OPERATIONS ON ANTI FUZZY GRAPHS

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Abstract: In 1965, Lofti A. zadeh introduced the notion of fuzzy sets to deal with uncertainty problems using the notion of fuzzy relation on fuzzy sets, the theory of fuzzy graphs was developed by Azriel Rosenfeld in 1975, since then many researchers worked on fuzzy graphs.

However, no notion about anti fuzzy relation has been developed. This motivated us to define the notion of anti fuzzy relation and anti fuzzy graphs in this paper. We illustrate the concepts with some examples. Also we define the operation of union, join, complement of anti fuzzy graphs and prove some of their properties.

Keywords: Fuzzy graph, anti fuzzy graph, strong anti fuzzy graph, regular anti fuzzy graphs.

Introduction: The theory of fuzzy graphs was developed by Azriel Rosenfeld in 1975 [4]. Some operations on fuzzy graphs have been studied in [1], [2]. The concept of regular fuzzy graph is investigated by Nagoorkani and Radha [3]. Sunitha and Vijayakumar [5] discussed about complement of fuzzy graphs. Whenever we discuss the fuzzy structures in any algebraic theory, analogously the notion of anti fuzzy structures has been studied. However in the theory of fuzzy graphs, no theory on anti fuzzy structures has been introduced. This motivated us to introduce the theory of anti fuzzy graphs.

Preliminaries

Definition 1.1: Let S_1 and S_2 be two sets, σ_1 and σ_2 be two fuzzy sets of S_1 and S_2 respectively. That is $\sigma_1 : S_1 \rightarrow [0,1]$ and $\sigma_2 : S_2 \rightarrow [0,1]$. A function $\mu : S_1 \times S_2 \rightarrow [0,1]$ is called a fuzzy relation on $\sigma_1 \times \sigma_2$ if for all x, y $\in S_1 \times S_2$,

$\mu(x,y) \leq \sigma(x) \wedge \sigma(y)$ where $\wedge$ denotes minimum.

Remark1.2:

If S_1 and S_2 are vertex sets of graphs, then μ can be interpreted as weight function on edges joining vertices of S_1 to vertices of S_2. Then the definition of fuzzy graph can be restricted to the case $S_1 = S_2$= S and applied to fuzzy set σ of S. Thus we have the definition of fuzzy graph as given in Rosenfeld [4].

Definition 1.3:

A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$ with $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for all u, v ϵ V where V is a finite nonempty set and $\wedge$ denotes minimum.

Definition 1.4:

Let $G_1 = (\sigma_1, \mu_1)$ on (V_1, E_1) and $G_2 = (\sigma_2, \mu_2)$ on (V_2, E_2) be two fuzzy graphs. Then the union of G_1 and G_2 is defined as

$G = G_1 \cup G_2 = (\sigma_1 \cup \sigma_2,\ \mu_1 \cup \mu_2)$ on (V,E) where $V = V_1 \cup V_2,\ E = E_1 \cup E_2$.

$$(\sigma_1 \cup \sigma_2)(u) = \begin{cases} \sigma_1(u), if u \in V_1 - V_2 \\ \sigma_2(u), if u \in V_2 - V_1 \\ \max\{\sigma_1(u), \sigma_2(u)\}\ if u \epsilon V_1 \cap V_2 \end{cases}$$

and $(\mu_1 \cup \mu_2)(u,v) =$

$$\begin{cases} \mu_1(u,v), if\ (u,v) \in E_1 - E_2 \\ \mu_2(u,v), if\ (u,v) \in E_2 - E_1 \\ \max\{\mu_1(u,v), \mu(u,v)\}\ if\ (u,v)\ \epsilon E_1 \cap E_2 \end{cases}$$

Definition 1.5: Let $G_1 = (\sigma_1, \mu_1)$ on (V_1, E_1) and $G_2 = (\sigma_2, \mu_2)$ on (V_2, E_2) be two fuzzy graphs. Let V = $V_1 \cup V_2$ and E = $E_1 \cup E_2 \cup E'$ where E' is the set of edges joining of the nodes of $V_1 and V_2$. Assume that $V_1 \cap V_2 \neq \emptyset$. Then the join of $G_1 and G_2$ is $G = (\sigma, \mu) = (\sigma_1 + \sigma_2, \mu_1 + \mu_2)$ on (V,E) is defined as

$(\sigma_1 + \sigma_2)(u) = (\sigma_1 \cup \sigma_2)(u)$ for all u $\in V_1 \cup V_2 and (\mu_1 + \mu_2)(u,v) =$

$$\begin{cases} (\mu_1 \cup \mu_2)(\mathrm{u,v}), \text{if } (\mathrm{u,v}) \epsilon E_1 \cup E_2 \\ \min\{\sigma_1(u) \vee \sigma_2(v)\}, if\ (\mathrm{u,v}) \epsilon E'. \end{cases}$$

Definition 1.6:

Let $G = (\sigma, \mu)$on (V,E) be a fuzzy graph. Then the complement $\bar{G}$ of $G$is defined as $\bar{G} = (\bar{\sigma}, \bar{\mu})$where $\bar{\sigma} = \sigma$ and $\bar{\mu}$ (u,v) = $(\sigma(u) \wedge \sigma(v))$ - $\mu(u,v)$ for all (u,v) $\in$ E.

2.Anti Fuzzy Graphs: In this section, we introduce the notion of anti fuzzy graphs and strong anti fuzzy graphs.

Definition 1.1:

Let S_1 and S_2 be two sets, σ_1 and σ_2 be two fuzzy sets of S_1 and S_2 respectively. That is $\sigma_1 : S_1 \rightarrow [0,1]$ and $\sigma_2 : S_2 \rightarrow [0,1]$. A function $\mu : S_1 \times S_2 \rightarrow [0,1]$ is called an anti fuzzy relation on $\sigma_1 \times \sigma_2$ if for all x, y $\in S_1 \times S_2$,

$\mu(x,y) \geq \sigma(x) \vee \sigma(y)$where V denote maximum.

Remark 2.2:

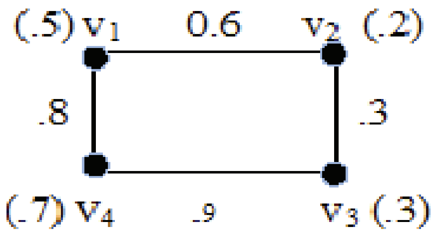
If S_1 and S_2 are vertex sets of graphs, then μ can be interpreted as weight function on edges joining vertices of S_1 to vertices of S_2. Then the definition of anti fuzzy graph can be restricted to the case $S_1 = S_2$= S and applied to fuzzy set σ of S. Thus we have the definition of anti fuzzy graph as follows.

Definition 2.3:

An antifuzzy graph $\mathcal{A} = (\sigma, \mu)$ is a pair of functions $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$ with $\mu(u,v) \geq \sigma(u) \vee \sigma(v)$for all u,v$\in$ V where V is a finite nonempty set and V denote maximum.

Example 2.4: Let V = $\{v_1, v_2, v_3, v_4\}$. Define $\sigma : V \rightarrow [0,1]$ by $\sigma(v_1)$ = 0.5,

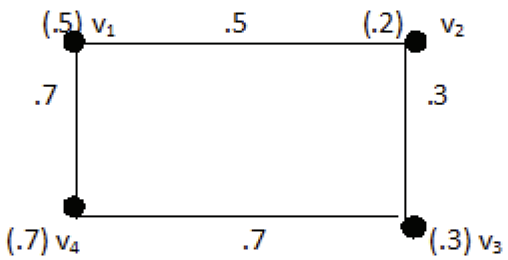
$\sigma(v_2) = 0.2$, $\sigma(v_3) = 0.3$ and $\sigma(v_4) = 0.7$. $\mu : V \times V \rightarrow [0,1]$ by $\mu(v_1, v_2) = 0.6, \mu(v_2, v_3) = 0.3, \mu(v_3, v_4) = 0.9$, $\mu(v_1, v_4) = 0.8$. Then the anti fuzzy graph $\mathcal{A} = (\sigma, \mu)$ can be represented as



Definition 2.5 :
The graph $\mathcal{A}^* = (V, E)$ is called the underlying crisp graph of the anti fuzzy graph $\mathcal{A}$ where $V = \{u|\sigma(u) \neq 0\}$ and E = $\{(u,v) \in V\times V | \mu(u, v) \neq 0\}$.

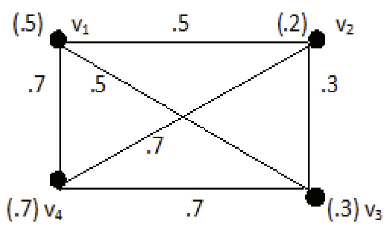
Definition 2.6:
An anti fuzzy graph $\mathcal{A} = (\sigma, \mu)$ is said to be strong if $\mu(u, v) = \sigma(u) \vee \sigma(v)$ for all (u, v) ∊ E.

Example 2.7: The graph given below is a strong anti fuzzy graph.



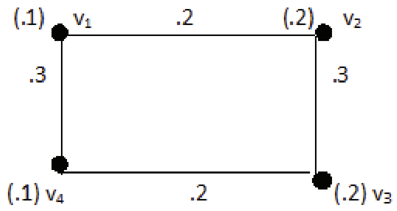
Definition 2.8: An anti fuzzy graph $\mathcal{A} = (\sigma, \mu)$ is said to be complete if the underlying graph $\mathcal{A}^*$ is complete and $\mu(u, v) = \sigma(u) \vee \sigma(v)$ for all u, v ∊ E.

Example 2.9: The graph given below is a complete anti fuzzy graph.



Definition 2.10: Let $\mathcal{A} = (\sigma, \mu)$ be an anti fuzzy graph on (V,E), the anti fuzzy degree of a node u ∊ V is defined as ($\mathcal{A}$ f d) (u) = $\sum_{v \neq u and v \in V} \mu(u, v)$, $\mathcal{A}$ is said to be a regular anti fuzzy graph if $(\mathcal{A} fd)(v) = k$ for all v∊V where k is a real constant. Then we say that $\mathcal{A}$ is k-regular anti fuzzy graph.

Example 2.11: The graph given below is 0.5 regular anti fuzzy graph.



3. Some operations on anti fuzzy graphs:

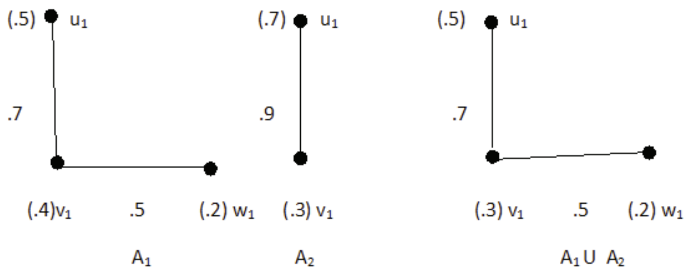
Definition 3.1:
Let $\mathcal{A}_1 = (\sigma_1, \mu_1)$ on (V_1, E_1) and $\mathcal{A}_2 = (\sigma_2, \mu_2)$ on (V_2, E_2) be two anti fuzzy graphs. Then the union of $\mathcal{A}_1$ and $\mathcal{A}_2$ is defined as
$\mathcal{A} = \mathcal{A}_1 \cup \mathcal{A}_2 = (\sigma_1 \cup \sigma_2, \ \mu_1 \cup \mu_2)$ on (V,E) where V $= V_1 \cup V_2$, $E = E_1 \cup E_2$,

$$(\sigma_1 \cup \sigma_2)(u) = \begin{cases} \sigma_1(u), if u \in V_1 - V_2 \\ \sigma_2(u), if u \in V_2 - V_1 \\ \min\{\sigma_1(u), \sigma_2(u)\} \ if u \epsilon V_1 \cap V_2 \end{cases}$$

and $(\mu_1 \cup \mu_2)(u, v) =$

$$\begin{cases} \mu_1(u, v), if \ (u, v) \in E_1 - E_2 \\ \mu_2(u, v), if \ (u, v) \in E_2 - E_1 \\ \min\{\mu_1(u, v), \mu_2(u, v)\} \ if \ (u, v) \ \epsilon E_1 \cap E_2 \end{cases}$$

Example 3.2:



Theorem 3.3: The union of two anti fuzzy graphs is again an anti fuzzy graph.

Proof:
Let $\mathcal{A}_1 = (\sigma_1, \mu_1)$ on (V_1, E_1) and $\mathcal{A}_2 = (\sigma_2, \mu_2)$ on (V_2, E_2) be two anti fuzzy graphs.
Let $\mathcal{A} = \mathcal{A}_1 \cup \mathcal{A}_2 = (\sigma_1 \cup \sigma_2, \ \mu_1 \cup \mu_2)$ on (V,E) where V = $V_1 \cup V_2 and E = E_1 \cup E_2$.
Case (I)
Let (u, v) ∈ $E_1 - E_2$.
Sub case (i)
Let u, v $\in V_1 - V_2$. Then
$(\sigma_1 \cup \sigma_2)(u) = \sigma_1(u) \ and \ (\sigma_1 \cup \sigma_2)(v) = \sigma_1(v)$
Now,
$(\mu_1 \cup \mu_2)$(u,v) = $\mu_1(u, v), since (u, v) \in E_1 - E_2$
$\geq \sigma_1(u) \vee \sigma_2(v)$
$= (\sigma_1 \cup \sigma_2)(u) \vee (\sigma_1 \cup \sigma_2)(v)$
Sub case (ii)
Let u ∈ $V_1 - V_2$, $v \epsilon V_1 \cap V_2$.
Therefore $(\sigma_1 \cup \sigma_2)(u) = \sigma_1(u)$ &
$(\sigma_1 \cup \sigma_2)(v) = \sigma_1(v) \wedge \sigma_2(v)$

Now,
$(\mu_1 \cup \mu_2)$(u,v) = $\mu_1(u,v)$, $since(u,v) \in E_1 - E_2$
$\geq \sigma_1(u) \vee \sigma_2(v)$
$\geq (\sigma_1 \cup \sigma_2)(u) \vee (\sigma_1 \cup \sigma_2)(v)$
since$\sigma_1(v) \geq (\sigma_1 \cup \sigma_2)(v)$
Sub case (iii)
Let u, v $\in V_1 \cap V_2$.
Therefore $(\sigma_1 \cup \sigma_2)(u) = \sigma_1(u) \wedge \sigma_2(u)$ &$(\sigma_1 \cup \sigma_2)(v) = \sigma_1(v) \wedge \sigma_2(v)$
Now,
$(\mu_1 \cup \mu_2)$(u,v) = $\mu_1(u,v), since(u,v) \in E_1 - E_2$
$\geq \sigma_1(u) \vee \sigma_1(v)$
$\geq (\sigma_1(u) \wedge \sigma_2(u)) \vee (\sigma_1(v) \wedge \sigma_2(v))$
since$\sigma_1(u) \geq \sigma_1(u) \wedge \sigma_2(u)$
$\sigma_1(v) \geq \sigma_1(v) \wedge \sigma_2(v)$
$= (\sigma_1 \cup \sigma_2)(u) \vee (\sigma_1 \cup \sigma_2)(v)$ since u, v $\in V_1 \cap V_2$
Thus $(\mu_1 \cup \mu_2)$(u,v) $\geq (\sigma_1 \cup \sigma_2)(u) \vee (\sigma_1 \cup \sigma_2)(v).forall(u,v) \in E_1 - E_2$
Case (II)
Let (u,v) $\in E_2 - E_1$.
As in case (i) we arrive at
$(\mu_1 \cup \mu_2)$(u,v)
$\geq (\sigma_1 \cup \sigma_2)(u) \vee (\sigma_1 \cup \sigma_2)(v)$
for all $(u,v) \in E_2 - E_1$
Case (III)
Let (u,v) $\in E_1 \cap E_2 \Rightarrow$ u, v $\in V_1 \cap V_2$.
Therefore $(\sigma_1 \cup \sigma_2)(u) = \sigma_1(u) \wedge \sigma_2(u)$ &$(\sigma_1 \cup \sigma_2)(v) = \sigma_1(v) \wedge \sigma_2(v)$
Then $(\mu_1 \cup \mu_2)$(u,v) = $\mu_1(u,v) \wedge \mu_2(u,v)$
$\geq (\sigma_1(u) \vee \sigma_1(v)) \wedge (\sigma_2(u) \vee \sigma_2(v))$
$\geq (\sigma_1(u) \wedge \sigma_2(u)) \vee (\sigma_1(v) \wedge \sigma_2(v))$
From the distributive law,
$(\sigma \vee \tau) \wedge \vartheta = (\sigma \wedge \tau) \vee (\tau \wedge \vartheta)$
$= (\sigma_1 \cup \sigma_2)(u) \vee (\sigma_1 \cup \sigma_2)(v)$
Hence$\mathcal{A} = (\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2)$ is also an anti fuzzy graph.

Theorem 3.4:
If $\mathcal{A}$is the union of two anti fuzzzy graphs $\mathcal{A}_1 = (\sigma_1, \mu_1)$ on (V_1, E_1) and $\mathcal{A}_2 = (\sigma_2, \mu_2)$ on (V_2, E_2), then every anti fuzzy subgraph (σ, μ) is union of anti fuzzy subgraph of $\mathcal{A}_1$and $\mathcal{A}_2$.

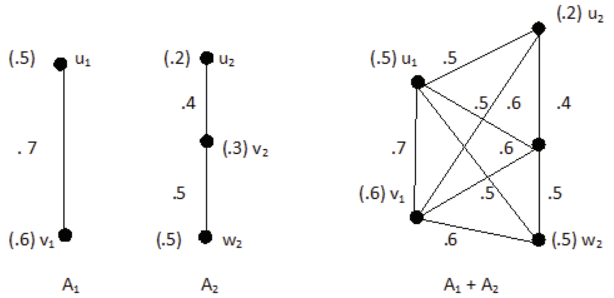
Proof:
Now, $\sigma_i(u) = \sigma(u)$ for all u $\in V_i$, i = 1,2 and $\mu_i(u,v) = \mu(u,v)$
if (u,v) $\in E_i$, i = 1,2
Then $\mu_i(u_i, v_i) = \mu(u_i, v_i)$
$\geq$ max{ $\sigma(u_i), \sigma(v_i)$}
= max{ $\sigma_i(u_i), \sigma_i(v_i)$} for all $(u_i, v_i) \in E_i$, i = 1,2
Thus (σ_i, μ_i) is an anti fuzzy subgraph of $\mathcal{A}_i$, i = 1,2.
Clearly $\sigma = \sigma_1 \cup \sigma_2$ and $\mu = \mu_1 \cup \mu_2$.

Theorem 3.5:
The union $\mathcal{A}_1 \cup \mathcal{A}_2$ of two strong anti fuzzy graphs $\mathcal{A}_1$and $\mathcal{A}_2$is again a strong anti fuzzy graph when$\mathcal{A}_1 \cap \mathcal{A}_2 = \emptyset$.

Proof:
Let $\mathcal{A}_1 = (\sigma_1, \mu_1)$ on (V_1, E_1) and $\mathcal{A}_2 = (\sigma_2, \mu_2)$ on (V_2, E_2)be two disjoint strong anti fuzzy graphs.
Then$\mu_1(u,v) = (\sigma_1(u) \vee \sigma_1(v))$ and $\mu_2(u,v) = (\sigma_2(u) \vee \sigma_2(v))$ $for\ all\ u\ \in V$
Let $\mathcal{A} = (\sigma_1 \cup \sigma_2, \mu_1 \cup \mu_2)$ on V = $V_1 \cup V_2$ and $E = E_1 \cup E_2$ be the union of $\mathcal{A}_1$and $\mathcal{A}_2$.
Let (u, v) $\in$ V.
Case (i)
Let (u, v) $\in V_1 - V_2$.
Then $(\sigma_1 \cup \sigma_2)(u) = \sigma_1(u)$ and $(\sigma_1 \cup \sigma_2)(v) = \sigma_1(v)$
Therefore
$(\mu_1 \cup \mu_2)$(u,v) = $\mu_1(u,v)$
= $\sigma_1(u) \vee \sigma_1(v)$ since $\mathcal{A}_1$is strong
= $(\sigma_1 \cup \sigma_2)$(u) $\vee$ $(\sigma_1 \cup \sigma_2)$(v)
Case (ii)
Let (u, v) $\in V_2 - V_1$.
Then $(\sigma_1 \cup \sigma_2)(u) = \sigma_2(u)$ and $(\sigma_1 \cup \sigma_2)(v) = \sigma_2(v)$
Therefore $(\mu_1 \cup \mu_2)$(u,v) = $\mu_2(u,v)$
= $\sigma_2(u) \vee \sigma_2(v)$ since $\mathcal{A}_2$is strong
= $(\sigma_1 \cup \sigma_2)$(u) $\vee$ $(\sigma_1 \cup \sigma_2)$(v)
Hence $\mathcal{A}_1 \cup \mathcal{A}_2$ is also strong anti fuzzy graph.

Definition 3.6:
Let $\mathcal{A}_1 = (\sigma_1, \mu_1)$ on (V_1, E_1) and $\mathcal{A}_2 = (\sigma_2, \mu_2)$ on (V_2, E_2) be two anti fuzzy graphs. Let V = $V_1 \cup V_2$ and E = $E_1 \cup E_2 \cup E'$ where E' is the set of edges joining of the nodes of V_1 and V_2. Assume that $V_1 \cap V_2 \neq \emptyset$. Then the join of $\mathcal{A}_1$ and $\mathcal{A}_2$ is
$\mathcal{A} = (\sigma, \mu) = (\sigma_1 + \sigma_2, \mu_1 + \mu_2)$ on (V,E) is defined as$(\sigma_1 + \sigma_2)(u) = (\sigma_1 \cup \sigma_2)(u)$ for all u $\in V_1 \cup V_2$ and
$(\mu_1 + \mu_2)\ (u,v) = \begin{cases} (\mu_1 \cup \mu_2)(u,v), \text{if } (u,v) \in E_1 \cup E_2 \\ \max\{\sigma_1(u), \sigma_2(v)\}, if\ (u,v) \in E'. \end{cases}$

Example 3.7:



Theorem 3.8:
Join of two anti fuzzy graphs is an anti fuzzy graph.

Proof:
Let $\mathcal{A}_1 = (\sigma_1, \mu_1)$ on (V_1, E_1) and $\mathcal{A}_2 = (\sigma_2, \mu_2)$ on (V_2, E_2) be two anti fuzzy graphs.
Let $\mathcal{A} = (\sigma_1 + \sigma_2, \mu_1 + \mu_2)$ on (V,E) be the join of $\mathcal{A}_1$ and $\mathcal{A}_2$.
Case(I)
Let (u, v) $\in E_1 \cup E_2$.
Then
$(\mu_1 + \mu_2)\ (u,v)$
$\geq (\sigma_1 + \sigma_2)$(u) $\vee$ $(\sigma_1 + \sigma_2)$(v)

follows from the theorem 3.3

Case (II)

Let (u, v) $\in E'$

Then $(\mu_1 + \mu_2)\ (u,v) = \max\{\sigma_1(u), \sigma_2(v)\}$

From the definition of join

$= \max\{ (\sigma_1 \cup \sigma_2)(u), (\sigma_1 \cup \sigma_2)(v)\}$

since u $\in V_1$, v $\in V_2$

$= \max\{ (\sigma_1 + \sigma_2)(u), (\sigma_1 + \sigma_2)(v)\}$

Hence $\mathcal{A}$ is also an anti fuzzy graph.

Theorem 3.9 :

If $\mathcal{A}$ is the join of two anti fuzzy subgraphs $\mathcal{A}_1$ *and* $\mathcal{A}_2$, then every strong anti fuzzy subgraph (σ, μ) of $\mathcal{A}$ is a join of a strong anti fuzzy subgraph of $\mathcal{A}_1$ and a strong anti fuzzy subgraph of $\mathcal{A}_{2.}$

Proof:

Define $\sigma_1, \mu_1, \sigma_2, \mu_2$ on V_1, E_1, V_2, E_2 respectively as follows:

$\sigma_i(u) = \sigma(u), for\ all$ u $\in V_i, i = 1,2$ and $\mu_i(u,v) = \mu(u,v)$, for all (u,v) $\in E_i, i = 1,2$

Then $\sigma = \sigma_1 \cup \sigma_2$as in theorem 2 and hence $\sigma = \sigma_1 + \sigma_2$.

If (u,v) $\in E_1 \cup E_2$, then

$\mu(u,v) = (\mu_1 \cup \mu_2)(u,v)$

$= (\mu_1 + \mu_2)\ (u,v)$ from definition of join.

If (u,v) $\in E'$, then u $\in V_1$ and v $\in V_2$

Now $(\mu_1 + \mu_2)\ (u,v) = \max\{\sigma_1(u), \sigma_2(v)\}$

$= \max\{ \sigma(u), \sigma(v)\}$

$= \mu(u,v)$since

(σ, μ) *is strong*.

Theorem 3.10:

Join of two strong anti fuzzy graphs is also a strong anti fuzzy graph.

Proof:

Let $\mathcal{A}_1 = (\sigma_1, \mu_1)$ on (V_1, E_1) and $\mathcal{A}_2 = (\sigma_2, \mu_2)$ on (V_2, E_2) be two strong anti fuzzy graphs.

Let $\mathcal{A}_1 + \mathcal{A}_2 = (\sigma_1 + \sigma_2, \mu_1 + \mu_2)$ on (V,E) be the join of $\mathcal{A}_1$ *and* $\mathcal{A}_2$.

Case (I):

Let (u,v) $\in E_1 \cup E_{2.}$

Sub case (i):

Let (u,v) $\in E_1 - E_2$

$(\mu_1 + \mu_2)(u,v) = (\mu_1 \cup \mu_2)(u,v)$ from the definition of join

$= \mu_1(u,v)$

$= \sigma_1(u) \vee \sigma_1(v)$

$= (\sigma_1 \cup \sigma_2)(u) \vee (\sigma_1 \cup \sigma_2)(v)$

$= (\sigma_1 + \sigma_2)(u) \vee (\sigma_1 + \sigma_2)(v)$

Subcase (ii)

Let (u,v) $\in E_2 - E_1$

$(\mu_1 + \mu_2)(u,v) = (\mu_2 \cup \mu_1)(u,v)$

$= \mu_2(u,v)$

$= \sigma_2(u) \vee \sigma_2(v)$

$= (\sigma_1 \cup \sigma_2)(u) \vee (\sigma_1 \cup \sigma_2)(v)$

$= (\sigma_1 + \sigma_2)(u) \vee (\sigma_1 + \sigma_2)(v)$

Case(II)

Let (u,v) $\in E'$

$(\mu_1 + \mu_2)\ (u,v) = \max\{\sigma_1(u), \sigma_2(v)\}$

$= \max\{(\sigma_1 \cup \sigma_2)(u), (\sigma_1 \cup \sigma_2)(v)\}$

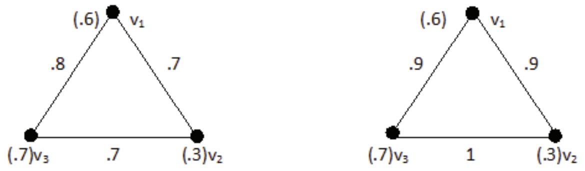
$= \max\{(\sigma_1 + \sigma_2)(u), (\sigma_1 + \sigma_2)(v)\}$

Hence $\mathcal{A}_1 + \mathcal{A}_2$ is also strong anti fuzzy graph.

Definition 3.11:

Let $\mathcal{A} = (\sigma, \mu)$on (V,E) be an anti fuzzy graph. Then the complement of $\bar{\mathcal{A}}$ of $\mathcal{A}$ is defined as $\bar{\mathcal{A}} = (\bar{\sigma}, \bar{\mu})$where $\bar{\sigma} = \sigma$ and $\bar{\mu}$ (u,v) = 1- $\mu(u,v) + (\sigma(u) \vee \sigma(v))$ for all (u,v) $\in$ E.

Example 3.12:



A $\bar{A}$

Theorem 3.13:

Complement of an anti fuzzy graph is an anti fuzzy graph.

Proof:

Let $\mathcal{A} = (\sigma, \mu)$on (V,E) be an anti fuzzy graph and $\bar{\mathcal{A}} = (\bar{\sigma}, \bar{\mu})$ *be its complement*.

Since 1-$\mu(u,v) \geq 0$ and $\sigma(u) \vee \sigma(v) \geq 0$, $\bar{\mu}$(u,v) $\geq$ 0.

$\bar{\mu}(u,v) = 1 - \mu(u,v) + \sigma(u) \vee \sigma(v)$

$\leq$ 1- $(\sigma(u) \vee \sigma(v)) + (\sigma(u) \vee \sigma(v))$

since $\mu(u,v) \geq \sigma(u) \vee \sigma(v)$

= 1 $\quad -\mu(u,v) \leq -(\sigma(u) \vee \sigma(v))$

$\bar{\mu}(u,v) = 1 - \mu(u,v) + (\sigma(u) \vee \sigma(v))$

= 1- $\mu(u,v) + (\bar{\sigma}(u) \vee \bar{\sigma}(v))$,

since $\sigma = \bar{\sigma}$

$\geq \bar{\sigma}(u) \vee \bar{\sigma}(v)$

Thus $\bar{\mathcal{A}}$ is also an anti fuzzy graph

Theorem 3.14 :

If $\mathcal{A}$ is a complete anti fuzzy graph, then its complement $\bar{\mathcal{A}}$ is regular.

Proof:

Let $\mathcal{A} = (\sigma, \mu)$on (V,E) be a complete anti fuzzy graph.

Therefore $\mu(u,v) = (\sigma(u) \vee \sigma(v))$ for all (u,v) $\in$ E.

Now,

$\bar{\mu}$ (u,v) = 1- $\mu(u,v) + (\sigma(u) \vee \sigma(v))$

= 1- $\mu(u,v) + \mu(u,v)$

= 1 for all (u,v) $\in$ E.

Therefore $\sum_{v \neq u\ and\ v \in V} \bar{\mu}(u,v) = Constant$, for all u $\in$ V, since underlying graph of $\mathcal{A}$ is complete.

Therefore $\bar{\mathcal{A}}$ is regular anti fuzzy graph.

Theorem 3.15:

$\bar{\bar{\mathcal{A}}} = \mathcal{A}$ for any anti fuzzy graph $\mathcal{A}$.

Proof:

Let $\mathcal{A} = (\sigma, \mu)$ on (V,E) be any anti fuzzy graph. Then $\sigma = \bar{\sigma} = \bar{\bar{\sigma}}$.

Let (u,v) $\in$ E.

Now,

$\bar{\mu}$ (u,v) = 1- $\mu(u,v) + (\sigma(u) \vee \sigma(v))$

$\bar{\bar{\mu}}(u,v) = 1 - \bar{\mu}(u,v) + (\bar{\sigma}(u) \vee \bar{\sigma}(v))$

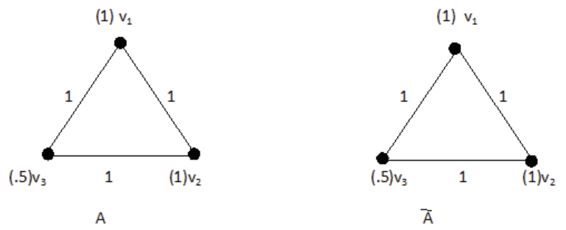
= 1- (1- $\mu(u,v) + (\sigma(u) \vee \sigma(v))) +$
$(\bar{\sigma}(u) \vee \bar{\sigma}(v))$
= $\mu(u,v)$ since $\sigma = \bar{\sigma}$
Thus $\bar{\bar{\mathcal{A}}} = \mathcal{A}$.

Theorem 3.16:
$\mathcal{A} \cup \bar{\mathcal{A}} = \bar{\mathcal{A}}$, if $\mathcal{A}$ is any complete anti fuzzy graph.

Proof:
Let $\mathcal{A} = (\sigma, \mu)$ on (V,E) be a complete anti fuzzy graph.
Let $\bar{\mathcal{A}} = (\bar{\sigma}, \bar{\mu})$ *be its complement.*
Let $\mathcal{A} \cup \bar{\mathcal{A}} = (\sigma_1, \mu_1)$.
Then $\sigma = \bar{\sigma} = \sigma_1$
From the definition of union,
$\mu_1(u,v)$ = max { $\mu(u,v), \bar{\mu}$ (u,v)}
= 1 since $\mu(u,v) = 1$,
for all (u,v) ∈ E.
= $\bar{\mu}$ (u,v)
Therefore $\mathcal{A} \cup \bar{\mathcal{A}} = \mathcal{A}$.

Definition 3.17 :
An anti fuzzy graph is self complementary, if $\mathcal{A} = \bar{\mathcal{A}}$.

Example 3.18 :



Theorem 3.19:
Let $\mathcal{A} = (\sigma, \mu)$ on (V,E) be an anti fuzzy graph. $\mathcal{A}$ is self complementary iff
$\mu(u,v) = \frac{1}{2}(1+ \sigma(u) \vee \sigma(v))$.

Proof:
Let $\mathcal{A} = (\sigma, \mu)$ on (V,E) be self complementary anti fuzzy graph. Therefore $\mathcal{A} = \bar{\mathcal{A}}$.
Then $\sigma = \bar{\sigma}$ *and* $\mu = \bar{\mu}$.
Let (u,v) ∈ E.
Thus
μ (u,v) = 1- $\mu(u,v) + (\sigma(u) \vee \sigma(v))$
2 μ (u,v) = 1 $+(\sigma(u) \vee \sigma(v))$
$\mu(u,v) = \frac{1}{2}(1+ \sigma(u) \vee \sigma(v))$,
for all (u,v) ∈ E.
Reversing the steps, converse holds.

Theorem 3.20:
If $\mathcal{A}$ is any anti fuzzy graph, then the underlying graph of $\bar{\mathcal{A}}$ is complete.

Proof:
Let $\mathcal{A} = (\sigma, \mu)$ on (V,E) be an anti fuzzy graph.
Now, $\bar{\mathcal{A}} = (\bar{\sigma}, \bar{\mu})$ where $\sigma = \bar{\sigma}$ and
$\bar{\mu}$ (u,v) = 1- $\mu(u,v) + (\sigma(u) \vee \sigma(v))$
When $\mu(u,v) \neq 0$, we have $\mu(u,v) > 0$, for all (u,v) ∈ V.
Therefore $\bar{\mu}$ (u,v) >0, whenever $\mu(u,v) \neq 0$.
When $\mu(u,v) = 0$, we have
$\bar{\mu}$ (u,v) = 1- $(\sigma(u) \vee \sigma(v)) > 0$
Therefore $\bar{\mu}$ (u,v) > 0 whenever $\mu(u,v) = 0$, for all (u,v) ∈ V.
Hence underlying graph of $\bar{\mathcal{A}}$ is complete.
Note: In case of fuzzy graph, the above result is not true

References:

1. Dr.S.Rose Mary, Decomposition of (1,2)* Ar-Continuous Function; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Issue 2 (2014), Pg 753-756
2. Morderson J.N.,Nair P.S, Fuzzy graphs and Fuzzy hypergraphs Physics – verlag Heidelberg 2000.
3. Navin Gulati, Rashi Bansal, Coefficient inequality for A Newly Constructed Subclass of Class of Starlike Analytic Functions; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 4 Issue 1 (2015), Pg 256-258
4. Nagoorkani A., Chandrasekaran V.T., A first look at Fuzzy graph theory 2010.
5. G.V.Sarada Devi, Duality for A Class of Non Smooth Generalised; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 3 Issue 1 (2014), Pg 485-487
6. Nagoorkani A., Radha K., On regular fuzzy graphs, Journal of Physical Sciences, Vol 12, 33- 40 (2008).
7. Sunita Chinmalli, Efficient Triple Connected Edge Domination Number Of A Graph; Mathematical Sciences International Research Journal : ISSN 2278-8697Volume 4 Issue 2 (2015), Pg 272-275
8. Rosenfeld A., Fuzzygraph, In Zadeh L.A., Shimura M.(Eds)., Fuzzy sets and their applications pp:77-95,1975 (Academic Press, Newyork).
9. K. Kavitha , N.G. David, Dominator Coloring of Vertex Switching of Graphs; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 3 Issue 1 (2014), Pg 494-498
10. Sunitha M.S. and Vijayakumar A., Complement of a Fuzzygraph, Indian Journal of Pure and Applied mathematics.
11. Renukadevi S Dyavanal , Rajalaxmi V Desai, Uniqueness Of -Difference And Differential Polynomials Of Entire Functions; Mathematical Sciences International Research Journal : ISSN 2278-8697Volume 4 Issue 2 (2015), Pg 267-271

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