CONFORMALLY FLAT SPHERICALLY SYMMETRIC PERFECT FLUID DISTRIBUTION IN *F*(*R*, *T*) GRAVITY

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Abstract: Spherically symmetric conformally flat space-time is considered in the presence of perfect fluid distribution in *f*(*R*, *T*) gravity proposed by Harko et al. (2011). An Exact solution of the field equations of the theory is obtained using barotropic equations of state. Some physical properties of the model are also discussed.

Keywords: f(R, T) gravity, perfect fluid distribution, conformally flat space time.

1. Introduction: It is well known that one of the greatest advancements in modern cosmology is the discovery of cosmic acceleration (Riess et al.1988, Perlmulter et al.1999). Also, Wilkinson Microwave Anisotropy Proble (W MAP) (Spergel 2007; Hawkins et al.2003; Eisenstein et al.2005) reveal that the cosmos at present is dominated by exotic energy component named as' dark energy'. The efforts made to explain this mysterious cosmic acceleration and dark energy can be grouped into two categories: introducing new ingredient of dark energy to the entire cosmic energy and modification of Einstein-Hilbert action to obtain modified theories of gravity such as $f(R)$(Nojiri and odintsov 2011) gravity and $f(R,T)$ gravity (Harko et al.2011) where R and T represent the scalar curvature and trace of the energy momentum tensor respectively.

2. A Brief Review of *f*(*R*, *T*) Gravity: In *f*(*R*, *T*) gravity proposed by Harko et al. (2011), gravitational Lagrangian is given by an arbitrary function of the Ricci Scalar R and of the trace T of the stress energy tensor T_{ij}. The field equations of this theory are derived from the Hilbert- Einstein type variational principle by taking the action:

$$S = \frac{1}{16\pi}\int[f(R,T) + L_m]\sqrt{-g}\ d^4x \quad (1)$$

where L_m is the matter Lagrangian density.

Stress energy tensor of matter is defined as

$$T_{ij} = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\, L_m)}{\delta^{ij}} \quad (2)$$

and the trace by $T = g^{ij}T_{ij}$ respectively. By assuming that L_m of matter depends only on the metric tensor components g^{ij}, we have obtain the field equations of $f(R,T)$ gravity as

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\theta_{ij} \quad (3)$$

Where

$$\theta_{ij} = -2T_{ij} + g_{ij}\, L_m - 2g^{lk}\, \frac{\partial^2 L_m}{\partial g^{ij}\, \partial g^{lm}} \quad (4)$$

Here $f_R = \frac{\delta f(R,T)}{\delta R}$, $f_T = \frac{\delta f(R,T)}{\delta T}$, $\Box = \nabla^i\nabla_j$ and ∇^i is the covariant derivative. It may be noted that when $f(R,T) = f(R)$ $Eq.(3)$ yields the field equations $f(R)$ gravity.

The problem of perfect fluids described an energy density ρ, pressure p and four velocity u^i is complicated since. There is no unique definition of the matter Lagrangian. However, here,
we assume that stress energy tensor of matter is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \quad (5)$$

and the matter Lagrangian can be taken as $L_m = -p$ and we have

$$u^i\nabla_j u_i = 0, \quad u^i u_i = 1 \quad (6)$$

Now with the use of Eq. (5) we obtain, for the variation of stress energy of perfect fluid,
the expression

$$\theta_{ij} = -2T_{ij} - p g_{ij} \quad (7)$$

Generally, the field equations also depend through the tensor θ_{ij}, on the physical nature of the matter field. Hence in the case of $f(R,T)$ gravity, depending on the nature of the matter source, we obtain several theoretical models corresponding to each choice of $f(R,T)$. Assuming

$$f(R,T) = R + 2f(T) \quad (8)$$

as a first choice where $f(T)$ is an arbitrary function of the trace of stress energy tensor of matter, we get the gravitational field equations of $f(R,T)$ gravity from Eq.(3) as(Harko et al.2011)

$$R_{ij} - \frac{1}{2}g_{ij}R = 8\pi T_{ij} - 2f'(T)T_{ij} - 2f'(T)\theta_{ij} + f(T)g_{ij} \quad (9)$$

where the prime denotes differentiation with respect to the arguments. If the mater source is perfect fluid then the field equations of $f(R,T)$ gravity, in view of Eq.(7), become

$$R_{ij} - \frac{1}{2}g_{ij}R = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij} \quad (10)$$

There have been several investigations in $f(R,T)$ gravity with different physical distributions. Noteworthy among them are exact models with perfect and viscous fluid distributions discussed by Reddy et al.(2012), Harko et al (2011), Reddy et al. (2013), Reddy and Santhikumar (2013), Reddy et.al.(2014).

Considerable interest has been shown to the study of physical properties of space times which are conformal to certain well known gravitational fields. A number of conform ally flat physically significant space-times are known like Schwarzschild internal solution and Lamaitre consmological universe. Singh and Roy (1966), Singh and Abdusattar(1974) and Roy and Raj Bali (1978) have discussed solutions of Einstein field equations representing spherically symmetric conformally flat perfect fluid distributions. Also, Reddy and Venkateswarlu (1988), Reddy et al. (1988), Reddy and Venkateswarlu (1987) Rao and Reddy(1982), Reddy(1979a,1979b) have investigated spherically symmetric conformally flat models in Brans-Dicke(1961), and other scalar-tensor theories of gravitations formulated subsequently.

Motivated by the above investigations, in this paper, we obtain a spherically symmetric conformally flat perfect fluid distribution model in $f(R,T)$ gravity.

3. Metric & Field Equations: We consider the conformally flat metric in spherical polar coordinates

$$ds^2 = e^{\alpha}\left(dr^2 + r^2 d\theta^2 + r^2\sin^2\theta - dt^2\right) \qquad (11)$$

where α is a function of r and t only. We number the coordinate as $x^1 = r, x^2 = \theta, x^3 = \phi$ and $x^4 = t$.

The energy momentum tensor for perfect fluid distribution is given by

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij} \qquad (12)$$

where ρ is the energy density and p is the isotropic pressure of fluid distributions and u^i is the four velocity of the fluid satisfying

$$e^{-\alpha}\left(\frac{3}{4}\alpha'^2 + \frac{2\alpha'}{r}\right) - e^{-\alpha}\left(\ddot{\alpha} + \frac{\dot{\alpha}^2}{4}\right) = p(8\pi + 6\lambda) - \lambda\rho (13)$$

$$e^{-\alpha}\left(\alpha'' + \frac{\alpha'^2}{4} + \frac{\alpha'}{r}\right) - e^{-\alpha}\left(\ddot{\alpha} + \frac{\dot{\alpha}^2}{4}\right) = p(8\pi + 6\lambda) - \lambda\rho (14)$$

$$e^{-\alpha}\left(\alpha'' + \frac{\alpha'^2}{4} + \frac{2\alpha'}{r}\right) - e^{-\alpha}\left(\frac{3\dot{\alpha}^2}{4}\right) = -\rho(8\pi + 2\lambda) + 5p\lambda (15)$$

$$e^{-\alpha}\left(\dot{\alpha}' - \frac{\dot{\alpha}\alpha'}{2}\right) = 0 (16)$$

$$g^{ij}u_i u_j = -1 \qquad (13)$$

Using comoving coordinate system the field equations (10) of $f(R,T)$ gravity, with the help of (12), for the metric (11), when $\boldsymbol{f(T) = \lambda T}$, take the form

where an overhead dot indicates partial differentiation with respect to t and a prime indicates partial differentiation with respect to r. Here ρ and p are also functions of r and t.

4. Solutions of Field Equations: It can be seen that the field equations $(13)-(16)$ reduce to the following independent equations

$$e^{-\alpha}\left(\alpha'' - \frac{\alpha'^2}{2} - \frac{\alpha'}{r}\right) = 0 \qquad (17)$$

$$e^{-\alpha}\left(\alpha'' + \frac{\alpha'^2}{4} + \frac{2\alpha'}{r}\right) - e^{-\alpha}\left(\frac{3\dot{\alpha}^2}{4}\right) = -\rho(8\pi + 2\lambda) + 5p\lambda \qquad (18)$$

$$e^{-\alpha}\left(\dot{\alpha}' - \frac{\dot{\alpha}\alpha'}{2}\right) = 0 \qquad (19)$$

To solve the above set of highly non-linear field equations, we use the barotropic equations of state

$$p = \varepsilon\rho \qquad (20)$$

It is well known that when $\varepsilon = 0$, we have dust model, $\varepsilon = 1$, we get stiff fluid model and when $\varepsilon = \frac{1}{3}$, we obtain a radiation model in $f(R,T)$ gravity.

Now from Eq. (17) we obtain

$$e^{\alpha} = \left[g(t) - f(t)r^2\right]^{-2} \qquad (21)$$

Using Eq. (21) in Eq. (19), we obtain $\dot{f} = 0$, which in turn gives that f is a constant and hence we write

$$f = k = \text{const.} \qquad (22)$$

Hence from Eqs. (21) and (22), we get

$$e^{\alpha} = \left[g(t) - kr^2\right]^{-2} \qquad (23)$$

where $g(t)$ is an arbitrary function of t.

Now using (23) in (11) we get the model as

$$ds^2 = \left[g(t) - kr^2\right]^{-2}\left[dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2 - dt^2\right] \qquad (24)$$

5. Discussion: Eq. (24) represents a spherically symmetric conform ally flat perfect fluid model in $f(R,T)$ gravity. It may be observed that when $k = 0$, we obtain a non-static conform ally flat model. Also when $g = 0$, we will have a static conform ally flat model. The energy density and pressure in the model are given by:

$$\rho = \frac{1}{\left[8\pi - \lambda\left(5\varepsilon+2\right)\right]}\left[\frac{3}{2}\dot{g}^2 - 20k^2r^2 + 4k\left(g+2\right)\right] \quad (25)$$

$$p = \frac{\varepsilon}{\left[8\pi - \lambda\left(5\varepsilon+2\right)\right]}\left[\frac{3}{2}\dot{g}^2 - 20k^2r^2 + 4k\left(g+2\right)\right] \quad (26)$$

The non-vanishing component of the flow vector u^4 is given by

$$u_4 = \left(g\left(t\right) - kr^2\right)^{-1} \quad (27)$$

The expression for scalar expansion Φ is (Ellis 1971)

$$\Phi = u^i_{;i} = 3\dot{g}\left(t\right)$$

Hence we find that the expansion is time dependent only. It can also be seen that the components of rotation and shear tensors vanish. Hence the model (24) represents a conformally flat model with a perfect fluid source which is expanding non-rotating and shear free. It may be noted that at the centre of the sphere (r=0) density and pressure are functions of cosmic time t.

6. Conclusions: In this paper, we have investigated a spherically symmetric conform ally flat perfect fluid model in $f\left(R,T\right)$ gravity formulated by Harko et al. (2011). To obtain a determinate solution we have used barotropic equation of state for a perfect fluid. We have also observed that the model is expanding with time, non-rotating and shear free.

References

1. Rises, A. et al., Astron .J. 116, 1009(1998)
2. Perlmutter, S. et al., Astrophys. J. 517, 5(1999)
3. Harko, T; et al., Phys. Rev. D84, 024020(2011)
4. Nojiri,S., Odintsov, S.D., Phy.Rep.505,59(2011)
5. Reddy, D.R.K., et al., Astrophys. Space Sci.342,,249(2012)
6. Reddy, D.R.K., et al., Int.J.Theor.Phys.52,239(2013)
7. Reddy, D.R.K., Santhikumar, R., Astrophy. Space Sci.344,253(2013)
8. Reddy, D.R.K., et al., Astrophys. Space Sci. 350, 375(2014)
9. Spergel, D.N.et al., Astrophys.J. Suppl. Ser 170, 377(2007)
10. Hawkins, E., et al.Mon.Not.Roy.Astron.Soc.346,1250088(2003)
11. Eisentein, D.J., et al., Astrophys.J.633,560(2005)
12. Singh, K.P., Roy, S.R., Proc. Natnl. Inst. Sci. India 32A, 223(1966)
13. Singh, K.P., Abdusattar, Gen.Relativ. Gravit.5,115(1974)
14. Roy, S.R., Raj Bali, Ind.J.Pur appl.Math.9, 871(1978)
15. Reddy, D.R.K., Venkateswarlu, R., Astrophy. Space Sci.136,183((1987)
16. Rao, V.U.M., Reddy, D.R.K., Gen. Relativ.Gravit.14, 1017 (1982)
17. Reddy, D.R.K., J. Math. Phys.20, 1413(1979a)
18. Reddy, D.R.K. , J. Math. Phys.20, 23 (1979b)
19. Reddy, D.R.K., Venkateswarlu, R., Astrophy. Space Sci. 147,115(1988)
20. Reddy, D.R.K. et al., Astrophy. Space Sci.141, 181(1988

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