FUZZY ANALYSIS OF M^X/M/1/MWV WITH BREAKDOWN QUEUEING MODEL

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Abstract: In this paper we analyse the bulk arrival multiple working vacations queuing model with breakdown policies by using fuzzy numbers. The bulk service rule is applied. The most general bulk service rule is introduced by Neuts[14]. The batches are served according to FCFS discipline. The basic idea is to convert all these fuzzy numbers into crisp values by applying Robust Ranking Technique. Further Robust Ranking technique is used to find the expected mean queue length. Moreover, the analytical results are numerically illustrated under crisp environment for the different values of the parameters.

Keywords: Bulk Service, Robust Ranking Technique, Multiple Working Vacations, Fuzzy number and Crisp value.

I. **Introduction:** Fuzzy is one of the paradigmatic changes in Science and Mathematics in this century, fuzzy concerns the concept of uncertainty. Uncertainty is an important commodity in various fields. This important role of uncertainty began the second stage of the transition from the traditional view to the modern view of uncertainty which was characterized by the emergence of several new theories distinct from probability theory.

Fuzzy set were introduced by Lotfi A. Zadeh[10] as an extension of the classical notation of set. Fuzzy set theory permits the gradual assessment of the membership of elements in a set, described with the aid of a membership function valued in the real unit interval [0,1].Fuzzy sets have been able to provide solution to many real world problems. In logic, fuzzy concepts which in their application are neither completely true nor completely false or which are partially true or partially false. A fuzzy variable is a value which could lie in a probable range defined by quantitative limits or parameters, and be usefully described with imprecise categories.

A fuzzy set can be defined mathematically by assigning to each possible individual in the universe of discourse a value representing its grade of membership in fuzzy set. This grade corresponds to the degree to which that individual is similar or compatible with the concept represented by the fuzzy set. Thus individuals any belong in the fuzzy set to a greater or lesser degree as indicated by a larger or smaller membership grade, these membership grades are very often represented by real number values ranging in the closed interval between 0 and 1.

Li and Lee[9] investigated analytical results for two fuzzy queues using a general approach based on Zadeh's extension principle. Nagi and Lee[13] proposed a procedure using α-cut and two variables simulations to analyze fuzzy queues, using parametric programming. Kao et al[8] proposed a general approach for queuing systems in a fuzzy environment based on Zadeh's extension principle. Moreover Julia Rose Mary and Shanmuha Priya[6] have discussed $FM^X_{(m,n)}/G_{sos}/1$ with fuzzy breakdowns and fuzzy multiple vacations, they derived the membership function ot total average cost using Zadeh extension principle. Robust Ranking Technique has been discussed by researchers like Choobinesh and Li[2], S.P.Chan[1], A. Nagoor Gani and V.Ashok Kumar[12] have analyzed bulk arrival fuzzy queue with fuzzy outputs. Kao applied α-cut approach to reduce a fuzzy queue into family of crisp queue. Julia Rose Mary and Majula Christina[4] studied Fuzzy parameters on total average cost for $M^X_{(m,N)}$/M/1/BD/MV. B.Palapandi and G.Geedhamani[15] have analyzed evaluations of performance measures of bulk arrival queue with fuzzy using Robust Ranking Technique. Recently Julia Rosy Mary and Angel Jenita[3] have studied the cost analysis for bi level threshold policy and single vacation of an unreliable server with fuzzy parameters and also Optimal operating policy of $FM^{[X]}$/FM/1 multiple working vacation queuing system using Robust Ranking Technique. Julia Rose Mary and Pavithra[7] studied the FM/M(a,b)/1 with multiple working vacations queuing model in Robust Ranking Technique. Julia Rose Mary and Maria Remona[5] have analyzed the expected queue length of M^X/M/1/MWV/BD Queuing Model.

With the help of these available literatures we determine the fuzzy performance measure for M^X/M/1/MWV/BD Queuing Model.

II. **Model Description:** In this model it is assume the arrival rate λ with Poisson distribution, service rate μ for busy period and service rate μ_v for vacation period with exponential distribution of vacation parameters η, breakdown parameter α and breakdown repair parameter β. Their fuzzy set can be represented as $\bar{\lambda}, \bar{\mu}, \bar{\mu}_v, \bar{\eta}, \bar{\alpha}, and\ \bar{\beta}$.(i.e.,) $\bar{\lambda} = \{p, \theta_{\bar{\lambda}}(p) / p \in S(\bar{\lambda})\}$, $\bar{\mu} = \{q, \theta_{\bar{\mu}}(q) / q \in S(\bar{\mu})\}$, $\bar{\mu}_v = \{r, \theta_{\bar{\mu}_v}(r) / r \in S(\bar{\mu}_v)\}$, $\bar{\eta} = \{t, \theta_{\bar{\eta}}(t) / t \in S(\bar{\eta})\}$, $\bar{\alpha} = \{a, \theta_{\bar{\alpha}}(a) / a \in S(\bar{\alpha})\}$ $\bar{\beta} = \{b, \theta_{\bar{\beta}}(b) / b \in S(\bar{\beta})\}$.

Here $\theta_m(n)$ and S(m) denote the membership function and support of a where m = $\bar{\lambda}, \bar{\mu}, \bar{\mu}_v, \bar{\eta}, \bar{\alpha}, \bar{\beta}$ are fuzzy numbers and n = p,q,r,t,u,v are the crisp values corresponding to arrival rate, service rate for busy period, service rate for vacation period, vacation parameter, breakdown parameter and breakdown repair parameter respectively.

On the basis of the concept of α-cut we develop a mathematical programming approach for deriving the α-cuts of $\bar{\lambda},\bar{\mu},\bar{\mu}_v,\bar{\eta},\bar{\alpha},\bar{\beta}$ as crisp intervals which are given by

$$\bar{\lambda}(\omega)=\{p\in P/\theta_{\bar{\lambda}}(p)\ge\omega\},\qquad \bar{\mu}(\omega)=\{q\in Q/\theta_{\bar{\mu}}(q)\ge\omega\},$$
$$\bar{\mu}_v(\omega)=\{r\in R/\theta_{\bar{\mu}_v}(r)\ge\omega\},$$
$$\bar{\eta}(\omega)=\{t\in T/\theta_{\bar{\eta}}(t)\ge\omega\},\qquad \bar{\alpha}(\omega)=\{a\in A/\theta_{\bar{\alpha}}(a)\ge\omega\},$$
$$\bar{\beta}(\omega)=\{b\in B/\theta_{\bar{\beta}}(b)\ge\omega\}.$$

where 0<ω≤1. Hence a fuzzy queue can be reduced to a family crisp queues with different α-cuts as $\{\lambda(\omega)/0<\omega\le1\},\{\mu(\omega)/0<\omega\le1\},\{\mu_v(\omega)/0<\omega\le1\}$, $\{\eta(\omega)/0<\omega\le1\},\{\alpha(\omega)/0<\omega\le1\}, and \{\beta(\omega)/0<\omega\le1\}$.

Let the confidence interval of the fuzzy sets λ(ω), μ(ω), $\mu_v(\omega)$, η(ω), α(ω), β(ω) be $[l_\lambda(\omega),u_\lambda(\omega)]$, $[l_\mu(\omega),u_\mu(\omega)],[l_{\mu_v}(\omega),u_{\mu_v}(\omega)],[l_\eta(\omega),u_\eta(\omega)]$, $[l_\alpha(\omega),u_\alpha(\omega)],[l_\beta(\omega),u_\beta(\omega)]$.

Then the expected queue length (L_q) is given by

$$L_q=\frac{\left(1-\frac{\alpha\rho}{\beta(1-\rho)}\right)}{\left(1+\frac{\alpha D}{\beta C}\right)}$$
$$\left\{\frac{\rho\mu}{\beta C^2}\left[C^2+\alpha\left(z_1-\frac{D}{\lambda E(X)}\right)(D-C)+\alpha D\left(\frac{C}{\beta}-\frac{1}{\rho}\right)-1\right]+\right.$$
$$\left.\left(1+\frac{\alpha D}{\beta C}\right)\left[\frac{\lambda\left(E(X)+E(X^2)\right)}{2\mu(1-\rho)}+\frac{\lambda E(X)-\mu_V}{\eta}+\frac{z_1\mu(1-\rho)}{C}\right]\right\}$$

where C = $\mu(1-z_1)+\lambda z_1(X(z_1)-1)$, D = $\lambda z_1(X(z_1)-1)$ and $\rho=\frac{\lambda E(X)}{\mu}$.

In the above formula by applying the fuzzy variable for arrival rate, service rate for busy period, service rate for vacation, vacation parameter, breakdown parameter and repair parameter then we get,

Then the expected queue length (L_q) is given by

$$L_q=\frac{\left(1-\frac{a\rho}{b(1-\rho)}\right)}{\left(1+\frac{aD}{bC}\right)}$$
$$\left\{\frac{\rho\mu}{bC^2}\left[C^2+a\left(z_1-\frac{D}{\rho\mu}\right)(D-C)+aD\left(\frac{C}{b}-\frac{1}{\rho}\right)-1\right]+\right.$$
$$\left.\left(1+\frac{aD}{bC}\right)\left[\frac{\lambda\left(E(X)+E(X^2)\right)}{2\mu(1-\rho)}+\frac{\lambda E(X)-\mu_V}{\eta}+\frac{z_1\mu(1-\rho)}{C}\right]\right\}$$

where C = $q(1-z_1)+pz_1(X(z_1)-1)$, D = $pz_1(X(z_1)-1)$ and $\rho=\frac{pE(X)}{q}$.

Now we apply the Robust Ranking Technique to the required formula.

II. **Robust Ranking Method:** To find the characteristics of system interest interms of crisp value we defuzzyify the numbers into crisp ones by a fuzzy number ranking method. Robust ranking technique by Nagarajan & Solai Raju[11] satisfies compensation, linearity and additive properties results which are consistent with human intuition. By giving a convex fuzzy number p, the Robust Ranking Index is defined by, $R(\bar{p})=\int_0^1 0.5(p_\alpha^L+p_\alpha^U)d\alpha$

where $(p_\alpha^L+p_\alpha^U)$ is the α-level cut of the fuzzy number $\bar{p}$. In this paper we use this method for ranking the fuzzy numbers. The Robust ranking index $R(\bar{p})$ gives the representative value of the fuzzy number $\bar{p}$. It satisfies the linearity and additive property.

III. **Numerical Example:** Consider **M^x/M/1/MWV/BD** Queuing System. The corresponding parameters such as arrival rate,service rate for busy, service rate for vacation, breakdown parameter, repair parameter and vacation parameter are fuzzy numbers.

Let us consider the parameters as,λ=[0.1,0.2,0.3,0.4],μ=[0.1,0.15,0.2,0.25], μ_v=[0.0,0.05,0.1,0.15], η=[0.01,0.02,0.03,0.04],α=[0.03,0.035,0.04,0.045],β=[0.1,0.15,0.2,0.25]whose intervals of confidence are[0.1+γ,0.4-γ], [0.1+ γ,0.25- γ],[0.0+ γ,0.15- γ], [0.01+ γ,0.04- γ], [0.03+ γ,0.045- γ], [0.1+ γ,0.25- γ] respectively. Now we evaluate R(0.1,0.2,0.3,0.4) by applying Robust Ranking Method. The membership function of the Trapezoidal Fuzzy number (0.1,0.2,0.3,0.4) is

$$\theta(p)=\begin{cases}\frac{p-0.1}{0.1} & ,0.1\le p\le0.2\\ 0.2 & ,0.2\le p\le0.3\\ \frac{0.4-p}{0.1} & ,0.3\le p\le0.4\\ 0 & otherwise\end{cases}$$

The α-cut of the fuzzy numbers (0.1,0.2,0.3,0.4) is (0.1+0.1γ,0.4-0.1γ) for which

$$R(\bar{\lambda})=R(0.1,0.2,0.3,0.4)$$
$$=\int_0^1 0.5(0.1+0.4)dp$$
$$=\int_0^1 0.5(0.5)dp$$
$$=0.25$$

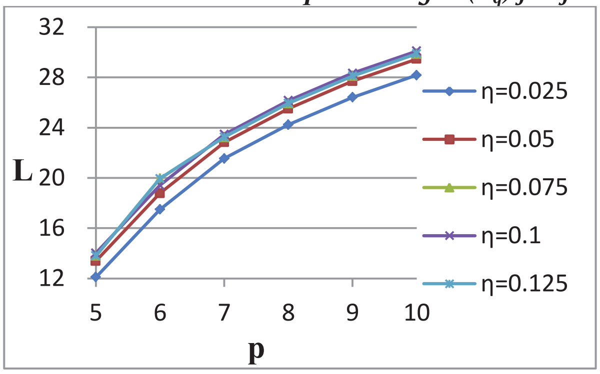
Similarly by calculating the α-cut of the fuzzy numbers forμ=[0.1,0.15,0.2,0.25],μ_v=[0.0,0.05,0.1,0.15], η=[0.01,0.02,0.03,0.04],α=[0.03,0.035,0.04,0.045],β=[0.1,0.15,0.2,0.25], we get the Robust Ranking Indices for the fuzzy numbers $\bar{\lambda},\bar{\mu},\bar{\mu}_v,\bar{\eta},\bar{\alpha}$ and $\bar{\beta}$ as $R(\bar{\lambda})=0.25, R(\bar{\mu})=0.2, R(\bar{\mu}_v)=0.1, R(\bar{\eta})=0.025, R(\bar{\alpha})=0.04$ $and\ R(\bar{\beta})=0.2$. Moreover p denotes the batch size.

Thus the mean queue length(L_q) is calculated and tabulated.

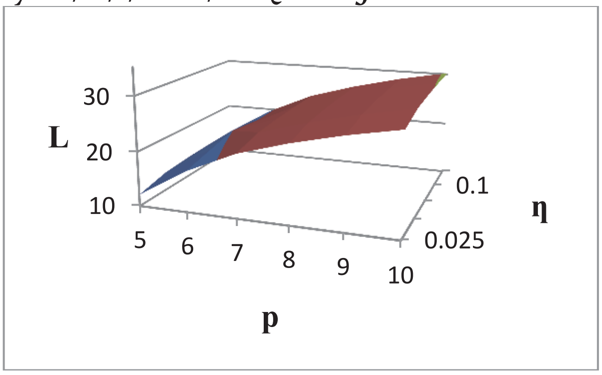
TABLE 1: ***Mean queue length (L_q) for fuzzy $M^X/M/1/MWV/BD$ Queuing Model.***

λ=0.25	μ=0.2	μ_v =0.1	α=0.04	β=0.2	
η / p	0.025	0.05	0.075	0.1	0.125
p=5	12.1005	13.3805	13.8045	14.0205	14.1485
p=6	17.4899	18.7699	19.9400	19.4100	19.5380
p=7	21.5419	22.8219	23.2459	23.4619	23.5899
p=8	24.2328	25.5130	25.9370	26.1530	26.2810
p=9	26.4128	27.6928	28.1168	28.3328	28.4608
p=10	28.1756	29.4556	29.8796	30.0956	30.2236

Mean queue length (L_q) for fuzzy $M^X/M/1/MWV/BD$ Queuing Model.



Graph (1)



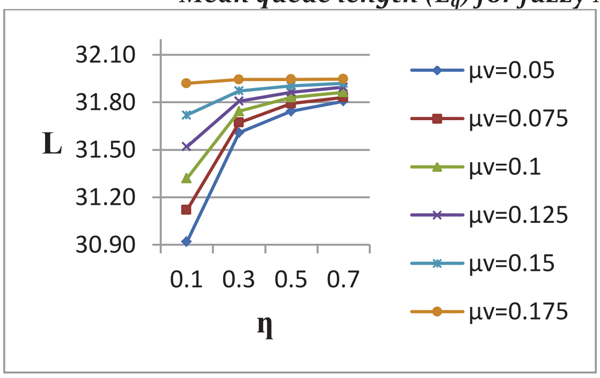
Graph (1A)

The above tabulated values are shown in the graph 1. From the table (1) and graph (1 & 1A) we find that when the vacation parameter 'η' and the batch size 'p' increases, then the expected number of customers in the system also increases.By making few changes in the values of fuzzy numbers and proceeding similarly, the Robust Ranking Indices for the fuzzy numbers $\bar{\lambda}, \bar{\mu}, \bar{\mu}_v, \bar{\eta}, \bar{\alpha}$ and $\bar{\beta}$ are calculated for $R(\bar{\lambda}) = 0.23$, $R(\bar{\mu}) = 0.2$, $R(\bar{\mu}_v) = 0.05$, $R(\bar{\eta}) = 0.1$, $R(\bar{\alpha}) = 0.04$ and $R(\bar{\beta}) = 0.2$.

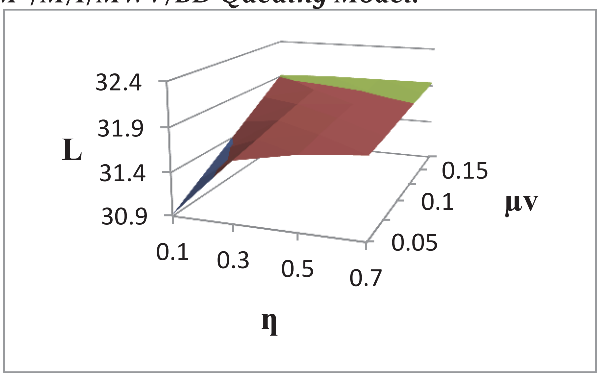
TABLE 2: ***Mean queue length (L_q) for fuzzy $M^X/M/1/MWV/BD$ Queuing Model.***

λ=0.23 β=0.2	μ=0.2	p=10	α=0.04	
η / μ_v	0.1	0.3	0.5	0.7
0.05	30.9196	31.6076	31.7436	31.8076
0.075	31.1196	31.6716	31.7916	31.8316
0.1	31.3196	31.7436	31.8316	31.8636
0.125	31.5196	31.8076	31.8636	31.8956
0.15	31.7196	31.8716	31.9036	31.9196
0.175	31.9196	31.9436	31.9444	31.9468

Mean queue length (L_q) for fuzzy $M^X/M/1/MWV/BD$ Queuing Model.



Graph (2)



Graph (2A)

Thus by ploting the above table values in the graph (2 & 2A), we find from the table(2) and graph (2 & 2A) that when the vacation parameter 'η' and the service rate during vacation 'μ_v' increases, then the expected number of customers in the system also increases.

V. **Conclusion:** Thus from this paper we have analyzed the queuing model for bulk arrival under multiple working vacations with breakdowns policies in Robust Ranking Techniques. And their values are calculated for two various Robust Ranking Indices of the fuzzy numbers and the calculated values are displayed into two-dimensional and three-dimensional graphs respectively. Thus fuzzy analyses explains the concept of uncertainty to**M^X/M/1/MWV/BD** Queuing Model. It helps to began a profitable transition in varies field.

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