PAIRWISE INTUITIONISTIC FUZZY REGULAR VOLTERRA SPACES

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Abstract: The aim of this paper is to introduce the concepts of pairwise intuitionistic fuzzy regular Volterra spaces and pairwise intuitionistic fuzzy weakly regular Volterra space. In this regard, the notion of pairwise intuitionistic fuzzy regular $G_\delta -$ set and $F_\sigma -$ set is introduced. Using it, we investigate several characterizations ofpairwise intuitionistic fuzzy regular Volterra spaces and weakly regular Volterra spaces.

Keywords: Intuitionistic fuzzy dense set, pairwise intuitionistic fuzzy regular $G_\delta -$set, pairwise intuitionistic fuzzy regular $F_\sigma -$set, pairwise intuitionistic fuzzy volterra space.

1. Introduction: The ususal notion of set topology was generalised with the introduction of fuzzy topology by C.L.Chang[7] based on the concept of fuzzy sets invented by L.A.Zadeh[14]. Today fuzzy topology has been firmly established as one of the basic disciplines of fuzzy mathematics. In 1989, A. Kandil [10] introduced the concept of fuzzy bitopological spaces. The concepts of Volterra spaces have been studied extensively in classical topology in [11,12,13]. The concept of Volterra spaces in intuitionistic fuzzy setting was introduced by Sharmila.S and Arockiarani [13]. In this paper, the concepts of pairwise intuitionistic fuzzy regular $G_\delta -$set and $F_\sigma -$ set are introduced and studied. By means of this we introduce the concepts of pairwise intuitionistic fuzzy regular volterra spaces and pairwise intuitionistic fuzzy weakly regular volterra space.

2. Preliminaries:

Definition 2.1 [2].An intuitionistic fuzzy set (IFS, in short) A in X is an object having the form $A=\{x,\mu_A(x),\upsilon_A(x)/x\in X\}$where the functions $\mu_A: X\to I$ and $\upsilon_A: X\to I$ denote the degree of membership (namely $\mu_A(x)$)and the degree of non-membership (namely $\upsilon_A(x)$) of each element $x\in X$ to the set A on a nonempty set X and $0\le \mu_A(x)+\upsilon_A(x)\le 1$ for each $x\in X$. Obviously every fuzzy set A on a nonempty set X is an IFS's A and B be in the form $A=\{x,\mu_A(x),1-\mu_A(x)/x\in X\}$

Definition 2.2 [2].Let X be a nonempty set and the IFS's A and B be in the form $A=\{x,\mu_A(x),\upsilon_A(x)/x\in X\}$, $B=\{x,\mu_B(x),\upsilon_B(x)/x\in X\}$ and let $A=\{A_j : j\in J\}$ be an arbitrary family of IFS's in X. Then we define

(i) $A\subseteq B$ if and only if $\mu_A(x)\le\mu_B(x)$ and $\upsilon_A(x)\ge\upsilon_B(x)$ for all $x\in X$.

(ii) A=B if and only if $A\subseteq B$ and $B\subseteq A$.

(iii) $\overline{A}=\{x,\upsilon_A(x),\mu_A(x)/x\in X\}$.

(iv) $A\cap B=\{x,\mu_A(x)\cap\mu_B(x),\upsilon_A(x)\cup\upsilon_B(x)/x\in X\}$

$A\cup B=\{x,\mu_A(x)\cup\mu_B(x),\upsilon_A(x)\cap\upsilon_B(x)/x\in X\}$ $1_\sim=\{\langle x,1,0\rangle x\in X\}$ and $0_\sim=\{\langle x,0,1\rangle x\in X\}$.

Definition 2.3 [7].An intuitionistic fuzzy topology (IFT, in short) on a nonempty set X is a family τ of an intuitionistic fuzzy set (IFS, in short) in X satisfying the following axioms:

(i) $0_\sim, 1_\sim\in\tau$.

(ii) $A_1\cap A_2\in\tau$ for any $A_1, A_2\in\tau$.

(iii) $\bigcup A_j\in\tau$ for any $A_j : j\in J\subseteq\tau$.

The complement $\overline{A}$ of intuitionistic fuzzy open set (IFOS, in short) in intuitionistic fuzzy topological space (IFTS, in short) (X,τ) is called an intuitionistic fuzzy closed set(IFCS, in short).

Definition 2.4 [7].Let (X,τ) be an IFTS and $A=\{x,\mu_A(x),\upsilon_A(x)\}$ be an IFS in X. Then the fuzzy interior and closure of A are denoted by

(i) $cl(A)=\bigcap$ {K: K is an IFCS in X and $A\subseteq K$}.

(ii) $\text{int}(A)=\bigcup$ {G: G is an IFOS in X and $G\subseteq A$}.

Definition 2.5 [7].Let (X,τ) be any IFTS. Let A be an IFSs in (X,τ). Then the intuitionistic fuzzy closure operator satisfy the following properties:
(i) $1-IFcl(A)=IF\operatorname{int}(1-A)$
(ii) $1-IF\operatorname{int}(A)=IFcl(1-A)$

Definition 2.6 [6]. An IFS A in an intuitionistic fuzzy bitopological space (IFBTS, in short) (X,τ_1,τ_2) is called a *pairwise IF regular open set* in (X,τ_1,τ_2) if $IF\operatorname{int}\tau_1 IFcl\tau_2(A)=A=IF\operatorname{int}\tau_2 IFcl\tau_1(A)$.

Definition 2.7 [6]. An IFS A in an IFBTS (X,τ_1,τ_2) is called a *pairwise IF regular closed set* in (X,τ_1,τ_2) if $IFcl\tau_1 IF\operatorname{int}\tau_2(A)=A=IFcl\tau_2 IF\operatorname{int}\tau_1(A)$.

Definition 2.8 [8].An IFS A in IFTS (X,τ) is called *intuitionistic fuzzy dense* if there exists no IFCS B in (X,τ) such that $A\subseteq B\subseteq 1_{\sim}$.

Definition 2.9 [8].If A is an *intuitionistic fuzzy nowhere dense* set in (X,τ) then $\overline{A}$ is an IF dense set in (X,τ).

Definition 2.10 [8]. Let A be an IFS in (X,τ). If A is an IFCS in (X,τ) with $IF\operatorname{int}(A)=0_{\sim}$, then A is an *intuitionistic fuzzy nowhere dense set* in (X,τ).

Definition 2.11 [8]. Let (X,τ) be an IFTS. An IFS A in (X,τ) is called *intuitionistic fuzzy first category* if $A=\bigcup_{i=1}^{\infty}B_i$, where B_i's are intuitionistic fuzzy nowhere dense sets in (X,τ). Any other IFS in (X,τ) is said to be of *intuitionistic fuzzy second category.*

Definition 2.12 [8]. An IFTS (X,τ) is called intuitionistic fuzzy first category if the IFS $1_{\sim}$ is an *intuitionistic fuzzy first category* in (X,τ). That is $1_{\sim}=\bigcup_{i=1}^{\infty}A_i$, where A_i's are IF nowhere dense sets in (X,τ). otherwise (X,τ) is said to be of *intuitionistic fuzzy second category*.If A is an IF first category set in (X,τ), then 1-A is called *IF residual set* in (X,τ)

Definition 2.13 [2]. Let (X,τ) be a FTS and be a fuzzy set in X. Then λ is called fuzzy G_δ if $\lambda=\bigcap_{i=1}^{\infty}\lambda_i$ where each $\lambda_i\in T$.

Definition2.14 [2].Let (X,τ) be a FTS and be a fuzzy set in X. Then λ is called fuzzy F_σ if $\lambda=\bigcup_{i=1}^{\infty}\lambda_i$ where each $\lambda_i\in T$.

Definition 2.15 [13]. An intuitionistic fuzzy topological space (X,τ) is called IFVolterra space, if $IFcl(\bigcap_{i=1}^{N}A_i)=1_{\sim}$ where A_i 's are IFdense and IF $G_\delta-$sets in (X,τ).

3. Pairwise if regular $G_\delta-$sets and $F_\sigma-$sets

Definition 3.1. Let (X,τ_1,τ_2) be *IFBTS.* An IFS A in (X,τ_1,τ_2) is called a *pairwise IF regular* $G_\delta-set$ if $A=\bigcap_{k=1}^{\infty}(IF\operatorname{int}_{\tau_i}(IFcl_{\tau_j}(A_k)))$, ($i\neq j$ and *i,j* = 1,2), where A_k*'s* are IFSs in (X,τ_1,τ_2).

Definition 3.2. Let (X,τ_1,τ_2) be IFBTS. An IFS A in (X,τ_1,τ_2) is called a *pairwise IF regular* $F_\sigma-set$ if $A=\bigcup_{k=1}^{\infty}(IFcl_{\tau_i}(IF\operatorname{int}_{\tau_j}(A_k)))$, ($i\neq j$ and *i,j* = 1,2), where A_k*'s* are IFSs in (X,τ_1,τ_2).

Proposition 3.3. If A is a pairwise IF regular $G_\delta-set$ in an IFBTS (X,τ_1,τ_2) if and only if *1-A* is a *pairwise IF regular* $F_\sigma-set$ in (X,τ_1,τ_2).

Proof. Let A be a pairwise IF regular $G_\delta-set$ in (X,τ_1,τ_2) then, ($i\neq j$ and *i,j* = 1,2), where A_k*'s* are IFSs in (X,τ_1,τ_2). Then

$A=\bigcap_{k=1}^{\infty}(IF\operatorname{int}_{\tau_i}(IFcl_{\tau_j}(A_k)))$, ($i\neq j$ and $i,j=1,2$), where A_k's are IFSs in (X,τ_1,τ_2). Now

$1-A=1-\bigcap_{k=1}^{\infty}(IF\operatorname{int}_{\tau_i}(IFcl_{\tau_j}(A_k)))=\bigcup_{k=1}^{\infty}(1-IF\operatorname{int}_{\tau_i}(IFcl_{\tau_j}(A_k)))=\bigcup_{k=1}^{\infty}(1-IFcl_{\tau_i}(IF\operatorname{int}_{\tau_j}(1-A_k)))$. Let $B_k=1-A_k$. Hence

$1-A=\bigcup_{k=1}^{\infty}(IFcl_{\tau_i}IF\operatorname{int}_{\tau_j}(1-A_k))$, ($i\neq j$ and $i,j=1,2$) implies that 1-*A*is a *pairwise IF regular* $F_\sigma-set$ in (X,τ_1,τ_2).

Conversely, let A be a pairwise *IF regular* $F_\sigma-set$ in (X,τ_1,τ_2). Then

$A=\bigcup_{k=1}^{\infty}(IFcl_{\tau_i}(IF\operatorname{int}_{\tau_j}(A_k)))$, ($i\neq j$ and $i,j=1,2$), where A_k's are IFSs in (X,τ_1,τ_2). Now

$1-A=1-\bigcup_{k=1}^{\infty}(IFcl_{\tau_i}IF\operatorname{int}_{\tau_j}(A_k))=\bigcap_{k=1}^{\infty}(1-IFcl_{\tau_i}IF\operatorname{int}_{\tau_j}(A_k))=\bigcap_{k=1}^{\infty}(IFcl_{\tau_i}IF\operatorname{int}_{\tau_j}(1-A_k))$.

Let $B_k=1-A_k$. Hence

$1-A=\bigcap_{k=1}^{\infty}(IF\operatorname{int}_{\tau_i}IFcl_{\tau_j}(A_k))$ ($i\neq j$ and $i,j=1,2$) implies that 1-A is a pairwise IF regular $G_\delta-set$in (X,τ_1,τ_2).

Proposition 3.4. Let (X,τ_1,τ_2) be an IFBTS

(i) If A is a pairwise IFOS in (X,τ_1,τ_2), then $IFcl_{\tau_i}(A)$, (i=1,2) is a pairwise IFRCS in (X,τ_1,τ_2).

(ii) IF B is a pairwise IFCS in (X,τ_1,τ_2), then $IF\operatorname{int}_{\tau_i}(B)$, (i=1,2) is a pairwise IFROS in (X,τ_1,τ_2).

Proof.(i). Let A be a pairwise IFOS in (X,τ_1,τ_2) and $IF\operatorname{int}_{\tau_j}IFcl_{\tau_i}(A)\subseteq IFcl_{\tau_i}(A)$, ($i\neq j$ and $i,j=1,2$) implies that $IFcl_{\tau_i}IF\operatorname{int}_{\tau_j}IFcl_{\tau_i}(A)\subseteq IFcl_{\tau_i}IFcl_{\tau_i}(A)=IFcl_{\tau_i}(A)$ Hence $IFcl_{\tau_i}IF\operatorname{int}_{\tau_j}IFcl_{\tau_i}(A)\subseteq IFcl_{\tau_i}(A)$..................I.

Since A is a pairwise IFOS, we have $A=IF\operatorname{int}_{\tau_j}(A)$, (j=1,2). Now $A=IF\operatorname{int}_{\tau_j}(A)\subseteq IF\operatorname{int}_{\tau_j}IFcl_{\tau_i}(A)$ implies that $A\subseteq IF\operatorname{int}_{\tau_j}IFcl_{\tau_i}(A)$. Hence $IFcl_{\tau_i}(A)\subseteq IFcl_{\tau_j}IF\operatorname{int}_{\tau_i}IFcl_{\tau_i}(A)$...............II

From I and II we have $IFcl_{\tau_j}IF\operatorname{int}_{\tau_i}IFcl_{\tau_i}(A)=IFcl_{\tau_i}(A)$, ($i\neq j$ and $i,j=1,2$). Therefore $IFcl_{\tau_i}(A)$ is a pairwise IFRCS in (X,τ_1,τ_2).

(ii). Let B be a pairwise IFCS in (X,τ_1,τ_2). Then *1-B* is a pairwise IFOS in (X,τ_1,τ_2). By (i) $IFcl_{\tau_i}(1-B)$ is a pairwise IFRCS in (X,τ_1,τ_2). Then $1-IF\operatorname{int}_{\tau_i}(B)$ is a pairwise IFRCS in (X,τ_1,τ_2). Hence $IF\operatorname{int}_{\tau_i}(B)$ is a pairwise IFROS in (X,τ_1,τ_2).

Proposition 3.5. Let (X,τ_1,τ_2) be an IFBTS,

(i) If A is a pairwise IFregular $G_\delta-$setin (X,τ_1,τ_2) then $A=\bigcap_{k=1}^{\infty}\delta_k$, where $(\delta_k)'s$ are pairwise IFROS in (X,τ_1,τ_2).

(ii) IF B is a pairwise IFregular $F_\sigma-$set in (X,τ_1,τ_2) then $B=\bigcup_{k=1}^{\infty}(\eta_k)$, where $(\eta_k)'s$ are pairwise IFRCS in (X,τ_1,τ_2).

Proof. Let A be a pairwise IFregular G_δ −set in (X,τ_1,τ_2), then $A=\bigcap_{k=1}^{\infty}(IF\operatorname{int}_{\tau_i} IFcl_{\tau_j}(A_k))$, $(i \neq j$ and $i,j=1,2)$, where (A_k)'s are in (X,τ_1,τ_2). Take $\delta_k = IF\operatorname{int}_{\tau_i} IFcl_{\tau_j}(A_k)$. Now $IF\operatorname{int}_{\tau_i} IFcl_{\tau_j}(A_k) = IF\operatorname{int}_{\tau_i} IFcl_{\tau_j}(IF\operatorname{int}_{\tau_i} IFcl_{\tau_j}(A_k)) \subseteq IF\operatorname{int}_{\tau_i} IFcl_{\tau_j} IFcl_{\tau_j}(A_k)$

$= IF\operatorname{int}_{\tau_i} IFcl_{\tau_j}(A_k) = \delta_k$ ....I. Also

$IF\operatorname{int}_{\tau_i} IFcl_{\tau_j}(\delta_k) = IF\operatorname{int}_{\tau_i} IFcl_{\tau_j}(IF\operatorname{int}_{\tau_i} IFcl_{\tau_j}(A_k)) \supseteq IF\operatorname{int}_{\tau_i} IF\operatorname{int}_{\tau_i} IFcl_{\tau_j}(A_k)$

$= IF\operatorname{int}_{\tau_i} IFcl_{\tau_j}(A_k) = \delta_k$. Hence $IF\operatorname{int}_{\tau_i} IFcl_{\tau_j}(\delta_k) \geq \delta_k$ II

From I and II, we have $IF\operatorname{int}_{\tau_i} IFcl_{\tau_j}(\delta_k) = \delta_k$. Hence (δ_k)'s are pairwise IFROSs in (X,τ_1,τ_2). Therefore $A=\bigcap_{k=1}^{\infty}\delta_k$, where $(\delta_k)'s$ are pairwise IFROS in (X,τ_1,τ_2).

(ii) Let B is a pairwise IF regular F_σ −set in (X,τ_1,τ_2). Then by Proposition 3.3., *1-B*is a pairwise IF regular G_δ −set in (X,τ_1,τ_2). By (i), $1-B=\bigcap_{k=1}^{\infty}(\delta_k)$, where the IFS $(\delta_k)'s$ are pairwise IFROSs in (X,τ_1,τ_2). Now $B=\bigcup_{k=1}^{\infty}(1-\delta_k)$. Let $1-\delta_k=\eta_k$. Hence $B=\bigcup_{k=1}^{\infty}(\eta_k)$, where the IFSs $(\eta_k)'s$ are pairwise IFRCSs in (X,τ_1,τ_2).

Proposition 3.6. If A is a pairwise IFregular G_δ −set in (X,τ_1,τ_2), then A is a pairwise IF G_δ −set in (X,τ_1,τ_2).

Proof. Let A be a pairwise IFregular G_δ −set in (X,τ_1,τ_2), then by Proposition 3.5, $A=\bigcap_{k=1}^{\infty}(\delta_k)$, where the IFSs $(\delta_k)'s$ are pairwise IFROSs in (X,τ_1,τ_2). Since every pairwise IFROS in (X,τ_1,τ_2) is a IFOS in (X,τ_1,τ_2), $(\delta_k)'s$ are pairwise IFOSs in (X,τ_1,τ_2). Hence $A=\bigcap_{k=1}^{\infty}(\delta_k)$, $(\delta_k)'s$ are pairwise IFOSs in (X,τ_1,τ_2), implies that A is a pairwise IF regular G_δ −set in (X,τ_1,τ_2).

Proposition 3.7. If A is a pairwise IFregular F_σ −set in (X,τ_1,τ_2), then A is an IF F_σ −set in (X,τ_1,τ_2).

Proof. Let A be a pairwise IFregular F_σ −set in (X,τ_1,τ_2), then by Proposition 3.5, $A=\bigcup_{k=1}^{\infty}(\eta_k)$, where the IFSs $(\eta_k)'s$ are pairwise IFROSs in (X,τ_1,τ_2). Since every pairwise IFRCS in (X,τ_1,τ_2) is a IFCS in (X,τ_1,τ_2), $(\eta_k)'s$ are pairwise IFCSs in (X,τ_1,τ_2). Hence $A=\bigcup_{k=1}^{\infty}(\eta_k)$, $(\eta_k)'s$ are pairwise IFCSs in (X,τ_1,τ_2), implies that A is a pairwise IF regular F_σ −set in (X,τ_1,τ_2).

Proposition 3.8. If A is a pairwise IFregular G_δ −set in (X,τ_1,τ_2). Then $A\subseteq\bigcap_{k=1}^{\infty} IFcl_{\tau_i} IFcl_{j_i}(A_k)$, $(i \neq j$ and $i,j=1,2)$.

Proof. Let A is a pairwise IFregular G_δ −set in (X,τ_1,τ_2). Then then $A=\bigcap_{k=1}^{\infty}(IF\operatorname{int}_{\tau_i} IFcl_{\tau_j}(A_k))$, $(i \neq j$ and $i,j=1,2)$, where (A_k)'s are in (X,τ_1,τ_2). Now $A=\bigcap_{k=1}^{\infty}(IF\operatorname{int}_{\tau_i} IFcl_{\tau_j}(A_k)) \subseteq \bigcap_{k=1}^{\infty} IFcl_{\tau_j}(A_k)$. Hence $A\subseteq\bigcap_{k=1}^{\infty} IFcl_{\tau_j}(A_k) \subseteq \bigcap_{k=1}^{\infty} IFcl_{\tau_i} IFcl_{\tau_j}(A_k)$. Therefore $A\subseteq\bigcap_{k=1}^{\infty} IFcl_{\tau_i} IFcl_{j_i}(A_k)$.

Proposition 3.9. If A is a pairwise IFregular F_σ −set in (X,τ_1,τ_2). Then $\bigcup_{k=1}^{\infty} IF\operatorname{int}_{\tau_i} IF\operatorname{int}_{j_i}(A_k) \subseteq A$,

$(i \neq j$ and $i, j = 1,2)$.

Proof. Let A is a pairwise IFregular $F_\sigma -$set in (X, τ_1, τ_2). Then $A = \bigcup_{k=1}^{\infty}(IFcl_{\tau_i} IF\text{int}_{\tau_j}(A_k))$, $(i \neq j$ and $i, j = 1,2)$, where (A_k)'s are in (X, τ_1, τ_2). Now $A = \bigcup_{k=1}^{\infty}(IFcl_{\tau_i} IF\text{int}_{\tau_j}(A_k)) \supseteq \bigcup_{k=1}^{\infty} IF\text{int}_{\tau_j}(A_k)$. Hence $\bigcup_{k=1}^{\infty} IF\text{int}_{\tau_j}(A_k) \subseteq A$. Then

$\bigcup_{k=1}^{\infty} IF\text{int}_{\tau_i} IF\text{int}_{\tau_j}(A_k) \subseteq \bigcup_{k=1}^{\infty} IFcl_{\tau_j}(A_k) \subseteq A$.Therefore $\bigcup_{k=1}^{\infty} IF\text{int}_{\tau_i} IF\text{int}_{j_i}(A_k) \subseteq A$.

Proposition 3.10. If $IF\text{int}_{\tau_j}(A) = 0$, (j=1,2) for a pairwise IF regular $F_\sigma -$set in (X, τ_1, τ_2), then A is a pairwise IF first category set in (X, τ_1, τ_2).

Proof. Let A be a a pairwise IF regular $F_\sigma -$set in (X, τ_1, τ_2). $A = \bigcup_{k=1}^{\infty}(IFcl_{\tau_i} IF\text{int}_{\tau_j}(A_k))$, $(i \neq j$ and $i, j = 1,2)$, where (A_k)'s are IFSs in (X, τ_1, τ_2). Now $IF\text{int}_{\tau_j}(A) = 0$ implies that

$IF\text{int}_{\tau_j}(\bigcup_{k=1}^{\infty}(IFcl_{\tau_i} IF\text{int}_{\tau_j}(A_k)) = 0$. But

$\bigcup_{k=1}^{\infty}(IF\text{int}_{\tau_j}(IFcl_{\tau_i} IF\text{int}_{\tau_j}(B_k))) \subseteq IF\text{int}_{\tau_j}(\bigcup_{k=1}^{\infty}(IFcl_{\tau_i} IF\text{int}_{\tau_j}(B_k))$. Then we have

$\bigcup_{k=1}^{\infty}(IF\text{int}_{\tau_j}(IFcl_{\tau_i} IF\text{int}_{\tau_j}(B_k))) = 0$. This implies that $IF\text{int}_{\tau_j}(IFcl_{\tau_i} IF\text{int}_{\tau_j}(B_k)) = 0$. Also $IF\text{int}_{\tau_j}(IFcl_{\tau_i}(IFcl_{\tau_i}(IF\text{int}_{\tau_j}(B_k))) = IF\text{int}_{\tau_j}(IFcl_{\tau_i}(IF\text{int}_{\tau_j}(B_k) = 0$ and hence $IFcl_{\tau_i}(IF\text{int}_{\tau_j}(B_k))$ is a pairwise IF nowhere dense set in (X, τ_1, τ_2). Therefore A is a pairwise IF first category set in (X, τ_1, τ_2).

Definition 3.11. An IFBTS (X, τ_1, τ_2) is said to be a *pairwise IF strongly irresolvable space* if $IFcl_{\tau_1} IF\text{int}_{\tau_2} = 1_\sim = IFcl_{\tau_2} IF\text{int}_{\tau_1}$ for each pairwise IFdense set A in (X, τ_1, τ_2).

Proposition 3.12. If $IF\text{int}_{\tau_1} IF\text{int}_{\tau_2} = 0_\sim$ and $IF\text{int}_{\tau_2} IF\text{int}_{\tau_1} = 1_\sim$ for an IFS A in a pairwise IF strongly irresolvable space (X, τ_1, τ_2), then $IF\text{int}_{\tau_1}(A) = 1_\sim$ and $IF\text{int}_{\tau_2}(A) = 1_\sim$ in (X, τ_1, τ_2).

Proof. Let $IF\text{int}_{\tau_1} IF\text{int}_{\tau_2} = 0_\sim$ and $IF\text{int}_{\tau_2} IF\text{int}_{\tau_1} = 0_\sim$ for an IFS A in a pairwise IF strongly irresolvable space (X, τ_1, τ_2). Hence $1 - IF\text{int}_{\tau_1} IF\text{int}_{\tau_2} = 1_\sim$ and $1 - IF\text{int}_{\tau_2} IF\text{int}_{\tau_1} = 1_\sim$. This implies that $IFcl_{\tau_1} cl_{\tau_2}(1 - A) = 1_\sim$ and $IFcl_{\tau_2} cl_{\tau_1}(1 - A) = 1_\sim$. That is *1-A*is a pairwise IF dense set in (X, τ_1, τ_2). Since (X, τ_1, τ_2) is a pairwise IF strongly irresolvable space, for the pairwise IF dense set *1-A* in (X, τ_1, τ_2), we have $IFcl_{\tau_1} \text{int}_{\tau_2}(1 - A) = 1_\sim$ and $IFcl_{\tau_2} \text{int}_{\tau_1}(1 - A) = 1_\sim$. Then we have $IF\text{int}_{\tau_1} cl_{\tau_2}(A) = 0_\sim$ and $IF\text{int}_{\tau_2} cl_{\tau_1}(A) = 0_\sim$. Hence $IF\text{int}_{\tau_1}(A) \subseteq IF\text{int}_{\tau_1} IFcl_{\tau_2}(A) = 0_\sim$ and $IF\text{int}_{\tau_2}(A) \subseteq IF\text{int}_{\tau_2} IFcl_{\tau_1}(A) = 0_\sim$ implies that $IF\text{int}_{\tau_1}(A) \subseteq 0_\sim$ and $IF\text{int}_{\tau_2}(A) \subseteq 0_\sim$. That is $IF\text{int}_{\tau_1}(A) = 0_\sim$ and $IF\text{int}_{\tau_2}(A) = 0_\sim$.

Proposition 3.13. If $IFcl_{\tau_1} IFcl_{\tau_2} = 1_\sim$ and $IFcl_{\tau_2} IFcl_{\tau_1} = 1_\sim$ for an IFS A in a pairwise IF strongly irresolvable space (X, τ_1, τ_2), then $IFcl_{\tau_1}(A) = 1_\sim$ and $IFcl_{\tau_2}(A) = 1_\sim$ in (X, τ_1, τ_2).

Proof. Let $IFcl_{\tau_1} IFcl_{\tau_2} = 0_\sim$ and $IFcl_{\tau_2} IFcl_{\tau_1} = 0_\sim$ for an IFS A in a pairwise IF strongly irresolvable space (X, τ_1, τ_2). Hence $1 - IFcl_{\tau_1} IFcl_{\tau_2} = 1_\sim$ and $1 - IFcl_{\tau_2} IFcl_{\tau_1} = 1_\sim$. This implies that

$IF\operatorname{int}_{\tau_1}\operatorname{int}_{\tau_2}(1-A)=1_\sim$ and $IF\operatorname{int}_{\tau_2}\operatorname{int}_{\tau_1}(1-A)=1_\sim$. Now by Proposition 3.12., $IF\operatorname{int}_{\tau_1}(1-A)=0_\sim$ and $IF\operatorname{int}_{\tau_2}(1-A)=0_\sim$ and hence $1-IFcl_{\tau_1}(A)=0_\sim$ and $1-IFcl_{\tau_2}(A)=0_\sim$. Therefore $IFcl_{\tau_1}(A)=1_\sim$ and $IFcl_{\tau_2}(A)=1_\sim$ in (X,τ_1,τ_2).

Proposition 3.14. If the pairwise IF regular $G_\delta-$set A is pairwise IF dense in a pairwise IF strongly irresolvable space (X,τ_1,τ_2), then A is a pairwise IF residual set in (X,τ_1,τ_2).

Proof. Let A be a pairwise IF regular $G_\delta-$set with $IFcl_{\tau_1}IFcl_{\tau_2}=1_\sim$ and $IFcl_{\tau_2}IFcl_{\tau_1}=1_\sim$. Since (X,τ_1,τ_2) is a pairwise IF strongly irresolvable space and by Proposition 3.13, $IFcl_{\tau_1}(A)=1_\sim$ and $IFcl_{\tau_2}(A)=1_\sim$ in (X,τ_1,τ_2). That is, $IFcl_{\tau_i}(A)=1_\sim$, (i=1,2). Now *1-A* is a pairwise IF regular $F_\sigma-$set with $IF\operatorname{int}_{\tau_1}(1-A)=0_\sim$. Then by Proposition 3.10, *1-A*is a pairwise IF first category set in (X,τ_1,τ_2). Therefore A is a pairwise IF residual set in (X,τ_1,τ_2).

4. Pairwise intuitionistic fuzzy regular volterra spaces:

Definition 4.1. An IFBTS (X,τ_1,τ_2) is called a pairwise IF regular Volterra space if $IFcl_{\tau_i}(\bigcap_{k=1}^{N}(A_k))=1_\sim$, (i=1,2), where ($A_k$)'s are pairwise IF dense and pairwise IF regular $G_\delta-$sets in (X,τ_1,τ_2).

Proposition 4.2. If $IF\operatorname{int}_{\tau_i}(\bigcup_{k=1}^{N}(A_k))=0_\sim$, (i=1,2), where the IFSs (A_k)'s are pairwise IF regular $F_\sigma-$sets with $IF\operatorname{int}_{\tau_i}(A_k)=0_\sim$, (i=1,2) in an IFBTS (X,τ_1,τ_2), then (X,τ_1,τ_2) is a pairwise IF regular Volterra space.

Proof. Suppose that $IF\operatorname{int}_{\tau_i}(\bigcup_{k=1}^{N}(A_k))=0_\sim$, (i=1,2), where the IFSs (A_k)'s are pairwise IF regular $F_\sigma-$sets with $IF\operatorname{int}_{\tau_i}(A_k)=0_\sim$. Now $1-IF\operatorname{int}_{\tau_i}(\bigcup_{k=1}^{N}(A_k))=1_\sim$. Then we have $IFcl_{\tau_i}(1-\bigcup_{k=1}^{N}(A_k))=1_\sim$. This implies that $IFcl_{\tau_i}(\bigcap_{k=1}^{N}(1-A_k))=1_\sim$. Since ($A_k$)'s are pairwise IF regular $F_\sigma-$sets in (X,τ_1,τ_2), by Proposition 3.3, ($1-A_k$)'s are pairwise IF regular $G_\delta-$sets in (X,τ_1,τ_2). Also $IF\operatorname{int}_{\tau_i}(A_k)=0_\sim$ implies that $1-IF\operatorname{int}_{\tau_i}(A_k)=1_\sim$. Then we have $IFcl_{\tau_i}(1-A_k)=1_\sim$, i=1,2. Then $IFcl_{\tau_1}IFcl_{\tau_2}(1-A_k)=IFcl_{\tau_1}(1)=1_\sim$ and $IFcl_{\tau_2}IFcl_{\tau_1}(1-A_k)=IFcl_{\tau_2}(1)=1_\sim$. Hence ($1-A_k$)'s are pairwise IF dense sets in (X,τ_1,τ_2). Let $B_k=1-A_k$. Then (B_k)'s are pairwise IF dense and pairwise IF regular $G_\delta-$sets in (X,τ_1,τ_2). Therefore (X,τ_1,τ_2) is a pairwise IF regular Volterra space.

Proposition 4.3. An IFBTS (X,τ_1,τ_2) is a pairwise IF Volterra space, then (X,τ_1,τ_2) is a pairwise IF regular Volterra space.

Proof. Let (X,τ_1,τ_2) be a pairwise IF Volterra space. Now, consider $IFcl\tau_i(\bigcap_{k=1}^{N}(A_k))$. (i=1,2) where the IFSs (A_k)'s are pairwise IF dense and pairwise IF regular $G_\delta-$sets in (X,τ_1,τ_2). By Proposition 3.6, the pairwise IF regular $G_\delta-$sets (A_k)'s are pairwise IF $G_\delta-$sets in (X,τ_1,τ_2). Hence in $IFcl\tau_i(\bigcap_{k=1}^{N}(A_k))$, ($A_k$)'s are pairwise IF dense and pairwise IF $G_\delta-$sets in (X,τ_1,τ_2). Since (X,τ_1,τ_2) is a pairwise IF Volterra space, $IFcl\tau_i(\bigcap_{k=1}^{N}(A_k))=1_\sim$. Hence we have $IFcl\tau_i(\bigcap_{k=1}^{N}(A_k))=1_\sim$, where ($A_k$)'s are pairwise IF dense and pairwise IF regular $G_\delta-$sets in (X,τ_1,τ_2). Therefore (X,τ_1,τ_2) is a pairwise IF regular Volterra space.

Proposition 4.4. If the pairwise IF strongly irresolvable space (X,τ_1,τ_2) is a pairwise IF regular Volterra space, then $IFcl\tau_i(\bigcap_{k=1}^{N}(A_k)) = 1_{\sim}$, (i=1,2) where the IFSs (A_k)'s are pairwise IF residual sets in (X,τ_1,τ_2).

Proof. Let (X,τ_1,τ_2) be a pairwise IF regular Volterra space. Then $IFcl\tau_i(\bigcap_{k=1}^{N}(A_k)) = 1_{\sim}$ dense and pairwise IF regular in (X,τ_1,τ_2), (i=1,2) where the IFSs (A_k)'s are pairwise IF dense and pairwise IF regular G_δ −sets in (X,τ_1,τ_2). By Proposition 3.14, (A_k)'s are pairwise IF residual sets in (X,τ_1,τ_2). Hence $IFcl\tau_i(\bigcap_{k=1}^{N}(A_k)) = 1_{\sim}$, (i=1,2) where the IFSs (A_k)'s are pairwise IF residual sets in G_δ −sets (X,τ_1,τ_2).

Proposition 4.5. If the pairwise IF strongly irresolvable space (X,τ_1,τ_2) is a pairwise IF regular Volterra space, then $IF\operatorname{int}_{\tau_i}(\bigcup_{k=1}^{N}(A_k)) = 0_{\sim}$, (i=1,2) where the IFSs (A_k)'s are pairwise IF first category sets in (X,τ_1,τ_2).

Proof. Let (X,τ_1,τ_2) be a pairwise IF regular Volterraspace .Then $IFcl\tau_i(\bigcap_{k=1}^{N}(A_k)) = 1_{\sim}$, (i=1,2) where the IFSs (A_k)'s are pairwise IF dense and pairwise IF regular G_δ −sets in (X,τ_1,τ_2). Now $1 - IFcl\tau_i(\bigcap_{k=1}^{N}(A_k)) = 0_{\sim}$ implies that $IF\operatorname{int}\tau_i(1 - \bigcap_{k=1}^{N}(A_k)) = 0_{\sim}$. Then we have

$IF\operatorname{int}\tau_i(\bigcup_{k=1}^{N}(1 - A_k)) = 0_{\sim}$, (i=1,2).

By Proposition 3.3, the IFSs (A_k)'s are pairwise IF regular G_δ −sets implies that (1-A_k)'s are pairwise IF F_σ −sets in (X,τ_1,τ_2). Since (X,τ_1,τ_2) is a pairwise IF strongly irresolvable space and by Proposition 3.13, $IFcl_{\tau_1}(A_k) = 1_{\sim}$ and $IFcl_{\tau_2}(A_k) = 1_{\sim}$ in (X,τ_1,τ_2). That is, $IFcl_{\tau_i}(A_k) = 1_{\sim}$, (i=1,2), which implies $1 - IFcl_{\tau_i}(A_k) = 0_{\sim}$. Then $IF\operatorname{int}_{\tau_i}(1 - A_k) = 0_{\sim}$. Hence the IFSs (1-$A_k$)'s are pairwise IF F_σ −sets with $IF\operatorname{int}_{\tau_i}(1 - A_k) = 0_{\sim}$. Therefore by Proposition 3.10, (1-A_k)'s are pairwise IF first category sets in (X,τ_1,τ_2). Let B_k=1-A_k . Hence we have $IF\operatorname{int}\tau_i(\bigcup_{k=1}^{N}(A_k)) = 0_{\sim}$, i=1,2 where the IFSs ($A_k$)'s are pairwise IF first category sets in (X,τ_1,τ_2).

5. Pairwise intuitionistic fuzzy weakly regular volterra spaces:

Definition 5.1. An IFBTS (X,τ_1,τ_2) is called a *pairwise IF weakly regular Volterra space* if $\bigcap_{k=1}^{N}(A_k) \neq 0_{\sim}$, where ($A_k$)'s are pairwise IF dense and pairwise IF regular G_δ −sets in (X,τ_1,τ_2).

Proposition 5.2. If an IFBTS (X,τ_1,τ_2) is a pairwise IF regular Volterra space, then (X,τ_1,τ_2) is a pairwise IF weakly regular Volterra space.

Proof. Let (X,τ_1,τ_2) be a pairwise IF regular Volterra space. Then $IFcl(\bigcap_{k=1}^{N}(A_k)) = 1_{\sim}$, where ($A_k$)'s are pairwise IF dense and pairwise IF regular G_δ −sets in (X,τ_1,τ_2). This implies that $\bigcap_{k=1}^{N}(A_k) \neq 0_{\sim}$ in (X,τ_1,τ_2). Otherwise, if $\bigcap_{k=1}^{N}(A_k) \neq 0_{\sim}$, then $IFcl_{\tau_i}(\bigcap_{k=1}^{N}(A_k)) = IFcl_{\tau_i}(0) = 0_{\sim} \neq 1_{\sim}$, a contradiction. Therefore (X,τ_1,τ_2) is a pairwise IF weakly regular Volterra space.

Proposition 5.3. If an IFBTS (X,τ_1,τ_2) is a pairwise IF weakly Volterra space, then (X,τ_1,τ_2) is a pairwise IF weakly regular Volterra space.

Proof. Let (A_k)'s *(k=1 to N)*be pairwise IF dense and pairwise IF regular G_δ −sets in a pairwise IF weakly Volterra space (X,τ_1,τ_2). Then by Proposition 3.6, the pairwise IF regular G_δ −sets (A_k)'s in (X,τ_1,τ_2), are pairwise IF G_δ −sets in (X,τ_1,τ_2). Hence (A_k)'s are pairwise IF dense and pairwise G_δ −sets in (X,τ_1,τ_2). Since (X,τ_1,τ_2) is a pairwise IF weakly Volterra space, $\bigcap_{k=1}^{N}(A_k) \neq 0_\sim$. Therefore (X,τ_1,τ_2) is a pairwise IF weakly regular Volterra space.

Proposition 5.4. If A is a pairwise IF nowhere dense set in an IFBTS (X,τ_1,τ_2), then *1-A* is a pairwise IF dense set in (X,τ_1,τ_2).

Proof. Let A be a pairwise IF nowhere dense set in (X,τ_1,τ_2). Then we have $IF\operatorname{int}_{\tau_1} IFcl_{\tau_2}(A) = IF\operatorname{int}_{\tau_2} IFcl_{\tau_1}(A) = 0_\sim$. Now $1 - IF\operatorname{int}_{\tau_1} IFcl_{\tau_2}(A) = 1 - 0 = 1_\sim$. Then $IFcl_{\tau_1}(1 - IFcl_{\tau_2}(A)) = 1_\sim$, which implies that $IFcl_{\tau_1}(IF\operatorname{int}_{\tau_2}(1-A)) = 1_\sim$. But $IFcl_{\tau_1}(IF\operatorname{int}_{\tau_2}(1-A)) \subseteq IFcl_{\tau_1}(IFcl_{\tau_2}(1-A))$. Hence $1_\sim \subseteq IFcl_{\tau_1}(IFcl_{\tau_2}(1-A))$. That is $IFcl_{\tau_1}(IFcl_{\tau_2}(1-A)) = 1_\sim$. Also $1 - IF\operatorname{int}_{\tau_2} IFcl_{\tau_1}(A) = 1 - 0 = 1_\sim$. Then we have $IFcl_{\tau_2}(1 - IFcl_{\tau_1}(A)) = 1_\sim$. But $IFcl_{\tau_2}(IF\operatorname{int}_{\tau_1}(1-A)) \subseteq IFcl_{\tau_2} IFcl_{\tau_1}(1-A)$. Hence $1_\sim \subseteq IFcl_{\tau_2} IFcl_{\tau_1}(1-A)$. That is $IFcl_{\tau_2} IFcl_{\tau_1}(1-A) = 1_\sim$. Hence we have $IFcl_{\tau_1} IFcl_{\tau_2}(1-A) = IFcl_{\tau_2} IFcl_{\tau_1}(1-A) = 1_\sim$. Therefore *1-A* is a pairwise IF dense set in (X,τ_1,τ_2).

Proposition 5.5. If $\bigcup_{k=1}^{N}(A_k) \neq 1_\sim$, where ($A_k$)'s are pairwise IF nowhere dense and pairwise IF regular F_σ −sets in an IFBTS (X,τ_1,τ_2), then (X,τ_1,τ_2) is a pairwise IF weakly regular Volterra space.

Proof. Let (A_k)'s (k=1 to N) be pairwise IF nowhere dense and pairwise IF regular F_σ −sets in an IFBTS (X,τ_1,τ_2) such that $\bigcup_{k=1}^{N}(A_k) \neq 1_\sim$. Then, we have $1 - \bigcup_{k=1}^{N}(A_k) \neq 0_\sim$. This implies that $\bigcap_{k=1}^{N}(1 - A_k) \neq 0_\sim$. Since ($A_k$)'s are pairwise IF nowhere dense sets, we have by Proposition 5.5., Let (1-A_k)'s are pairwise IF dense sets in (X,τ_1,τ_2). Also, since (A_k)'s are pairwise IF regular F_σ −sets, by Proposition 3.3, (1-A_k)'s are pairwise IF regular G_δ −sets in (X,τ_1,τ_2). Therefore (X,τ_1,τ_2) is a pairwise IF weakly regular Volterra space.

Proposition 5.6. If each pairwise IF nowhere dense set is a pairwise IF regular F_σ − set in a pairwise IF second category space (X,τ_1,τ_2), then (X,τ_1,τ_2) is a pairwise IF weakly regular Volterra space.

Proof. Let (X,τ_1,τ_2) be a pairwise IF second category space in which each pairwise IF nowhere dense set is a pairwise IF regular F_σ −set. Since (X,τ_1,τ_2) is a pairwise IF second category space, $\bigcup_{k=1}^{\infty}(A_k) \neq 1_\sim$, where ($A_k$)'s are pairwise IF nowhere dense sets in (X,τ_1,τ_2). By hypothesis, (A_k)'s are pairwise IF regular F_σ −sets in (X,τ_1,τ_2). Let us take the first N(A_k)'s as (B_k)'s in (X,τ_1,τ_2). Then $\bigcup_{k=1}^{N}(B_k) \subseteq \bigcup_{k=1}^{\infty}(A_k)$ implies that $\bigcup_{k=1}^{N}(B_k) \neq 1_\sim$. Thus $\bigcup_{k=1}^{N}(B_k) \neq 1_\sim$, where ($A_k$)'s are pairwise IF nowhere dense and pairwise IF regular F_σ −sets in (X,τ_1,τ_2). Therefore by Proposition 5.5, (X,τ_1,τ_2) is a pairwise IF weakly regular Volterra space.

Proposition 5.7. If $\bigcup_{k=1}^{N}(A_k) = 1_\sim$, where ($A_k$)'s are pairwise IF regular F_σ −sets in a pairwise IF weakly regular Volterra space (X,τ_1,τ_2), then there exists atleast one $A_k$in (X,τ_1,τ_2) with $IF\operatorname{int}_{\tau_i}(A_k) \neq 0_\sim$, (i=1,2).

Proof. Suppose that $IF\operatorname{int}_{\tau_i}(A_k) = 0_\sim$ for all k=1 to N in (X,τ_1,τ_2). Then $1 - IF\operatorname{int}_{\tau_i}(A_k) = 1_\sim$. This will imply that $IF\operatorname{int}_{\tau_i}(1 - A_k) = 1_\sim$. Since ($A_k$)'s are pairwise IF regular F_σ −setsin (X,τ_1,τ_2), (1-A_k)'s are pairwise IF regular G_δ − sets in (X,τ_1,τ_2). Then $\bigcap_{k=1}^{N}(1-A_k)=1-(\bigcup_{k=1}^{N}(A_k))=1-1=0_\sim$. Hence we will have $\bigcap_{k=1}^{N}(1-A_k)=0_\sim$, where (1-$A_k$)'s are pairwise IF dense and pairwise IF regular G_δ − sets in (X,τ_1,τ_2) and this will imply that (X,τ_1,τ_2) will not be an IF weakly regular Volterra space, a contradiction to the hypothesis. Hence there must be atleast one A_k in (X,τ_1,τ_2) with $IF\,\text{int}_{\tau_i}(A_k)\neq 0_\sim$.

Proposition 5.8. If $\bigcup_{k=1}^{N}(A_k)=1_\sim$, where ($A_k$)'s are pairwise IF regular F_σ − sets such that $IF\,\text{int}_{\tau_i}(A_k)\neq 0_\sim$, (i=1,2) for atleast one A_k in an IFBTS (X,τ_1,τ_2), then (X,τ_1,τ_2) is a pairwise IF weakly regular Volterra space.

Proof. Suppose that $\bigcup_{k=1}^{N}(A_k)=0_\sim$, where ($A_k$)'s are pairwise dense and pairwise IF regular G_δ − sets in (X,τ_1,τ_2). Then $1-\bigcap_{k=1}^{N}(A_k)=1_\sim$ and hence $\bigcup_{k=1}^{N}(1-A_k)=1_\sim$, where (1-$A_k$)'s are pairwise IF regular F_σ − sets in (X,τ_1,τ_2) such that $IF\,\text{int}_{\tau_i}(1-A_k)=0_\sim$ for all k=1 to N in (X,τ_1,τ_2) contradiction to the hypothesis. Hence $\bigcap_{k=1}^{N}(A_k)\neq 0_\sim$. Therefore (X,τ_1,τ_2) is a pairwise IF weakly regular Volterra space.

Proposition 5.9. If(A_k)'s are pairwise IF dense and pairwise IF regular G_δ − sets in an IFBTS (X,τ_1,τ_2), such that $\bigcap_{k=1}^{N}(A_k)$ is not a pairwise IF nowhere dense set in (X,τ_1,τ_2), then (X,τ_1,τ_2) is a pairwise IF weakly regular Volterra space.

Proof. Suppose that the IFBTS (X,τ_1,τ_2) is not a pairwise IF weakly regular Volterra space, then we have $\bigcap_{k=1}^{N}(A_k)=0_\sim$. This will imply that $IF\,\text{int}_{\tau_i}\,IFcl_{\tau_j}\bigcap_{k=1}^{N}(A_k)=0_\sim$, $(i\neq j, i,j=1,2)$, where (A_k)'s are pairwise IF dense and pairwise IF regular G_δ − sets in an IFBTS (X,τ_1,τ_2), a contradiction. Therefore (X,τ_1,τ_2) is a pairwise IF weakly regular Volterra space.

Proposition 5.10. If the pairwise IF strongly irresolvable space (X,τ_1,τ_2) is a pairwise IF weakly regular Volterra space, then $\bigcap_{k=1}^{N}(A_k)\neq 0_\sim$, where ($A_k$)'s are pairwise IF residual sets in (X,τ_1,τ_2).

Proof. Proof is obvious from Proposition 4.4 and 5.2.

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