ON NANO GENERALIZED β OPEN SETS IN NANO TOPOLOGICAL SPACES

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Abstract: The aim of this paper is to introduce Nano generalized β open sets in Nano topological spaces and also we introduce and study the properties of Nano g β interior and Nano g β closure.

Keywords: Nano generalized β interior, Nano generalized β closure and Nano generalized β open sets.

1.Introduction: The notion of Nano topology was introduced by Lellis Thivagar [7] which was defined in terms of approximations and boundary regions of a subset of an universe using an equivalence relation on it and he also defined Nano closed set, Nano interior and Nano closure. Levine [8] introduced generalized closed sets as a generalization of closed sets in topological spaces. Abd El Monsef et al. [1] introduced the notion of β-open set in topology, and the equivalent notion of semi-pre open set was given independently by Andrijevic[2], and further investigation of Nano β open sets was given by Gnanmbal[6].In this paper we introduce Nano generalized β open set and some of its properties are investigated.

2. Preliminaries: Definition: 2.1[7]Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space.

Let $X \subseteq U$. Then,

- The lower approximation of X with respect to R is the set of all objects which can be for certainly classified as X with respect to R and is denoted by $L_R(X)$.

  $L_R(X) = \cup\{R(x) : R(x) \subseteq X, x \in U\}$

  where $R(x)$ denotes the equivalence class determined by $x \in U$.
- The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$.

  $U_R(X) = \cap\{R(x) : R(x) \cap X \neq \phi, x \in U\}$
- The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not$-X$ with respect to R and is denoted by $B_R(X)$.

$B_R(X) = U_R(X) - L_R(X)$

Property: 2.2 [7] If (U,R) is an approximation space and $X,Y \subseteq U$, then

1) $L_R(X) \subseteq X \subseteq U_R(X)$
2) $L_R(\phi) = U_R(\phi) = \phi$
3) $L_R(U) = U_R(U) = U$
4) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
5) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
6) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
7) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
8) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
9) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$
10) $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$
11) $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$

Definition: 2.3 [7] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 2.2 $\tau_R(X)$ satisfies the following axioms:

i) U and $\phi \in \tau_R(X)$.

ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X , $(U, \tau_R(X))$ is called the Nano topological space. Elements of the Nano topology are known as Nano open sets in U . Elements of $[\tau_R(X)]^C$ are called Nano closed sets with $[\tau_R(X)]^C$ being called dual Nano topology of $\tau_R(X)$.

Definition: 2.5 [7] If $(U, \tau_R(X))$ is a Nano topological space with respect X where $X \subseteq U$ and if $A \subseteq U$, then

- The Nano interior of a set A is defined as the union of all Nano open subsets contained in A and is denoted by $N\operatorname{int}(A)$. $N\operatorname{int}(A)$ isthe largest Nano open subset of A.
- The Nano closure of a set A is defined as the intersection of all Nano closed sets containing A and is denoted by $Ncl(A)$. $Ncl(A)$ is the smallest Nano closed set containing A.

Definition: 2.6 [3][6][7]Let $(U,\tau_R(X))$ be a Nano topological space and $A \subseteq U$. Then A is said to be

(i) Nano semi open if $A \subseteq Ncl(N\operatorname{int}(A))$

(ii) Nano pre open if $A \subseteq N\operatorname{int}(Ncl(A))$

(iii) Nano α open if $A \subseteq N\operatorname{int}[Ncl(N\operatorname{int}(A))]$

(iv) Nano regular open if $A = N\operatorname{int}(Ncl(A))$

(v) Nano b open if $A \subseteq Ncl(N\operatorname{int}(A)) \cup N\operatorname{int}(Ncl(A))$

(vi) Nano β open(Nano semi-pre open) if $A \subseteq Ncl[N\operatorname{int}(Ncl(A))]$

$NSO(U,X)$, $NPO(U,X)$, $N\alpha O(U,X)$, $NRO(U,X)$, $NBO(U,X)$ and $N\beta O(U,X)$ respectively, denote the families of all Nano semi open, Nano pre open, Nano α open,Nano regular open, Nano b open and Nano β open subsets of U.

Let $(U,\tau_R(X))$be a Nano topological space and $A \subseteq U$. Then A is said to beNano semi closed, Nano pre closed, Nano α closed, Nano regular closed, Nano b closed and Nano β closed if its complement is respectively Nano semi open, Nano pre open, Nano α open, Nano regular open, Nano b open and Nano β open.

Definition: 2.7 [10]If $(U,\tau_R(X))$ is a Nano topological space with respect X where $X \subseteq U$ and if $A \subseteq U$, then

- The Nano β interior of a set A is defined as the union of all Nano β open subsets contained in A and is denoted by $N\beta\operatorname{int}(A)$. $N\beta\operatorname{int}(A)$is the largest Nano β open subset of A.
- The Nano β closure of a set A is defined as the intersection of all Nano β closed sets containing A and is denoted by $N\beta cl(A)$. $N\beta cl(A)$is the smallest Nano β closed set containing A.

Definition: 2.9 [4] A subset A of $(U,\tau_R(X))$ is called Nano generalized closed set (briefly Ng closed) if $Ncl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in $(U,\tau_R(X))$.

Definition: 2.8 [10]A subset A of Nano topological space $(U,\tau_R(X))$ is called Nano generalized β closed set (briefly Ngβ closed) if $N\beta cl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in $(U,\tau_R(X))$.

3.Nanogeneralized β Open Sets: Throughout this paper $(U,\tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$, R is an equivalence relation on U, U/R denotes the family of equivalence classes of U by R **Definition: 3.1** A subset A of a Nano topological space$(U,\tau_R(X))$ is called Nano generalized β open (briefly Ng β open), if its compliment A^C is Nano g β closed.

The collection of all Nano gβ open subsets of is denoted by $Ng\beta O(U,X)$.

Theorem: 3.2 Every Nano β open set in $(U,\tau_R(X))$ is Nano g β open set in $(U,\tau_R(X))$.

Proof: Assume that A is a Nano β open. Then A^C is Nano β closed. We know that every Nano β closed is Nano gβ closed. Hence A^C is Nano gβ closed.Therefore A is Nano gβ openin U.

Remark: 3.3 The converse of the above theorem need not be true which can be seen from the following example.

Example: 3.4 Let $U = \{a,b,c,d,e\}$ with $U/R = \{\{a,b\},\{c,e\},\{d\}\}$and $X = \{a,d\}$. Then the Nano topology is defined as $\tau_R(X) = \{U,\phi,\{d\},\{a,b,d\},\{a,b\}\}$. Here the set $\{c\}$is Nano gβ open but not Nano β open in U.

The following theorem can also be proved in a similar way.

Theorem: 3.5 Let$(U,\tau_R(X))$be a Nano topological space and $A \subseteq U$. Then

(i) Every Nano open set is Nano gβ open set.

(ii) Every Nano semi open set is Nano gβ open set.

(iii) Every Nano pre open set is Nano gβ open set.

(iv) Every Nano α open set is Nano gβ open set.

(v) Every Nano regular open set is Nano gβ open set.

(vi) Every Nano b open set is Nano gβ open set.

(vii) Every Nano g open set is Nano gβ open set.

(viii) Every Nano gs open set is Nano gβ open set.

(ix) Every Nano α g open set is Nano g β open set.

(x) Every Nano g r open set is Nano g β open set.

Remark: 3.6 Reverse implications of the above theorem 3.6 need not be true which can be seen from the following example.

Example: 3.7 Let $U=\{a,b,c,d,e\}$ with $U/R=\{\{a,b\},\{c,e\},\{d\}\}$and $X=\{a,d\}$. Then the Nano topology is defined as $\tau_R(X)=\{U,\phi,\{d\},\{a,b,d\},\{a,b\}\}$. Here the set $\{a,c,e\}$is Nano g β open but not Nano open, Nano semi open, Nano pre open, Nano α open, Nano regular open, Nano b open, Nano g open, Nano g s open, Nano α g open, Nano g r open in U.

Remark: 3.8 Let A be a subset of a Nano topological space $(U,\tau_R(X))$. Then

i) $(N\beta cl(A))^C = N\beta \operatorname{int}(A^C)$

ii) $(N\beta \operatorname{int}(A))^C = N\beta cl(A^C)$.

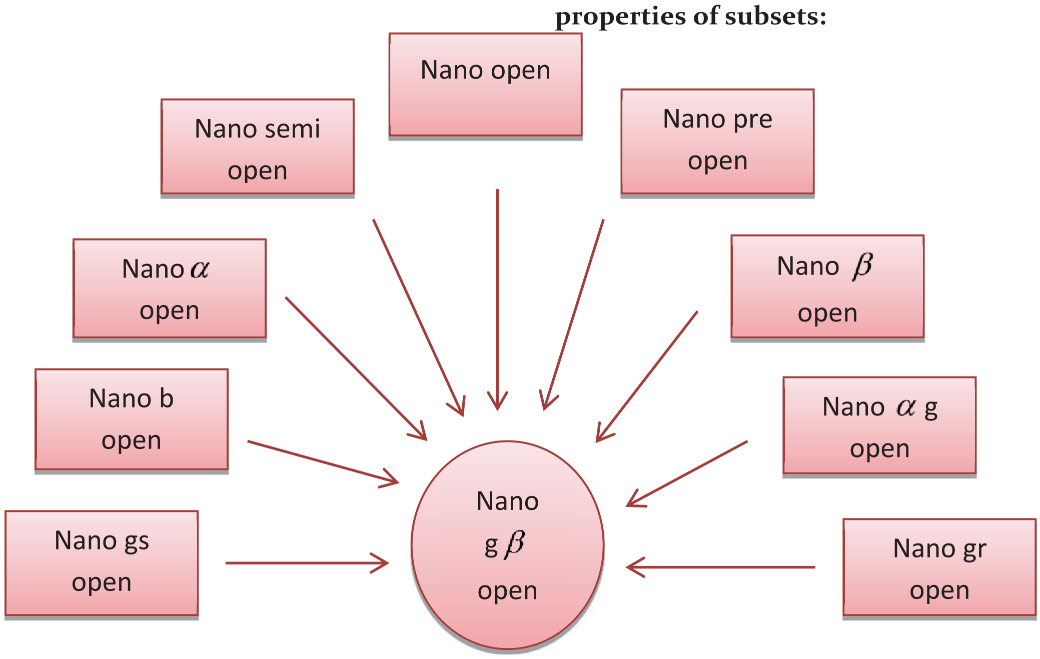
Theorem: 3.9 The union of two Nano g β open sets need not be Nano g β open set which can be seen from the following example.

Example: 3.10 Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{b\},\{c\},\{d\}\}$and $X=\{a,d\}$. Then the Nano topology is defined as $\tau_R(X)=\{U,\phi,\{a,d\}\}$. Here the sets $\{b\}$ and $\{c\}$are Nano g β open sets but $\{b\}\cup\{c\}=\{b,c\}$ is not Nano g β open set in U.

Theorem: 3.11 The intersection of two Nano g β open sets need not be Nano g β open set which can be seen from the following example.

Example: 3.12 Let $U=\{a,b,c,d\}$ with $U/R=\{\{a,b\},\{c\},\{d\}\}$and $X=\{a,c\}$. Then the Nano topology is defined as $\tau_R(X)=\{U,\phi,\{c\},\{a,b,c\},\{a,b\}\}$. Here the sets $\{a,d\}$ and $\{c,d\}$are Nano g β open sets but $\{a,d\}\cap\{c,d\}=\{d\}$ is not Nano g β open set in U.

We have the following implications for the properties of subsets:



Theorem: 3.13A subset A of U is $Ng\beta$ open if and only if $F\subseteq N\beta\operatorname{int}(A)$ whenever F is Nano closed and $F\subseteq A$.

Proof: (Necessity) Let A be $Ng\beta$ open set. Let F be Nano closedand $F\subseteq A$, then $A^C\subseteq F^C$ whenever F^C is Nano open. Since A^C is Nano g β closed set $N\beta cl(A^C)\subseteq F^C$, by remark 3.8 $(N\beta\operatorname{int}(A))^C\subseteq F^C$. Thus $F\subseteq N\beta\operatorname{int}(A)$.

(Sufficiency) If F is Nano closed set with $F\subseteq N\beta\operatorname{int}(A)$and $F\subseteq A$. Then $(N\beta\operatorname{int}(A))^C\subseteq F^C$.Thus $N\beta cl(A^C)\subseteq F^C$. Hence A^C is Nano g β closed set and A is Nano g β open setin U.

Theorem: 3.14 If $N\beta\operatorname{int}(A)\subseteq B\subseteq A$and A is Nano g β open, then B is also Nano g β open.

Proof: Let A is Nano g β open set and $N\beta\operatorname{int}(A)\subseteq B\subseteq A$then

$A^C \subseteq B^C \subseteq (N\beta\,\mathrm{int}(A))^C$ that is $A^C \subseteq B^C \subseteq N\beta cl(A^C)$, by remark 3.8. Since A^C is Nano g β closed, B^C is also Nano g β closed. Therefore B be Nano g β open.

Theorem: 3.15 If a set A is Nano g β open in U, then $G = U$ whenever G is Nano open and $N\beta\,\mathrm{int}(A) \cup A^C \subseteq G$.

Proof: Suppose that A be Nano g β open in U. Let G is Nano open set and $N\beta\,\mathrm{int}(A) \cup A^C \subseteq G$. Thus $G^C \subseteq (N\beta\,\mathrm{int}(A) \cup A^C)^C = (N\beta\,\mathrm{int}(A))^C \cap A$ $= N\beta cl(A^C) - A^C$, by remark 3.8 that is $(N\beta\,\mathrm{int}(A))^C = N\beta cl(A^C)$. Now G^C is Nano closed and A^C is Nano g β closed, then $N\beta cl(A^C) - A^C$ cannot contain any non-empty Nano closed set by theorem 3.13[10]. Therefore $G^C = \phi$. That is $G = U$.

Theorem: 3.16 If a subset A of a Nano topological space $(U, \tau_R(X))$ is both Nano closed and Nano g β open then it is Nano β open.

Proof: Let A be Nano closed and Nano g β open in β. Now $N\beta\,\mathrm{int}(A) \subseteq A$, then by theorem 3.13, we have $A \subseteq N\beta\,\mathrm{int}(A)$. Therefore $N\beta\,\mathrm{int}(A) = A$ which means that A is Nano β open.

4. Properties of N G β Interior And Ng β Closure: Definition: 4.1 Let $(U, \tau_R(X))$ be a Nano topological space and $A \subseteq U$ then Nano g β interior is defined as

$$Ng\beta\,\mathrm{int}(A) = \cup\{B : B \text{ is } Ng\beta \text{ open}, B \subset A\}$$

Clearly $Ng\beta\,\mathrm{int}(A)$ is the largest Nano g β open set over U which is contained in A.

Definition: 4.2 Let $(U, \tau_R(X))$ be a Nano topological space and $A \subseteq U$ then Nano g β closure is defined as

$$Ng\beta cl(A) = \cap\{F : F \text{ is } Ng\beta \text{ closed}, A \subset F\}$$

Clearly $Ng\beta cl(A)$ is the smallest Nano g β closed set over U which contains A.

Theorem: 4.3 Let A and B betwo subsets of Nano topological space $(U, \tau_R(X))$. Then

i) A is $Ng\beta$ open set if $A = Ng\beta\,\mathrm{int}(A)$

ii) $Ng\beta\,\mathrm{int}(U) = U$ and $Ng\beta\,\mathrm{int}(\phi) = \phi$

iii) $Ng\beta\,\mathrm{int}(Ng\beta\,\mathrm{int}(A)) = Ng\beta\,\mathrm{int}(A)$

iv) $A \subset B \Rightarrow Ng\beta\,\mathrm{int}(A) \subset Ng\beta\,\mathrm{int}(B)$

Proof: i) Let A be $Ng\beta$ open set of U. Always $Ng\beta\,\mathrm{int}(A) \subseteq A$, by the assumption A is $Ng\beta$ open set contained by itself. But $Ng\beta\,\mathrm{int}(A)$ is the largest $Ng\beta$ open set contained in A.Hence $A \subseteq Ng\beta\,\mathrm{int}(A)$.Therefore $A = Ng\beta\,\mathrm{int}(A)$.

ii)Since ϕ and U are $Ng\beta$ open sets. By theorem 4.3(i), $Ng\beta\,\mathrm{int}(\phi) = \phi$ & $Ng\beta\,\mathrm{int}(U) = U$.

iii) Since $Ng\beta\,\mathrm{int}(A)$ is $Ng\beta$ open set. Then by theorem 4.3(i) we have $Ng\beta\,\mathrm{int}(Ng\beta\,\mathrm{int}(A)) = Ng\beta\,\mathrm{int}(A)$.

iv) Given $A \subset B$. By definition $Ng\beta\,\mathrm{int}(A) \subset A \subset B$ that is $Ng\beta\,\mathrm{int}(A) \subset B$. But $Ng\beta\,\mathrm{int}(B)$is the largest $Ng\beta$ open set contained in B that is $Ng\beta\,\mathrm{int}(B) \subset B$. Therefore $Ng\beta\,\mathrm{int}(A) \subset Ng\beta\,\mathrm{int}(B)$.

Theorem: 4.4 Let A and B be two subsets of Nano topological space $(U, \tau_R(X))$. Then

i) A is a $Ng\beta$ closed set if $A = Ng\beta cl(A)$

ii) $Ng\beta cl(U) = U$ and $Ng\beta cl(\phi) = \phi$

iii) $Ng\beta cl(Ng\beta cl(A)) = Ng\beta cl(A)$

iv) $A \subset B \Rightarrow Ng\beta cl(A) \subset Ng\beta cl(B)$

Proof: Proof is similar to the theorem 4.3.

Theorem: 4.5 Let A be a subset of a Nano topological space $(U, \tau_R(X))$. Then

i) $(Ng\beta cl(A))^C = Ng\beta\,\mathrm{int}(A^C)$

ii) $(Ng\beta\,\mathrm{int}(A))^C = Ng\beta cl(A^C)$.

Proof: i)

$$(Ng\beta cl(A))^C = \cap\{F : F \text{ is } Ng\beta \text{ closed}, A \subset F\}$$
$$= \cup\{F^C : F^C \text{is } Ng\beta \text{ open}, F^C \subseteq A^C\}$$
$$= Ng\beta\,\mathrm{int}(A^C)$$

ii) Proof is similar.

Theorem: 4.6 Let A and B be two subsets of a Nano topological space $(U, \tau_R(X))$, then

i) $Ng\beta\,\mathrm{int}(A \cup B) \supset Ng\beta\,\mathrm{int}(A) \cup Ng\beta\,\mathrm{int}(B)$

$Ng\beta\,\mathrm{int}(A \cap B) \subset Ng\beta\,\mathrm{int}(A) \cap Ng\beta\,\mathrm{int}(B)$

ii) $Ng\beta cl(A \cap B) \subset Ng\beta cl(A) \cap Ng\beta cl(B)$

iii) $Ng\beta cl(A \cup B) \supset Ng\beta cl(A) \cup Ng\beta cl(B)$.

Proof: i)Let A and B are subsets of U clearly $A \subset A \cup B$ and $B \subset A \cup B$ .By theorem 4.3 (iv), $Ng\beta\,\mathrm{int}(A) \subset Ng\beta\,\mathrm{int}(A \cup B)$ and $Ng\beta\,\mathrm{int}(B) \subset Ng\beta\,\mathrm{int}(A \cup B)$. Hence $Ng\beta\,\mathrm{int}(A) \cup Ng\beta\,\mathrm{int}(B) \subset Ng\beta\,\mathrm{int}(A \cup B)$.

Proof is similar for ii),iii) and iv)

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