SOME MORE RESULTS ON STRONG AND WEAK TRIPLE CONNECTED DOMINAITON IN FUZZY GRAPHS

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Abstract: In this paper we discuss the theoretical approach of strong triple connected domination number and weak triple connected domination number in fuzzy graphs and prove several results on isomorphism of fuzzy triple connected graphs. We conclude this paper by relate strong triple connected graphs and weak triple connected graphs.

Keywords: strong triple connected domination number, weak triple connected domination number, fuzzy graphs
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1. **Introduction:** A fuzzy subset of a nonempty set V is a mapping σ: V → [0,1]. A fuzzy relation on V is a fuzzy subset of V × V. A fuzzy graph G = (σ,μ) is a pair of function σ : V → [0,1] and μ : V X V → [0,1] where μ(u,v) ≤ σ(u) Λ σ(v) for all u,v ∈ V. The underlying crisp graph of G = (σ , μ) is denoted by G^* = (V,E) where V={u∈ V/ σ(u) > 0} and E= {(u,v) ∈ V X V, μ(u,v)>0}. The order p and size of the fuzzy graph G = (σ , μ) are defined by p = $\sum_{u\in V}\sigma(u)$ and q = $\sum_{uv\in E}\mu(u,v)$. Let G be a fuzzy graph on V and S ⊆ V then fuzzy cardinality of s is defined by $\sum_{u\in S}\sigma(u)$. The strength of the connectedness between two nodes u and v in a fuzzy graph G, μ^k(u,v) = sup {μ(u,u_1)Λμ(u_1,u_2)Λ........ Λμ(u_{k-1},v)}. An arc (u,v) is said to be a strong arc if μ^∞(u,v) = sup {μ^k(u,v), k = {1,2,.....}. If μ(u,v)=0 for every v ∈ V then u is called an isolated node.

Definition 1.1 Let G(v, σ,μ)be a fuzzy graph and S ⊆ V, is a dominating set if for every u ∈ V – S then there exists v ∈ S such that (i) (u,v) is a strong arc (ii) σ(u) ≤ σ(v) (iii) ⟨S⟩ is triple connected. The minimum cardinality taken over all triple connected fuzzy graphs is called triple connected fuzzy domination number and is denoted by $\gamma_{ftc}(G)$. The γ_{ftc} number is the sum of membership values σ(u) of the vertices of the minimum triple connected fuzzy graphs. For example,

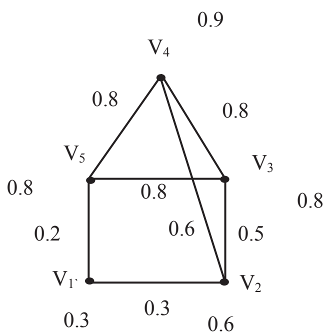


Figure 1.1
Fuzzy Triple connected dominating set G(v, σ,μ)

In figure 1.1 strong arcs are {v_3v_4, v_3v_5, v_4v_5} and D = {v_1, v_2, v_3} and hence $\gamma_{ftc}(G)$ = 1.7

Definition 1.2 Let G = (V, σ , μ) be the fuzzy graph, for any u,v∈ V, u strongly dominates v if μ(u,v)=σ(u)Λσ(v) and d(u) ≥ d(v).
Similarly u weakly dominates v if μ(u,v)=σ(u)Λσ(v) and d(u) ≤ d(v). A set S ⊆ V is a STC dominating set if every vertex in V – S is strongly dominated by at least one vertex in S. The minimum cardinality taken over tall STC of fuzzy graph is denoted by $\gamma_{fstc}(G)$.
Similarly fuzzy WTC dominating set is denoted by $\gamma_{fwtc}(G)$.
Example for Fuzzy Strong and Weak Triple connected dominating set

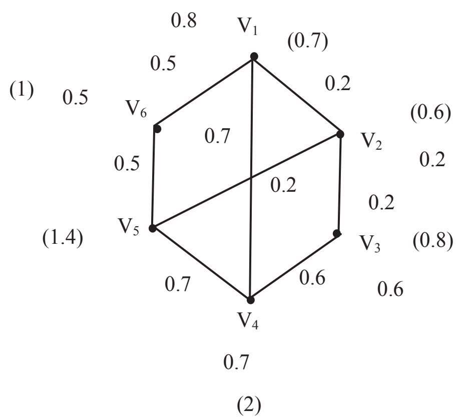


Figure 1.2
Weak triple connected dominating set G(v, σ,μ) with order and size

In Figure 1.2 o(G) = $\sum\sigma(u)$ = 3.8 , s(G) = $\sum\mu(u,v)$ = 3.6 , V(G) = {v_4,v_5,v_6,v_3,v_1,v_2}
$\gamma_{fstc}(G)$ = {v_4,v_5,v_6} =2.2, $\gamma_{fwtc}(G)$ = {v_1,v_2,v_3} = 1.6

2. Isomorphism of Fuzzy Triple connected Graphs: A homomorphism of fuzzy graphs $\varphi : G \to G'$ is a map $\psi : S \to S'$ satisfying σ(x) ≤ σ'(ϕ(x)) for all x ∈ S and μ(x,y) ≤ μ'(ϕ(x) , ϕ(y)) for all x,y ∈ S. An isomorphism $\varphi : G \to G'$ is a map $\psi : S \to S'$ which is bijective that satisfies σ(x) = σ'(ϕ(x)) for all x ∈ S and μ(x,y) = μ'(ϕ(x) , ϕ(y)) for all x,y ∈ S and is denoted by G ≅ G'

Theorem 2.1: Isomorphism between fuzzy Triple connected graphs is an equivalence relation.

Proof: Let G , G′ , G″ be the fuzzy triple connected graphs having sets (σ,μ), (σ′,μ′), (σ″,μ″) respectively.

To prove (i) G ~ G ⟹ G ~ G (ii) G ~ G′ ⟹ G′ ~ G (iii) G~ G′ and G′~ G″ ⟹ G~ G″

Define the relation ~ by defining the map $\varphi : G \to G'$ such that ϕ(x) = x. since ϕ is self mapping, it satisfies one to one and onto. Thus σ(x) = σ(ϕ(x)) = σ(x) and μ(x,y) = μ(ϕ(x),ϕ(y)) = μ(x,y) for all x,y ∈ S. Hence G ~ G and is reflexive. Consider $\varphi : G \to G'$ such that ϕ(x) = x′. To prove ϕ^{-1} : G′→ G is isomorphic.

Consider ϕ(x) = ϕ(y) ⟹ x′ = y′, thus ϕ is one to one. Let x′∈G′ then there exists x ∈ G such that ϕ(x) ∈ G, thus ϕ is onto. Hence ϕ is bijective map. Consider ϕ(x) = x′ gives σ(x) = σ′(x′) = σ′(ϕ(x)).

Also ϕ(x) = x′ ⟹ x = ϕ^{-1}(x′)

⟹ σ(x) = σ(ϕ^{-1}(x′))

⟹ σ′(ϕ(x)) =σ (ϕ^{-1}(x′))

⟹ σ′(x′) = σ(ϕ^{-1}(x′))

Also μ(ϕ^{-1}(x′), ϕ^{-1}(y′) = μ(x′, y′)

Thus ϕ^{-1} : G′→ G is isomorphic and G′ ≅ G and hence Symmetric.

Consider $\varphi : G \to G'$ and $: G' \to G''$. To prove G ≅ G″

Given $\varphi : G \to G'$ is an isomorphism then ϕ(x) = x′, σ(x) = σ′(x′) = σ′(ϕ(x))

μ(x,y) = μ(x′,y′) = μ(ϕ(x),ϕ(y))

similarly, $\psi : G' \to G''$ is an isomorphism then ψ(x′) = x″ , ψ′(x′) = σ″(ψ(x′)) and μ′(x′,y′) = μ″(ψ(x′), ψ(y′))

Consider σ(x) = σ′(x′) = σ″(ψ(x′)) = σ″(ψ(ϕ(x)))

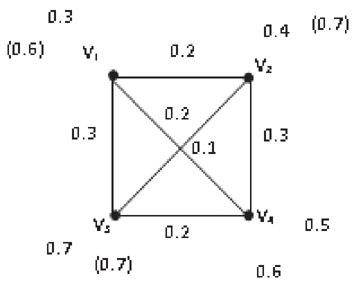
and μ(x,y) = μ(x′,y′) = μ″ (ψ (x′), ψ (y′)) = μ″ (ψ (ϕ(x)), ψ (ϕ(y))). Hence G~ G″

Theorem 2.2: For any isomorphic triple connected graphs their orders, size and degrees are same.

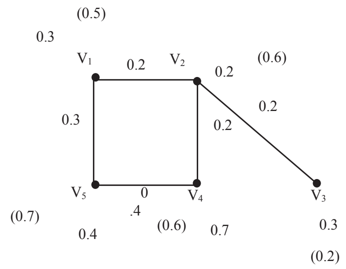
Proof: consider $\varphi : G \to G'$ is an isomorphism between G and G′. that is, σ(x) = σ′(ϕ(x)) and μ(x,y) = μ′(ϕ(x),ϕ(y)).

o(G) = $\sum \sigma(x) = \sum \sigma'(\varphi(x))$ = o(G′) , s(G) = $\sum \mu(u,v) = \sum \mu'(\varphi(x),\varphi(y))$ = s(G′) , d(u) = $\sum_{\substack{v \neq u \\ v \in S'}} \mu'(u,v)$ = d(ϕ(u)). Hence G is isomorphic to G′

Observation 2.3: The converse of the above theorem is not true when order, degrees and the size of the graphs are same.



G



G′

Figure 2.1

G is not isomorphic to G′ whereas order, size and degrees are same

In Figure. 2.1 (a), o(G) = 1.9 , s(G) = 1.3, d(G) = 2.6

In Figure 2.1(b), o(G′) = 1.9 , s(G′) = 1.3 , d(G′) = 2.6 but G is not isomorphic to G′.

Theorem 2.4: In isomorphic graphs the fuzzy triple connected domination number are same.

Proof: In isomorphic graphs o(G) = o(G′) , s(G) =s(G′) , d(G) = d(G′) and there is a one to one correspondence between the graphs. Define the isomorphic map $\varphi : G \to G'$ such as ϕ(x) = x′ and σ(x) = σ(x′) gives σ(x) = σ(ϕ(x)) , μ(x,y) = μ(x′,y′) = μ(ϕ(x),ϕ(y)) and d(x) = d(x′) = d(ϕ(x)). Hence $\gamma_{ftc}(G) = \gamma_{ftc}(G')$

Observation 2.5: For any connected graph G, 0.3 ≤ γ_{ftc} ≤ p

Proof: Consider P_3 minimum membership value as 0.1. Let σ(v_1) = 0.1, σ(v_2) = 0.1, σ(v_3) = 0.1.

Hence $\gamma_{ftc}(P_3)$ = {v_1,v_2,v_3}. If the membership values of P_3 are taken as maximum, then γ_{ftc} = p.

Theorem 2.6: For any complete graph G (V,σ,μ), $\gamma_{fwtc} \leq \gamma_{fstc}$.

Proof: Let G (V,σ,μ) be a complete fuzzy graph. Suppose all the σ values of K_n are equal then μ(u_i,v_i) are also same. That is, μ(u_i,v_i) = σ(u_i) Λ σ(v_i) = u_i . Thus, d(u_i) are same in all the vertices. Thus, any three vertices are treated as STC or WTC fuzzy domination number. Hence $\gamma_{fwtc} = \gamma_{fstc}$. Suppose, all the σ values of K_n are not equal. μ(u_i,v_i) = σ(u_i) Λ σ(v_i). if a vertex contains minimum value then p – 1 edges have minimum μ(u_i,v_i) value. Thus any three vertices having least μ(u_i,v_i) values are taken as γ_{fwtc} set. That is, γ_{fwtc} contain three vertices other than larger membership values. Same way, any three vertices having largest membership values are taken as γ_{fstc} set. That is, γ_{fstc} contain three vertices other than least membership values. Hence γ_{fstc} is strictly greater than γ_{fwtc}.

$\gamma_{fstc} > \gamma_{fwtc}$. Hence, $\gamma_{fwtc} \leq \gamma_{fstc}$.

Theorem 2.7: For any triple connected graph G,

(i) $\gamma_{fstc} \geq \gamma_{fwtc}$ (OR)

(ii) $\gamma_{fstc} \leq \gamma_{fwtc}$

Proof: For any fuzzy graph G, the least possible membership values are taken as WTC dominating set

and to satisfy triple connected condition all the in between vertices also added to γ_{fwtc} set. By the same way, the largest possible membership values are taken as γ_{fstc} set. Hence $\gamma_{fstc} \leq \gamma_{fwtc}$ or $\gamma_{fstc} \geq \gamma_{fwtc}$.
Let us prove this theorem by taking cycle graph. Consider $V(C_n) = \{v_1,v_2,......,v_{p-1},v_p\}$. since triple connected, we can discuss the cases by taking degrees of (v_1,v_2) and (v_p,v_{p-1})

Case (i): $d(v_1) \geq d(v_2)$ and $d(v_p) \geq d(v_{p-1})$
If $d(v_1) \geq d(v_2)$ and $d(v_p) \geq d(v_{p-1})$ and for strong triple connected set $v_1v_2 \in E(C_n)$ then $d(D) \geq d(V\text{-}D)$. Thus, $\{v_1,v_p\}$ also added to γ_{fstc} set. Hence γ_{fstc} = p. And in WTC set, $d(D) \leq d(V\text{-}D)$. Thus, $\{v_2,v_{p-1}\}$ also added to γ_{fwtc} set. Hence $\gamma_{fwtc} = \{v_2,v_3,......,v_{p-1}\} = \gamma_{fwtc} < \gamma_{fstc}$.

Case (ii): $d(v_1) \leq d(v_2)$ and $d(v_p) \leq d(v_{p-1})$.
In STC set, if $d(v_1) \leq d(v_2)$ then $\{v_2,v_3,......,v_{p-1}\}$ be in γ_{fstc} set. In WTC set if $d(v_1) \leq d(v_2)$ and $d(v_p) \leq d(v_{p-1})$ then $\{v_1,v_2,v_3,......,v_{p-1},v_p\}$ be in γ_{fwtc} set. Thus γ_{fwtc} = p and $\gamma_{fwtc} > \gamma_{fstc}$.

Case (iii): $d(v_1)$[or $d(v_p)$] $\leq d(v_2)$[or $d(v_{p-1})$ and $d(v_1)$[or $d(v_p)$] $\geq d(v_2)$[or $d(v_{p-1})$.
Consider, $d(v_1) \leq d(v_2)$ and $d(v_p) \geq d(v_{p-1})$. In STC set, if $uv \in E(G)$ gives $d(u) \geq d(v)$ then $u \in D$. Given $d(v_1) \leq d(v_2)$ and $d(v_p) \leq d(v_{p-1})$ then $\gamma_{fstc} = \{v_2,v_3,......,v_p\} = p - 1$
And $\gamma_{fwtc} = \{v_1,v_2,......,v_{p-1}\} = p - 1$
Similarly, $d(v_p) \leq d(v_{p-1})$ and $d(v_1) \geq d(v_2)$ then $\gamma_{fwtc} = \gamma_{fstc} = p - 1$.
Comparing the above inequalities, $\gamma_{fstc} \geq \gamma_{fwtc}$ (OR) $\gamma_{fstc} \leq \gamma_{fwtc}$

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