBd[JPInt(A,E)] =$\tilde{\Phi}$.(viii) Follows from (vii).(ix)Follows from Defnition 3.3 and Theorem 3.9(i).

Theorem 3.11: For any subset (A,E) of the soft topological space (X, τ, E),

1. JPFr($\tilde{\Phi}$)= $\tilde{\Phi}$.
2. JPFr($\tilde{X}$)= $\tilde{X}$.
3. JPInt(A,E) $\tilde{\cup}$JPFr(A,E)=JPCl(A,E).
4. JPInt(A,E) $\tilde{\cap}$JPFr(A,E)=$\tilde{\Phi}$.
5. JPBd(A,E)$\tilde{\subseteq}$ JPFr(A,E)$\tilde{\subseteq}$ JPCl(A,E).
6. JPFr(A,E)$\tilde{\subseteq}$ Fr(A,E).
7. JPFr[JPInt(A,E)] $\tilde{\subseteq}$ JPFr(A,E).
8. JPFr[JPCl(A,E)] $\tilde{\subseteq}$ JPFr(A,E).
9. JPFr(JPFr(A,E))$\tilde{\subseteq}$ JPFr(A,E).
10. JPFr(A,E)=JPCl(A,E)$\tilde{\cap}$ JPCl((A,E)^C).
11. (A,E) is soft JP closed iff (A,E)= JPInt(A,E) $\tilde{\cup}$JPFr(A,E).
12. (A,E) is soft JP closed iff JPBd(A,E) =JPFr(A,E).
13. $\tilde{X}$= JPInt(A,E)$\tilde{\cup}$ JPInt[(A,E)^C]$\tilde{\cup}$ JPFr(A,E).

Proof:(i),(ii),(iii),(iv),(v) are obivious from the Definition 3.4.(vi)Follows from Theorem 3.6(iii) and Theorem 3.8(iii).(vii) Follows from Defnition 3.4and Theorem 3.8(viii).(viii) Follows from Defnition 3.4and Theorem 3.6(vi).(ix) Follows from Defnition 3.4and the fact JPFr(A,E) is soft JP closed.(x) Follows from

Defnition 3.4and Theorem 3.9(i).(xi) Follows from (iii) and Theorem 3.6(v).(xii) Follows from Theorem 3.6(v).(xiii) Follows from (iii) and Theorem 3.9(iii).

Theorem 3.12: For any subset (A,E) of the soft topological space (X, τ, E),

1. JPExt($\tilde{\Phi}$)= $\tilde{\Phi}$.
2. JPExt($\tilde{X}$)= $\tilde{X}$.
3. If (A,E)$\tilde{\subseteq}$ (B,E) then JPExt(B,E)$\tilde{\subseteq}$ JPExt(A,E).
4. JPExt(A,E) is Soft JP Open.
5. Ext(A,E) $\tilde{\subseteq}$ JPExt(A,E)$\tilde{\subseteq}$(A,E)^C.
6. JPExt(A,E)=$\tilde{X}$\ JPCl(A,E).
7. $\tilde{X}$= JPInt(A,E) $\tilde{\cup}$JPFr(A,E) $\tilde{\cup}$JPExt(A,E).
8. JPExt[(A,E)$\tilde{\cup}$(B,E)]$\tilde{\subseteq}$JPExt(A,E)$\tilde{\cap}$JPExt(B,E).
9. JPExt(A,E)$\tilde{\cap}$JPExt(B,E)$\tilde{\subseteq}$ JPExt[(A,E)$\tilde{\cup}$(B,E)].

Proof:(i),(ii),(iii) are obivious from Defnition 3.5.(iv)Follows from Defnition 3.1& Defnition 3.5.(v) Follows from Theorem 3.7(viii).(vi) Follows from Theorem 3.9(iii).(vii) Follows from Defnition 3.5 and Theorem 3.11(xiii).(viii) Follows from (iii) and set theoretic properties.

Defnition3.13: Let (X, τ, E) be a Soft Topological Spaces over X. The **Soft JP Kernal** of (F, E) denoted by JPKer(F, E) is the intersection of all soft JP open sets containing(F,E).

JPKer (F, E) =$\tilde{\cap}${(O.E): (O, E) is soft JP open and (F,E) $\tilde{\subseteq}$(O,E)}.

Theorem3.14: Let (X, τ, E) be a Soft Topological Spaces over X. For any two subsets (A,E) and (B,E) of (X, τ, E),

1. (A,E)$\tilde{\subseteq}$ JPKer (A,E).
2. If (A,E)$\tilde{\subseteq}$ (B,E) then JPKer(A,E)$\tilde{\subseteq}$ JPKer(B,E).
3. JPKer(JPKer(A,E))= JPKer(A,E).
4. If (A,E)is soft JP open then JPKer(A,E)=(A,E).

Proof:(i)Follows from Defnition 3.13.(ii)Suppose (x,E)$\tilde{\notin}$ JPKer[(B,E)],Then there exists a soft JP Open set (U,E) such that (B,E)$\tilde{\subseteq}$ (U,E) and (x,E)$\tilde{\notin}$ (U,E).Since (A,E)$\tilde{\subseteq}$ (B,E), (x,E)$\tilde{\notin}$ JPKer[(A,E)].Then JPKer(A,E)$\tilde{\subseteq}$ JPKer(B,E).(iii)Follows from Defnition 3.13 and Theorem 3.14(i).(iv)By the Definition 3.13, (A,E)$\tilde{\in}$ SJPO(X, τ, E), Then JPKer (A,E)$\tilde{\subseteq}$ (A,E) and by (i), (A,E)$\tilde{\subseteq}$ JPKer (A,E). Then JPKer(A,E)=(A,E).

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