# ENUMERATION OF EDGE-GRACEFUL LABELINGS OF $K_3, K_4, K_5$

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**Abstract:** In this paper, we enumerate the distinct edge-graceful labelings of $K_3, K_4, K_5$

**Keywords:** Edge-graceful labeling, edge label, vertex label.

**Introduction:** In 1985, Lo [3] introduced the concept of edge-graceful graphs. A (p,q)-graph G is said to be edge-graceful if there exists a bijection such that the induced mapping defined by is also a bijection.A detailed list of edge-graceful graphs is given in Gallian Survey of Graph Labelings [2].

For a connected edge-graceful (p,q)-graph with , where and are integers, and , we have proved that every edge-graceful labeling of induces distinct edge-graceful labelings of [1].Applying this result, in this paper, we enumerate the distinct edge-graceful labelings of .

Known Results:Result 1[1]: Let f be an edge-graceful labeling of a connected (p,q)-graph G with , where and k is an integer with . Let and be the set of all permutations defined on the sets and respectively. For any p-tuple where and , define a labeling such that = if where if and if Then is also an edge-graceful labeling of G.

Result 2[1]: Let G be a edge-graceful connected (p,q)-graph with , where , and are integers. Then every edge-graceful labeling of G induces distinct edge-graceful labelings of G.

Enumeration of edge-graceful labelings: Let f be an edge-graceful labeling of a connected (p,q)-graph G with , where and k and r are integers with . In Result 1, the labeling induced by satisfies the relation for any edge e.

Result 2 is proved by enumerating the possible p-tubles mentioned in Result 1. We shall consider edge-graceful labelings induced by a single edge-graceful labeling f as a single category. Thus any edge-graceful labeling g with for every edge e, belongs to the same category containing f.

Hence, to enumerate edge-graceful labelings of $G$, it is enough to enumerate the categories and multiply by $[(k+1)!]^r.[k!]^{p-r}$.

**Main Results:**

**Theorem 1:** $K_3$ *has unique edge-graceful labeling and* $K_4$ *has 8 edge-graceful labelings.*

**Proof:**Clearly $K_3$ has the unique edge-graceful labeling in Fig.1. For the notational convenience, the edge labels $e_1, e_2, e_3, e_4, e_5, e_6$ and the vertex labels 0, 1, 2, 3 of $K_4$ are marked as in Fig.1.

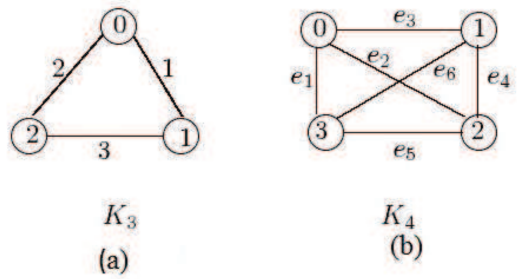


Fig . 1

Then we have the following system of linear congruences:

$e_1 + e_2 + e_3 \equiv 0$ (mod 4)(1)

$e_3 + e_6 + e_4 \equiv 1$ (mod4)

i.e. $e_4 + e_6 \equiv 1 - e_3$ (mod 4) (2)

$e_2 + e_4 + e_5 \equiv 2$ (mod 4)

i.e. $e_4 + e_5 \equiv 2 - e_2$ (mod 4)(3)

$e_1 + e_5 + e_6 \equiv 3$ (mod 4)

i.e. $e_5 + e_6 \equiv 3 - e_1$ (mod 4), (4)

where $1 \le e_i \le 6$ and $e_i \ne e_j$ for all $i \ne j$, $1 \le i, j \le 6$.

The number of distinct edge-graceful labelings of $K_4$ is the number of solutions of the above system of congruences and so our aim is to count the number of distinct solutions of the above system.
We shall partition the edge-graceful labelings into disjoint categories as follows:
First, we partition the set of all edge-graceful labelings of $K_4$ into disjoint categories according to the values in $\{e_1, e_2, e_3\}$ alone.
Next we further partition these categories according to $\{e_1, e_2, e_3, e_4, e_5, e_6\}$.
It is easy to note that the possible solutions of (1) that correspond to distinct categories are $A : \{e_1, e_2, e_3\} = \{1,3,4\}$, $B : \{e_1, e_2, e_3\} = \{1,2,5\}$, $C : \{e_1, e_2, e_3\} = \{2,4,6\}$. (For example,the solution $\{e_1, e_2, e_3\} = \{3,4,5\}$ is of category $A$ ).Adding all the congruences, we get $2\sum_{i=1}^{6} e_i \equiv 2$ (mod 4).

Then $\sum_{i=1}^{6} e_i \equiv 1 \, or \, 3$ (mod 4).

Now, using (1), $\sum_{i=4}^{6} e_i \equiv 1 \, or \, 3$ (mod 4).

**Case 1:** $\sum_{i=4}^{6} e_i \equiv 1$ (mod 4)

Then, from (2), (3) and (4), $e_5 \equiv e_3$ (mod 4), $e_6 \equiv e_2 - 1$ (mod 4) and $e_4 \equiv e_1 - 2$ (mod 4) .
In category *A*, the 6 possible values of $(e_1, e_2, e_3)$ are $A_1$ : **(1, 3, 4)** , $A_2$ : **(1, 4, 3)**, $A_3$ : **(3, 1, 4)**, $A_4$ : **(3, 4, 1)**, $A_5$ : **(4, 1, 3)**, $A_6$ : **(4, 3, 1)**.
In all the cases, $\{e_4, e_5, e_6\} = \{2,5,6\}$.
Since $e_5 \equiv e_3$ (mod 4),except in $A_4$ and $A_6$, we get a contradiction in all the other cases,.
$A_4 : (e_1, e_2, e_3) = (3,4,1)$
Now $e_4 \equiv 1 \equiv e_5 \equiv e_3$ (mod 4), a contradiction.
$A_6 : (e_1, e_2, e_3) = (4,3,1)$
Now $e_4 \equiv 2$ (mod 4), $e_5 \equiv 1$ (mod 4), $e_6 \equiv 2$ (mod 4),and so $(e_4, e_5, e_6) = (2,5,6)$ or $(6,5,2)$. Since $2 \equiv 6$, they are of same category; and so we can consider it as a same category with labeling **(4, 3, 1, 2, 5, 6).**Hence, there is only **one category of edge-graceful labeling in type A**.
In category B= $\{1,2,5\}$, $1 \equiv 5$ (mod 4), and so the six permutations can be further classified as follows: $B_1$ : **(1, 2, 5)**, (5, 2, 1); $B_2$ : **(2, 1, 5)**, (2, 5, 1); $B_3$ : **(1, 5, 2)**, (5, 1, 2).
We consider the highlighted cases only.
Now $\{e_4, e_5, e_6\} = \{3,4,6\}$.Since $e_5 \equiv e_3 \, (mod \, 4)$, $B_1$ and $B_2$ lead to contradiction.
$B_3 : (e_1, e_2, e_3) = (1,5,2)$
Now $e_4 \equiv 3$ (mod 4), $e_5 \equiv 2$ (mod 4), $e_6 \equiv 0$ (mod 4), and so $e_4$ = 3, $e_5$ = 6, $e_6$ = 4. Hence the edge-graceful labeling is **(1, 5, 2, 3, 6, 4).**
Hence there is only **one category of edge-graceful labeling in type** $B$ **.**
Since $e_5 \equiv e_3$ (mod 4), there is no edge-graceful labeling in type $C$ .
Thus, there are two categories of edge-graceful labelings in case 1.

**Case 2:** $\sum_{i=4}^{6} e_i \equiv 3$ (mod 4)

From (2), (3) and (4), $e_4 \equiv e_1$ (mod 4), $e_5 \equiv 2 + e_3$ (mod 4) and $e_6 \equiv 1 + e_2$ (mod 4). Since $e_4 \equiv e_1$ (mod 4), there is no solution when $e_1 \geq 3$ or $e_1 \equiv e_2 \, (mod \, 4)$ or $e_1 \equiv e_3$ (mod 4). Hence the only possible cases are $A_1$ :**(1, 3, 4),**

$A_2$ :**(1, 4, 3) and** $B_2$ : **(2,1,5)**. But inall these cases there is no solution. Hence in $K_4$, there are two categories of edge-graceful labeling. In $K_4$, $p=4$, $q=6=1\times4+2$. By Result 2, each edge-graceful labeling of $K_4$ induces four edge-graceful labeling. Hence there are $2\times4=8$ distinct edge-graceful labelings in $K_4$.

**Theorem 2:** $K_5$ *has 6560 distinct edge-graceful labelings.*

**Proof:**For the notational convenience, the edge labels $e_1$, $e_2$, $e_3$,..., $e_{10}$ and the vertex labels 0, 1, 2, 3, 4 of $K_5$ are marked as in Fig.2.

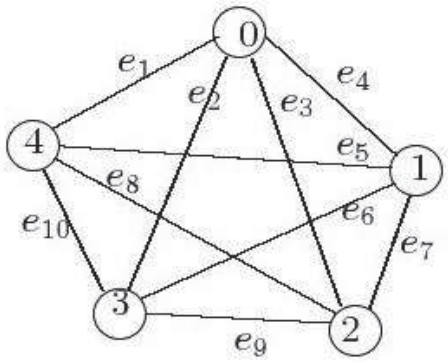


Fig.2

We have the following system of linear congruences:

$e_1+e_2+e_3+e_4\equiv0$ (mod 5)(1)

$e_4+e_5+e_6+e_7\equiv1$ (mod 5)

i.e. $e_5+e_6+e_7\equiv1-e_4$ (mod 5) (2)

$e_7+e_3+e_8+e_9\equiv2$ (mod 5)

i.e. $e_7+e_8+e_9\equiv2-e_3$ (mod 5)(3)

$e_9+e_6+e_2+e_{10}\equiv3$ (mod 5)

i.e. $e_9+e_6+e_{10}\equiv3-e_2$ (mod 5)(4)

$e_{10}+e_8+e_5+e_1\equiv4$ (mod 5)

i.e. $e_{10}+e_8+e_5\equiv4-e_1$ (mod 5), (5)

where $1\le e_i\le10$ and $e_i\ne e_j$ for all $i\ne j$, $1\le i,j\le10$.

Adding all the congruences, we get $\sum_{i=1}^{10}e_i\equiv0$ (mod 5). Now, using (1), $\sum_{i=5}^{10}e_i\equiv0$ (mod 5) ; and then using (2), we get

$e_8+e_9+e_{10}\equiv e_4-1$ (mod 5).(6)

From (3), (4), (5) and (6), we get

$e_7-e_{10}\equiv3-(e_3+e_4)$ (mod 5)(7)

$e_6-e_8\equiv4-(e_2+e_4)$ (mod 5) (8)

$e_5-e_9\equiv5-(e_1+e_4)$ $(mod\ 5)$ (9)

The possible solutions of (1) are $A=\{1,4,6,9\}$, $B=\{2,3,7,8\}$, $C=\{1,2,5,7\}$, $D=\{1,3,5,6\}$, $E=\{1,4,5,10\}$, $F=\{2,3,5,10\}$, $G=\{2,4,5,9\}$, $H=\{3,4,5,8\}$, $I=\{1,2,3,4\}$.

In category **A**, $1\equiv6$ (mod 5) and $4\equiv9$ (mod 5), and so the 24 permutations can be grouped into six categories as follows:

$A_1$ :**(1,4,6,9)** $A_2$ :**(1,4,9,6)** $A_3$ :**(1,6,4,9)**

$A_4$ :**(4,1,6,9)** $A_5$ :**(4,1,9,6)** $A_6$ :**(4,9,1,6)**.

Now $\{e_5, e_6, e_7, e_8, e_9, e_{10}\} = \{2,3,5,7,8,10\}$.

$A_1 : (e_1, e_2, e_3, e_4) = (1,4,6,9)$

$e_7 - e_{10} \equiv 3$ (mod 5) (7)

$e_6 - e_8 \equiv 1$ (mod 5)(8)

$e_5 - e_9 \equiv 0$ (mod 5)(9)

The modulo difference '1' is obtained for the pairs (3,2), (3,7), (8,2), (8,7). As $3 \equiv 8$ (mod 5) and $2 \equiv 7$ (mod 5), we can fix $(e_6, e_8) = (3,2)$. From the remaining labelings $\{5,7,8,10\}$, there is no solution for the congruences (7) and (9). Hence there is **no edge-graceful labeling in** $A_1$. $A_2 : (e_1, e_2, e_3, e_4) = (1,4,9,6)$

$e_7 - e_{10} \equiv 3$ (mod 5)(7)

$e_6 - e_8 \equiv 4$ (mod 5)(8)

$e_5 - e_9 \equiv 3$ (mod 5).(9)

From (8), $e_8 - e_6 \equiv 1$ (mod 5). As dealt in $A_1$, we can fix $e_6 = 2, e_8 = 3$. From the remaining labelings $\{5,7,8,10\}$, we have $\{(e_7, e_{10}), (e_5, e_9)\} = \{(8,5),(10,7)\}$. Hence the edge-graceful labelings are $(1,4,9,6,10,2,8,3,7,5)$ and $(1,4,9,6,8,2,10,3,5,7)$. It is easy to check that these **two labelings in** $A_2$ are of different categories. (For example, $e_5$ takes values 8 and 10, which are of different modulo 5.)

We deal all the other types as in $A$. The complete list of different categories in all the types are listed in Table I.

Hence on the whole, there are **205 edge-graceful labelings** of different categories and there are $205 \times 32 = 6560$ distinct edge-graceful labelings.

**Future Work:** Thefundamental concepts used in this paper can be approached in algorithmic manner and we have planned our further research in this direction.

**Table I**

| Type | No. of Categories | List of Labelings |
|---|---|---|
| *A* | 7 | (1,4,9,6,8,2,10,3,5,7), (1,4,9,6,10,2,8,3,7,5), (4,1,6,9,7,2,8,3,10,5), (4,1,6,9,5,2,10,3,8,7), (4,1,9,6,3,2,10,5,8,7), (4,1,9,6,2,5,8,3,7,10), (4,9,1,6,5,7,3,8,10,2) |
| B | 7 | (2,3,7,8,5,9,4,6,10,1), (2,3,8,7,6,9,4,5,10,1), (2,3,8,7,5,10,4,6,9,1), (3,2,7,8,10,9,4,5,6,1), (3,2,7,8,9,10,4,6,5,1), (3,8,2,7,4,10,5,6,9,1), (3,8,2,7,1,9,4,10,6,5) |
| C | 17 | (1,2,5,7,6,3,10,8,9,4), (1,2,7,5,9,8,4,6,10,3), (1,2,7,5,3,8,10,6,9,4), (1,5,2,7,6,10,3,8,9,4), (1,5,2,7,10,6,3,9,8,4), (2,1,5,7,6,9,4,8,10,3), (2,1,5,7,9,6,4,10,8,3), (2,1,5,7,9,4,6,8,3,10), (2,1,7,5,8,9,4,6,10,3), (2,1,7,5,9,8,4,10,6,3), (2,5,1,7,10,6,3,9,4,8), (2,5,7,1,3,8,4,10,6,9), (2,5,7,1,10,6,4,3,8,9), (2,7,5,1,6,4,10,3,9,8), (2,7,5,1,10,4,6,3,8,9), (5,1,2,7,6,4,9,3,8,10), (5,1,2,7,6,10,3,4,8,9) |
| *D* | 17 | (1,3,6,5,4,8,9,2,10,7), (1,3,6,5,2,10,9,4,8,7), (1,5,3,6,2,10,8,7,9,4), (1,5,3,6,10,2,8,9,7,4), (1,6,3,5,7,10,4,2,8,9), (3,1,5,6,4,2,9,10,8,7), (3,1,5,6,4,9,2,7,8,10), (3,1,6,5,9,8,4,10,7,2), (3,1,6,5,10,7,4,9,8,2), (3,1,6,5,9,2,10,4,7,8), (3,5,1,6,10,7,8,9,4,2), (3,5,1,6,8,2,10,9,7,4), (5,1,3,6,4,9,2,7,10,8), (5,1,3,6,2,9,4,7,8,10), (5,1,6,3,2,4,7,9,10,8) (5,3,1,6,4,2,9,7,10,8), (5,3,1,6,8,2,10,7,9,4) |
| | | (1,4,5,10,3,2,6,7,9,8), (1,4,5,10,6,3,2,8,7,9), (1,5,4,10,3,7,6,8,9,2), (1,5,4,10,7,3,6,9,8,2), (1,5,4,10,6,8,2,9,7,3), (1,5,4,10,8,6,2,7,9,3), (1,5,4,10,2,6,3,7,8,9), (1,5,4,10,6,2,3,8,7,9), (4,1,5,10,3,6,7,8,2,9), (4,1,5,10,3,7,6,9,2,8), (4,5,1,10,3,7,6,8,2,9), (5,1,4,10,7,6,3,8,2,9), |

| | | |
|---|---|---|
| *E* | 26 | (5,1,4,10,8,2,6,9,3,7), (5,1,10,4,7,8,2,9,6,3), (5,1,10,4,9,6,2,7,8,3),<br>(5,1,10,4,9,7,6,8,3, 2), (5,1,10,4,8,3,6,9,7,2), (5,1,10,4,8,6,3,2,7,9),<br>(5,1,10,4,2,7,3,8,6,9), (5,4,1,10,8,7,6,2,3,9), (5,4,1,10,2,8,6,3,7,9),<br>(5,4,10,1,7,2,6,3,8,9), (5,10,1,4,9,7,6,2,3,8), (5,10,1,4,7,8,2,3,6,9),<br>(5,10,4,1,2,6,7,8,3,9), (5,10,4,1,2,7,6,9,3,8) |
| *F* | 26 | (2,3,5,10,4,8,9,7,1,6), (2,5,3,10,7,8,6,9,4,1), (2,5,3,10,1,6,9,7,8,4),<br>(2,5,10,3,4,8,1,7,9,6), (2,5,10,3,1,8,4,7,6,9), (3,2,5,10,8,9,4,7,6,1),<br>(3,2,5,10,9,8,4,6,7,1), (3,2,5,10,4,6,1,9,7,8), (3,2,5,10,6,4,1,7,9,8),<br>(3,2,5,10,6,8,7,1,9,4), (3,2,5,10,8,6,7,9,1,4), (3,5,2,10,6,1,4,7,9,8),<br>(3,5,2,10,1,8,7,4,9,6), (3,5,10,2,1,4,9,7,6,8), (3,5,10,2,4,8,7,6,9,1),<br>(5,2,10,3,9,8,1,4,7,6), (5,2,10,3,8,6,4,7,1,9), (5,3,10,2,9,8,7,4,6,1),<br>(5,3,10,2,9,1,4,7,6,8), (5,10,2,3,6,8,4,7,9,1), (5,10,3,2,7,8,4,6,9,1),<br>(5,10,3,2,6,9,4,7,8,1), (5,10,3,2,7,6,1,4,9,8), (5,10,3,2,4,9,1,7,6,8),<br>(5,10,3,2,9,8,7,1,6,4), (5,10,3,2,1,6,7,9,8,4) |
| G | 17 | (2,4,9,5,8,1,7,6,10,3), (4,2,5,9,10,6,1,3,8,7), (4,2,5,9,3,8,1,10,6,7),<br>(4,2,9,5,3,8,10,6,7,1), (4,2,9,5,6,8,7,1,10,3), (4,5,9,2,1,8,10,6,7,3),<br>(4,5,9,2,1,10,8,3,7,6), (4,9,2,5,1,8,7,3,10,6), (4,9,2,5,7,8,1,3,6,10),<br>(4,9,5,2,7,6,1,3,8,10), (4,9,5,2,10,6,3,8,1,7), (5,2,4,9,3,8,1,10,7,6),<br>(5,4,2,9,3,1,8,10,7,6), (5,4,2,9,1,3,8,7,10,6), (5,4,9,2,6,10,3,7,8,1),<br>(5,4,9,2,10,6,3,8,7,1), (5,4,9,2,1,6,7,8,3,10) |
| H | 17 | (3,4,5,8,10,6,2,9,1,7), (3,5,4,8,9,7,2,6,10,1), (3,5,4,8,6,10,2,9,7,1),<br>(3,5,4,8,6,7,10,1,2,9), (3,5,8,4,9,2,1,7,6,10), (3,8,4,5,9,6,1,10,7,2),<br>(3,8,4,5,9,7,10,6,2,1), (3,8,5,4,9,7,1,10,6,2), (3,8,5,4,10,6,1,9,7,2),<br>(4,3,5,8,2,10,1,7,9,6), (4,3,5,8,10,2,1,9,7,6), (4,5,3,8,10,2,6,1,7,9),<br>(4,5,3,8,9,2,7,1,6,10), (5,3,4,8,7,9,2,6,10,1), (5,3,4,8,6,10,2,7,9,1),<br>(5,3,8,4,6,9,2,7,10,1), (5,3,8,4,7,9,1,2,6,10) |
| I | 71 | (1,2,3,4,5,9,8,6,10,7), (1,2,4,3,8,9,6,10,7,5), (1,2,4,3,10,7,6,8,9,5),<br>(1,2,4,3,10,5,8,6,9,7), (1,2,4,3,6,9,8,10,5,7), (1,2,4,3,6,7,5,8,10,9),<br>(1,2,4,3,8,10,5,6,7,9), (1,3,2,4,5,8,9,6,10,7), (1,3,2,4,5,9,8,7,10,6),<br>(1,3,4,2,9,5,10,6,7,8), (1,3,4,2,10,5,9,6,8,7), (1,3,4,2,7,9,8,5,10,6),<br>(1,3,4,2,8,9,7,5,6,10), (1,4,2,3,6,10,7,8,5,9), (1,4,2,3,6,9,8,7,5,10),<br>(1,4,2,3,5,7,6,10,9,8), (1,4,2,3,5,8,10,6,9,7), (1,4,3,2,7,9,8,6,5,10),<br>(1,4,3,2,7,8,9,10,5,6), (1,4,3,2,5,9,10,6,8,7), (1,4,3,2,5,10,9,7,8,6),<br>(1,4,3,2,6,10,8,7,9,5), (1,4,3,2,6,8,10,5,9,7), (2,1,3,4,7,9,6,10,8,5),<br>(2,1,3,4,9,7,6,8,10,5), (2,1,3,4,10,9,8,5,6,7), (2,1,3,4,9,10,8,6,5,7),<br>(2,1,3,4,7,10,5,6,8,9), (2,1,3,4,10,7,5,8,6,9), (2,3,1,4,5,10,7,8,6,9),<br>(2,3,1,4,5,9,8,7,6,10), (2,3,1,4,9,7,6,10,5,8), (2,3,1,4,9,8,10,6,5,7),<br>(2,3,4,1,8,5,7,10,6,9), (2,3,4,1,9,5,6,10,7,8), (2,4,1,3,5,6,7,9,10,8),<br>(2,4,3,1,7,10,8,6,5,9), (2,4,3,1,7,8,10,9,5,6), (2,4,3,1,10,6,9,7,8,5),<br>(2,4,3,1,10,9,6,5,8,7), (3,1,2,4,7,5,10,6,9,8), (3,1,2,4,8,5,9,6,10,7),<br>(3,1,2,4,6,9,7,5,8,10), (3,1,2,4,10,9,8,5,7,6), (3,1,4,2,5,8,6,7,10,9),<br>(3,2,1,4,8,9,5,6,10,7), (3,2,1,4,9,8,5,10,6,7), (3,2,1,4,5,9,8,6,7,10),<br>(3,2,1,4,9,5,8,7,6,10), (3,2,1,4,8,5,9,7,10,6), (3,2,1,4,5,8,9,10,7,6),<br>(3,2,4,1,10,7,8,6,9,5), (3,2,4,1,7,10,8,9,6,5), (3,2,4,1,6,9,10,8,5,7),<br>(3,2,4,1,9,6,10,5,8,7), (3,4,2,1,9,6,5,7,8,10), (3,4,2,1,7,8,5,9,6,10),<br>(4,1,2,3,6,5,7,10,8,9), (4,1,2,3,7,5,6,10,9,8), (4,1,3,2,6,10,8,9,7,5),<br>(4,1,3,2,9,7,8,6,10,5), (4,1,3,2,10,9,5,8,6,7), (4,1,3,2,8,6,5,10,9,7),<br>(4,2,1,3,5,10,8,6,7,9), (4,2,1,3,5,8,10,9,7,6), (4,2,1,3,8,6,9,7,5,10),<br>(4,2,1,3,8,9,6,10,5,7), (4,2,3,1,5,7,8,6,10,9), (4,2,3,1,5,9,6,8,10,7),<br>(4,3,1,2,8,6,5,7,9,10), (4,3,1,2,6,8,5,9,7,10) |

**References:**


1. A.P.Dhana Balan, RM.Sivagama Sundari, C.Santhi, Soft P-Sets and Soft Optimally Continuous Functions; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Issue 2 (2014), Pg 542-545
2. M.Lellis Thivagar, M.Davamani Christober, G.Jayaparthasarathy, M.Arockia Dasan, Weak Form of Separation Axioms In Extended Bitopological Ultra Spaces Support; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 4 Spl Issue (2015), Pg 1-7
3. K.Kayathri, R.Amutha,"Edge-graceful labelings of connected graphs", Electronic Notes in Discrete Mathematics, 53(2016), 287-296.
4. Paruchuri Venu Gopala Rao, Characteristic Function in Semi Near-Rings; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 3 Issue 1 (2014), Pg 363-364
5. J.A.Gallian, "A Dynamic Survey of Graph labelings", The Electronic journal of Combinatorics (2015) #D56, 206-214.
6. *A.Jayalakshmi, Ananth K. Atre,* on A Semigroup Whose Factorisable Elements Form A Band; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Spl Issue (2014), Pg 951-955
7. Sheng-Ping Lo, "On edge-graceful labelings of Graphs", Congressus Numerantium, 50 (1985), 231-241.
8. S. S. Sandhya, E. Ebin Raja Merly, B. Shiny, K-Super Geometric Mean Graphs; Mathematical Sciences International Research Journal : ISSN 2278-8697Volume 4 Issue 2 (2015), Pg 373-377
9. Gurmeet Singh, Rupinderjeet Kaur, A Possible Improvement in Spearman's Rank Correlation Coefficient, When Karl Pearson's Correlation Coefficient Is Negative; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 4 Issue 1 (2015), Pg 96-100



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