NOTE ON LOCAL FUNCTIONS IN IDEAL EXTENDED BITOPOLOGICAL SPACES

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Abstract: The ideal topology plays a vital role, which is the generalization of generaltopology. This paper focuses on to introduce ideal extended bitopological space, that is, theextended bitopological space together with an ideal. We, in addition to, derive several localfunctions and establish some of their basic properties. Apart from this, by the respective localfunctions, we define some weak form of Kuratowski's closure operators and derive correspondingnew topologies.

Keywords: Extended bitopological spaces,A^*, A^*_α,A^*_s,A^*_p,$(\tau_{(1,2)^*k})^*$-topology.

1. **Introduction:** The idea of bitopological space was introduced by J.C.Kelly [3] in 1963 that is a non emptysetequipped with two topologies. Further,many researcher develop the bitopological spacesby defining various type of open sets and also studied their properties. Lellis Thivagar et al. [6] introduced extended bitopological spaces and established some properties of weak form ofopen sets in these spaces. The Ideal concept in topological space was initiated by Kuratowski[5] and after several decades in 1990, Jankovic and Hamlett [1] investigated the properties ofideal topological spaces. The weak form of closed sets in ideal topological space by the helpof several local functions was developed by Khan and Noiri [2,7] and derived a decompositionof continuity. In this paper we introduce ideals in extended bitopological spaces called idealextended bitopological spaces and establish its basic properties. Also we introduce some localfunctions and using these, we obtain some new topologies.

2. **Preliminaries:** This section is to discuss some basic properties about ideal topological spaces andextended bitopological spaces which are useful in sequel.

Definition 2.1.[6] Let (X, τ_1, τ_2) be a bitopological space and$\tau_{1,2}O(X)\subset(\tau_{1,2})^+$. Then $(\tau_{1,2})^+$will be termed a simple extension of$\tau_{1,2}O(X)$ if and only if there exists an$A \notin\tau_{1,2}O(X)$ suchthat $(\tau_{1,2})^+= (\tau_{1,2})^+(A) =\{G_1\cup(G_2\cap A):G_1,G_2\in\tau_{1,2}O(X)\}$. We call $(X,(\tau_{1,2})^+(A))$ anextended bitopological space of (X, τ_1, τ_2) w.r.t A.

Definition 2.2.[6] Let $(X,(\tau_{1,2})^+(A))$ be an extended bitopological space and$S\subseteq X$. Then$(\tau_{1,2})^+$-closure of S is defined as $(\tau_{1,2})^+cl(S)=\cap\{F : S\subseteq F$and F is $(\tau_{1,2})^+$-closed$\}$and$(\tau_{1,2})^+$-interior of S is defined as $(\tau_{1,2})^+int(S)=\cup\{G: G\subseteq S$and G is $(\tau_{1,2})^+$-open$\}$.

Definition 2.3.[6] Let $(X,(\tau_{1,2})^+(A))$ be an extended bitopological space. A subset S of X iscalled

(i) $(1,2)^{*+}$-α-open if $S\subseteq(\tau_{1,2})^+int((\tau_{1,2})^+cl((\tau_{1,2})^+int(S)))$

(ii) $(1,2)^{*+}$-semi-open if $S\subseteq(\tau_{1,2})^+cl((\tau_{1,2})^+int(S))$ and

(iii) $(1,2)^{*+}$-pre-open if $S\subseteq(\tau_{1,2})^+int((\tau_{1,2})^+cl(S))$.

The collection of all $(1,2)^{*+}$-α-open sets,$(1,2)^{*+}$-semi-open sets and $(1,2)^{*+}$-pre-open sets of X are denoted by $(1,2)^{*+}\alpha O(X)$, $(1,2)^{*+}SO(X)$, and $(1,2)^{*+}PO(X)$ respectively.

Theorem 2.4.[6] Let X be an extended bitopological space. Then $S\subseteq X$is a $(1,2)^{*+}$-α-openif and only if S is a $(1,2)^{*+}$-semi-open and a $(1,2)^{*+}$-pre-open.

Definition 2.5. [5] An ideal I on a topological space (X, τ) is a non empty collection of subsets of X satisfying the following two conditions:

(i) If $A\in I$ and $B\subset A$, then $B\in I$.

(ii) If $A\in I$ and $B\in I$, then $A\cup B\in I$.

Let (X, τ) be a topological space and I an ideal of subsets of X. An ideal topological space is a topological space (X, τ) with an ideal I on X and is denoted by (X, τ, I).

Definition 2.6. For a subset A of X, $A^*(I, \tau) =\{x\in X: U\cap A \notin I$for every $U\in\tau(X, x)\}$iscalled the local function ofA[5] with respect to ideal Iand topologyτ, where $\tau(X, x) =\{U\in\tau: x\in U\}$. For every ideal topological space (X, τ, I), there exists a topology$\tau^*(I)$ [1, 2] finerthan τ, defined by $\tau^*(I) =\{U\subseteq X: cl^*(X-U) =X-U\}$which is generated by the base$\beta(I, \tau) =\{U-J: U\in\tau$ and$J\in I\}$andcl$^*(A) =A\cup A^*$is a Kuratowski closure operator forthe topologyτ^*.

Definition 2.7.[2] A subset Aof an ideal topological space (X, τ, I) is said to beI-generalizedclosed (brieflyIg-closed) if $A^*\subset U$wheneverA$\subset U$andUis open in X.

3. **Local Functions in Ideal Extended Bitopological Spaces:** In this section, we introduce ideal extended bitopological spaces and establish theproperties of the local function. Also from these local functions, we derive new topologies anddiscuss some properties of them.

Definition 3.1.An idealI on an extended bitopological space $(X,(\tau_{1,2})^+)$ is a non emptycollection of subsets ofXsatisfying the following two conditions:

(i) If A∈I and B⊂A, then B∈I.
(ii) If A∈I and B∈I, then A∪B∈I.
Then extended bitopological space $(X,(\tau_{1,2})^+)$ together with an ideal I on X is called an ideal extended bitopological space and is denoted by $(X,(\tau_{1,2})^+, I)$.
Definition 3.2. For a subset A of X,

1. $A^*(I,(\tau_{1,2})^+)$ ={x∈X: U∩A ∉ I for every U∈N(x)} is called the local function of A with respect to the ideal I and the extended bitopology, where N(x) ={U∈$(\tau_{1,2})^+$O(X) :x∈U}.
2. $A^*_\alpha(I,(\tau_{1,2})^+)$ ={x∈X: U∩A ∉ I for every U∈αN(x)} is called the α-local function of A with respect to the ideal I and the extended bitopology, where αN(x) = {U∈$(1,2)^{*+}$αO(X) :x∈U}.
3. $A^*_s(I,(\tau_{1,2})^+)$={x ∈ X: U∩A∉I for every U∈sN(x)} is called the semi-local function of A with respect to the ideal I and the extended bitopology, where sN(x) ={U∈$(1,2)^{*+}$SO(X) :x∈U}.
4. $A^*_p(I,(\tau_{1,2})^+)$ ={x∈X: U∩A∉I for every U∈pN(x)} is called the pre-local function of A with respect to the ideal I and the extended bitopology, where pN(x) = {U∈$(1,2)^{*+}$P O(X) :x∈U}.

We simply write A^* (resp. A^*_α, A^*_s, A^*_p) instead of $A^*(I,(\tau_{1,2})^+)$ (resp. $A^*_\alpha(I,(\tau_{1,2})^+)$, $A^*_s(I,(\tau_{1,2})^+)$, $A^*_p(I,(\tau_{1,2})^+)$). Obviously A^*_k is a proper subset of X for any k∈$(\tau_{1,2})^+$O(X) or $(1,2)^{*+}$αO(X) or $(1,2)^{*+}$SO(X) or $(1,2)^{*+}$PO(X). A subset A of X is said to be perfect if A=A^*_k.
Example 3.3. Let X={a, b, c, d}, τ_1O(X) = {∅, X,{a}}, τ_2O(X) = {∅, X,{a, b, c}}. Then $\tau_{1,2}$O(X) ={∅, X,{a},{a, b, c}}. Let A={b, c}∉$\tau_{1,2}$O(X), then $(\tau_{1,2})^+$(A) ={∅, X,{a},{b, c},{a, b, c}} is an extended bitopology and I={∅,{a}} is an ideal on X. Thus $(X,(\tau_{1,2})^+, I)$ is an ideal extended bitopological space on X. If B={b, c}⊂X, then A^*=A^*_α=A^*_p={b, c, d} and A^*_s={b, c}.
Theorem 3.4. Let $(X,(\tau_{1,2})^+, I)$ be an ideal extended bitopological space, then for every A, B⊆X and k ∈ $(\tau_{1,2})^+$O(X) or $(1,2)^{*+}$αO(X) or $(1,2)^{*+}$SO(X) or $(1,2)^{*+}$P O(X), the following statements are true:

1. If A⊆B⇒A^*_k⊆B^*_k.
2. $(A\cup B)^*_k = A^*_k \cup B^*_k$
3. $(A\cap B)^*_k \subseteq A^*_k \cap B^*_k$.
4. (iv) $A^*_k - B^*_k = (A-B)^*_k - B^*_k \subseteq (A-B)^*_k$.
5. $(A^*_k)^*_k \subseteq A^*_k$.
6. (vi) $A^*_k - (A^*_k)^*_k \subseteq (A - A^*_k)^*_k$.
7. A^*_k=kcl(A^*_k)⊆kcl(A) and A^*_k is a k-closed set in $(X,(\tau_{1,2})^+)$.

Proof. (i) If A⊆B and x∈A^*_k. Suppose that x ∉B^*_k, then there exists U∈kN(x) such that U∩B∈I. Since A⊆B, then U∩A∈I and also x∉A^*_k, which is contradicting the hypothesis. Thus $A^*_k \subseteq B^*_k$.
(ii) Since A⊆A∪B, B⊆A∪B, $A^*_k \subseteq (A\cup B)^*_k$, $B^*_k \subseteq (A\cup B)^*_k$. Thus $A^*_k \cup B^*_k \subseteq (A\cup B)^*_k$. Conversely, let x∈$(A\cup B)^*_k$. Then for every U∈kN(x) such that U∩(A∪B) =(U∩A)∪(U∩B)∉I. Therefore, U∩A∉I or U∩B∉I. This implies that x∈A^*_k or B^*_k. That is x∈$A^*_k \cup B^*_k$. Therefore, we have $(A\cup B)^*_k \subseteq A^*_k \cup B^*_k$. Thus we get $(A\cup B)^*_k = A^*_k \cup B^*_k$.
(iii) Since A∩B⊆A, B and by (i), we get $(A\cap B)^*_k \subseteq A^*_k \cap B^*_k$.
(iv) Since A= (A–B)∪(A∩B). By
(ii), $A^*_k = (A-B)^*_k \cup (A\cap B)^*_k$ and $A^*_k - B^*_k = A^*_k \cap (X - B^*_k) = (A-B)^*_k \cup (A\cap B)^*_k \cap (X-B^*_k) = ((A-B)^*_k \cap (X-B^*_k)) \cup ((A\cap B)^*_k \cap (X-B^*_k)) = ((A-B)^*_k \cap (X-B^*_k)) \cup \emptyset = (A-B)^*_k - B^*_k \subseteq (A-B)^*_k$.
(v) Let x∈$(A^*_k)^*_k$. Then, for every U∈kN(x), U∩A^*_k∉I and also U∩A^*_k≠∅. Then we can find a y such that y∈U∩A^*_k implies y∈A^*_k and U∈kN(y). Hence U∩A ∉I and x∈A^*_k. Thus, $(A^*_k)^*_k \subseteq A^*_k$.
(vi) By (iv), $A^*_k - (A^*_k)^*_k \subseteq (A - A^*_k)^*_k$.
(vii) We know that A^*_k⊆kcl(A^*_k). On the other hand, let x∈kcl(A^*_k). Then U∩A^*_k≠∅, for every U∈kN(x). Therefore, there exists some y∈U∩A^*_k and U∈kN(y). Since y∈A^*_k, U ∩A∉I and hence x∈A^*_k. Then, we have kcl(A^*_k) ⊆A^*_k. Again let x∈kcl(A^*_k) =A^*_k, then U∩A∉I for every kN(x). Then, U∩A≠∅, for every U∈kN(x). Therefore, x∈kcl(A). Hence A^*_k=kcl(A^*_k)⊆kcl(A).
Theorem 3.5. Let $(X,(\tau_{1,2})^+, I)$ be an ideal extended bitopological space, then for every A, B⊆X and for k∈$(\tau_{1,2})^+$O(X) or $(1,2)^{*+}$αO(X) or $(1,2)^{*+}$SO(X) or $(1,2)^{*+}$P O(X) the following statements are true:
(i) If U∈$(\tau_{1,2})^+$O(X)⇒U∩A^*_k=U∩$(U\cap A)^*_k \subseteq (U\cap A)^*_k$.
(ii) A^*_k=∅ if A∈I and so if J∈I⇒$(A-J)^*_k = A^*_k = (A\cup J)^*_k$.
(iii) If A–B, B–A∈I⇒$A^*_k = B^*_k$.
(iv) If I_1, I_2 are two ideals on X and $I_1 \subseteq I_2$, then $A^*_k(I_2) \subseteq A^*_k(I_1)$.

(v) If I_1, I_2 are two ideals on X, then $A_k^*(I_1\cap I_2) = A_k^*(I_1)\cup A_k^*(I_2)$.

Proof.(i) Assume that U∈$(\tau_{1,2})^+$O(X) and x∈U∩A_k^*. Then x∈U and x∈A_k^*. For any V ∈kN(x), V∩U∈kN(x) and V∩(U∩A) = (V∩U)∩A∉I. This shows that x∈$(U\cap A)_k^*$ and U∩A_k^*⊆$(U\cap A)_k^*$. Moreover, U∩A⊆A and by theorem 3.6(i), $(U\cap A)_k^*\subseteq A_k^*$ and U∩$(U\cap A)_k^*$⊆U∩A_k^*. Therefore, U∩A_k^*=U∩$(U\cap A)_k^*\subseteq(U\cap A)_k^*$.

(ii) By definition of A_k^*, if A∈I, then A_k^*=∅. Since J∈I, $(A\cup J)_k^*=A_k^*\cup J_k^* = A_k^*$. Also A∪J= (A–J)∪J implies that $(A\cup J)_k^*=(A-J)_k^*\cup J_k^*= (A-J)_k^*$.

(iii) Since A= (A–B)∪(A∩B), B= (B–A)∪(A∩B) and A–B, B–A∈I and by (ii), we have $A_k^*=(A\cap B)_k^* = B_k^*$.

(iv) Assume that $I_1\subseteq I_2$ and x∈$A_k^*(I2)$. Then for every U∈kN(x), U∩A∉I_2. This implies that U∩A∉I_1, for every U∈kN(x). Then x∈$A_k^*(I_1)$. Hence $A_k^*(I_2)\subseteq A_k^*(I_1)$.

(v) Let x∉$A_k^*(I_1)\cup A_k^*(I_2)$. Then x∉$A_k^*(I_1)$ and x∉$A_k^*(I_2)$. Now x∉$A_k^*(I_1)$ implies that there exists atleast one U∈kN(x) such that U∩A∈I_1. Again x∉$A_k^*(I_2)$ implies that there exists atleast one U∈kN(x) such that U∩A∈I_2. Therefore, there exists atleast one U∈kN(x) such that U∩A∈$I_1\cap I_2$, which implies that x∉$A_k^*(I_1\cap I_2)$. Thus, $A_k^*(I_1\cap I_2)\supseteq A_k^*(I_1)\cup A_k^*(I_2)$. Let x∈$A_k^*(I_1\cap I_2)$. Then for each U∈kN(x), U ∩A∉$(I_1\cap I_2)$. Since U∩A ∉$(I_1\cap I_2)$, either U∩A ∉I_1 or U∩A∉I_2. If U∩A∉I_1, then x∈$A_k^*(I_1)$, otherwise, x∈$A_k^*(I_2)$. Thus x∈$A_k^*(I_1)\cup A_k^*(I_2)$. Thus, $A_k^*(I_1\cap I_2)\subseteq A_k^*(I_1)\cup A_k^*(I_2)$ and hence $A_k^*(I_1\cap I_2) = A_k^*(I_1)\cup A_k^*(I_2)$.

Definition 3.6.Let $(X,(\tau_{1,2})^+, I)$ be an ideal extended bitopological space and for a subset A⊆X and fork∈$(\tau_{1,2})^+$O(X) or $(1,2)^{*+}\alpha$O(X) or $(1,2)^{*+}$SO(X) or $(1,2)^{*+}$P O(X), we define an closure operator kcl* by kcl*(A) =A∪A_k^*.

Theorem 3.7.In an ideal extended bitopological space $(X,(\tau_{1,2})^+,I)$ and k∈$(\tau_{1,2})^+$O(X) or $(1,2)^{*+}\alpha$O(X), the closure operator kcl* is Kuratowski's closure operator.

Proof.(i) kcl*(∅) =∅∪$\emptyset_k^*$=∅ and A⊆kcl*(A) for all A⊆X.

(ii) kcl*(A∪B) = (A∪B)∪$(A\cup B)_k^*$= (A∪B)∪(A_k^*∪B_k^*) =kcl*(A)∪kcl*(B).

(iii) For any A⊆X, kcl*(kcl*(A)) =kcl*(A∪A_k^*) = (A∪A_k^*)∪$(A\cup A_k^*)_k^*$=A∪A_k^*∪$(A_k^*)_k^*$=A∪A_k^*=kcl*(A).

Theorem 3.8.Let $(X,(\tau_{1,2})^+, I)$ be an ideal extended bitopological space and kcl* be the Kuratowski's closure operator. Then the collection $(\tau_{(1,2)^*}k)^*$={U⊆X: kcl*(X–U) =X–U} is a topology which is finer than $\tau_{(1,2)}*k$(where $\tau_{(1,2)^*}k$= $(\tau_{1,2})^+$O(X) or $(1,2)^{*+}\alpha$O(X)), called $(\tau_{(1,2)^*}k)^*$-topology.

Proof.(i) Since ∅⊆X, kcl*(X– ∅) =X and kcl*(X–X) =∅. Hence ∅, X ∈$(\tau_{(1,2)^*}k)^*$.

(ii) Assume that $\{Ui\}_{i=1}^{\infty}\in(\tau_{(1,2)^*}k)^*$, then kcl*(X–$U_i$) =X–$U_i$ for all i. That is, (X–U_i)∪$(X-Ui)_k^*$=X–U_i for all i. Therefore, $(X-Ui)_k^*$⊆X–U_i for all i. We have to prove that kcl*(X–$\bigcup_i U_i$) =X–$\bigcup_i U_i$. Now kcl*(X–$\bigcup_i U_i$) =kcl*($\cap_i$(X–U_i)) =($\cap_i$(X–U_i))∪$(\cap_i (X-\mathrm{Ui}))_k^*\supseteq\cap_i$(X–$U_i$). Also by hypothesis, ($\cap_i$(X–$U_i$))∪$(\cap_i (X-\mathrm{Ui}))_k^*$)⊆$\cap_i$(X–$U_i$). Hence kcl*($\cap_i$(X–$U_i$))⊆$\cap_i$(X–$U_i$). Thus, kcl*(X–$\bigcup_i U_i$) =X–$\bigcup_i U_i$.

(iii) Assume that $\{Ui\}_{i=1}^{n}\in(\tau_{(1,2)^*}k)^*$ then kcl*(X–U_i) =X–U_i for all i= 1,2, ..., n. That is,(X–U_i)∪$(X-Ui)_k^*$=X–U_i for all i= 1,2, ..., n. Therefore $(X-Ui)_k^*$⊆X–U_i for all i. We have to prove that kcl*(X–$\cap_i U_i$) =X–$\cap_i U_i$. Now kcl*(X–$\cap_i U_i$) =kcl*($\cup_i(X-Ui)$) =$\cup_i$(kcl*(X–U_i)) =$\cup_i$(X–U_i) = (X–$\cap_i U_i$). Hence the collection $(\tau_{(1,2)^*}k)^*$ forms a topology. If I ={∅}, then A_k^*=kcl(A). In this case, we have kcl*(A) = kcl(A) and $(\tau_{(1,2)^*}k)^*=\tau_{(1,2)^*}k$. If I=P(X), the power set of X, then A_k^*={∅} and kcl*(A) =A, for every A⊆X and hence $(\tau_{(1,2)^*}k)^*$(I) is the discrete topology. Thus the ideals {∅} and P(X) illustrate extreme cases, where $(\tau_{(1,2)^*}k)^*=\tau_{(1,2)^*}k$ and $(\tau_{(1,2)^*}k)^*$= discrete topology on X, respectively. Since for every ideal I on X, we have {∅} ⊆I⊆P(X), then by theorem 3.5(iv), we have $\tau_{(1,2)^*}k\subseteq(\tau_{(1,2)^*}k)^*$⊆discrete topology. Hence $(\tau_{(1,2)^*}k)^*$ is finer than $\tau_{(1,2)^*}k$.

Remark 3.9.The collection $(\tau_{(1,2)^*}s)^*$={U⊆X: $(\tau_{1,2})^+cl_s^*$(X–U) =X–U} is need not form a topology, for example, consider X={a, b, c, d}, τ_1O(X) ={∅, X,{a}}, τ_2O(X) ={∅, X}. Then $\tau_{1,2}$O(X) = {∅, X,{a}}. Let A={b} ∉$\tau_{1,2}$O(X), then $(\tau_{1,2})^+$(A) = {∅, X,{a},{b},{a, b}} is an extended bitopology and I ={∅,{d}} is an ideal on X. Thus $(X,(\tau_{1,2})^+, I)$ is an ideal extended bitopological space on X. Then collection $(\tau_{(1,2)^*}s)^*$={U⊆X: $(\tau_{1,2})^+cl_s^*$(X–U) =X–U}={∅,{a},{b},{a, b},{a, c},{a, d},{b, c},{b, d},{a, b, c},{a, b, d},{a, c, d},{b, c, d}, X}.If we take B={a, c}, C ={b, c} ∈$(\tau_{(1,2)^*}s)^*$, then B∩C={c}∉$(\tau_{(1,2)^*}s)^*$.

Remark 3.10.The collection $(\tau_{(1,2)^*}p)^*$={U⊆X: $(\tau_{1,2})^+cl_p^*$(X–U) =X–U} is need not form a topology, for example, consider X={a, b, c, d}, τ_1O(X) ={∅, X,{a}}, τ_2O(X) ={∅, X,{a, b, c}}. Then $\tau_{1,2}$O(X) = {∅, X,{a},{a, b, c}}. Let A={b,c}∉$\tau_{1,2}$O(X), then $(\tau_{1,2})^+$(A) ={∅, X,{a},{b, c},{a, b, c}} is an extended bitopology and I={∅,{a}} is an ideal on X. Thus $(X,(\tau_{1,2})^+, I)$ is an ideal extended bitopological space on X. Then collection $(\tau_{(1,2)^*}p)^*$={U⊆X: $(\tau_{1,2})^+cl_p^*$(X–U) =X–U}={∅,{a},{b},{c},{a, b},{a, c},{b, c},{b, d},{c, d},{a, b, c},{a, b, d},{a, c, d},{b, c, d}, X}. If we take B={a, b, d}, C ={a, c, d} ∈$(\tau_{(1,2)^*}p)^*$, then B∩C={a, d}∉$(\tau_{(1,2)^*}p)^*$.

Remark 3.11.We can easily deduct that

(i) $(\tau_{1,2})^{+} \subset (\tau_{(1,2)}{}^{*})^{*} \subset (\tau_{(1,2)}{}^{*}\alpha)^{*} \subset (\tau_{(1,2)}{}^{*}s)^{*}$.
(ii) $(\tau_{1,2})^{+} \subset (\tau_{(1,2)}{}^{*})^{*} \subset (\tau_{(1,2)}{}^{*}\alpha)^{*} \subset (\tau_{(1,2)}{}^{*}p)^{*}$.
Remark 3.12.LetJ_1andJ_2be two ideals on $(X,(\tau_{1,2})^{*},I)$ and$J_1 \subseteq J_2$, then $(\tau_{(1,2)}{}^{*}k)^{*}(J_1) \subseteq (\tau_{(1,2)}{}^{*}k)^{*}(J_2)$.
Proof.Proof is obviously from theorem 3.5(iv).
Definition 3.13. Let $(X,(\tau_{1,2})^{+}, I)$ be an ideal extended bitopological space and a subset$A \subseteq X$is said to be $(\tau_{(1,2)}{}^{*}k)^{*}$-closed (resp. $(\tau_{(1,2)}{}^{*}k)^{*}$-dense) if $A_k^{*} \subseteq A$(resp.$A_k^{*} \supseteq A$)andAis said to be $(\tau_{(1,2)}{}^{*}k)^{*}$-open if complement of Ais $(\tau_{(1,2)}{}^{*}k)^{*}$-closed. The set of all$(\tau_{(1,2)}{}^{*}k)^{*}$-closed sets and $(\tau_{(1,2)}{}^{*}k)^{*}$-open sets are respectively denoted by $(\tau_{(1,2)}{}^{*}k)^{*}C(X)$ and$(\tau_{(1,2)}{}^{*}k)^{*}O(X)$. Alsokint*(A) is the interior ofAin $(\tau_{(1,2)}{}^{*}k)^{*}O(X)$.
Theorem 3.14.Let $(X,(\tau_{1,2})^{+}, I)$ be an ideal extended bitopological space. Then the collection$\beta(I, \tau_{(1,2)}{}^{*}k) = \{V-J : V \in \tau_{(1,2)}{}^{*}k$ and$J \in I\}$is a basis for $(\tau_{(1,2)}{}^{*}k)^{*}$-topology (where$\tau_{(1,2)}{}^{*}k= (\tau_{1,2})^{+}O(X)$ or $(1,2)^{*+}O(X)$).
Proof.Let $(X,(\tau_{1,2})^{+},I)$ be an ideal extended bitopological space and$U \in (\tau_{(1,2)}{}^{*}k)^{*} \Leftrightarrow X-U$is $(\tau_{(1,2)}{}^{*}k)^{*}$-closed $\Leftrightarrow (X-U)_k^{*} \subseteq X-U \Leftrightarrow U \subseteq X-(X-U)_k^{*}$. Therefore, $x \in U$whichimplies that $x \notin (X-U)_k^{*}$. Then there exists a $V \in \tau_{(1,2)}{}^{*}k$such that$V \cap (X-U) \in I$. Letus take$J=V \cap (X-U) \in I$and we have$x \in V-J \subseteq U$where$V \in \tau_{(1,2)}{}^{*}kandJ \in I$. Hence$\beta(I, \tau_{(1,2)}{}^{*}k) =\{V-J: V \in \tau_{(1,2)}{}^{*}kandJ \in I\}$is a basis for $(\tau_{(1,2)}{}^{*}k)^{*}$-topology.
Conclusion: The ideal topological spaces was treated a lot by general topologist in recent years.In this paper, we introduced new types of local functions and constructed many ideal extendedtopologies on a non empty set and established the relationship between them. The conceptsof ideal extended bitopological spaces can be extend to the other applicable research areas oftopology such as nano topology, fuzzy topology, intuitionistic topology, and digital topology.

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