A STUDY ON WEAKLY ULTRA SEPARATION AXIOMS VIA $\hat{\mu}\beta$-KERNEL SET

J. SUBASHINI, K.INDIRANI

Abstract : The aim of this paper is to introduce a new class of sets namely $\hat{\mu}\beta$ kernel set and study their basic properties in topological spaces. We introduce and formulate some separation axioms by using $\hat{\mu}\beta$-kernel set and $\hat{\mu}\beta$ closed set.

Keywords: $\hat{\mu}\beta$ closed set, $\hat{\mu}\beta$- kernel set and $\hat{\mu}\beta$-R_i - spaces, i=0,1.

1.Introduction: The notion of R_0 topological spaces is introduced by shanin [6] in 1943. Later Davis[3] rediscovered it and studied some properties of this weak separation axiom. In the same paper, Davis also introduced the notion of R_1 topological space which are independent of both T_0 and T_1 but strictly weaker than T_2. In 1970, Levine [4] introduced the concept of generalized closed set in topological spaces. In 2000, Veerakumar [8] introduced several generalized closed sets namely g* closed set,*g closed set, α*g closed set, *gs closed set,$\hat{g}$closed set, μ closed set, μs closed set. Pious Missier and Sucila [5] introduced $\hat{\mu}$ closed set and their continuity. Andrijevic[1] introduced semi preopen set(β open set) in general topology. In 2012, Al-Swidi and Mohammed [2] introduced the separation axioms via kernel set in topological spaces. The purpose of this paper is to introduce the concept of $\hat{\mu}\beta$ – kernel set and to study some of its properties in topological spaces. In addition to this some $\hat{\mu}\beta$ – separation axioms are formulate to find the relation among $\hat{\mu}\beta$ –R_i-spaces for all i=0,1 are also determined.

2. Preliminaries

Throughout this paper (X, τ) or simply X denotes a topological space in which separation axioms are not assumed unless otherwise stated. For a subset A of a topological space (X, τ), int(A), cl(A), and A^c represents the interior of A, the closure of A and the complement of A in X respectively.

Definition2.1 [7]: (i) A subset A of X is called μ closed set if cl(A)⊆U whenever A⊆ U and U is $g\alpha$* open in X.

(ii)$\hat{\mu}$ closed set if scl(A)⊆U whenever A⊆ U and U is μ open in X.

(iii) $\hat{\mu}\beta$ closed set if $\hat{\mu}$cl(A)⊆ U, whenever A⊆ U and U is β open in X.

Definition 2.2 : Let A be a subset of a space X. Then $\hat{\mu}\beta$ closure of A denoted by $\hat{\mu}\beta$cl(A) is defined as the intersection of all $\hat{\mu}\beta$ closed sets containing A.

3.$\hat{\mu}\beta$-Kernel and $\hat{\mu}\beta$-R_i – Spaces, i= 0,1.

Definition 3.1: The intersection of all $\hat{\mu}\beta$ – open subsets of X containing A is called the $\hat{\mu}\beta$ – kernel of A and it is denoted as $\hat{\mu}\beta$ – ker(A).

Definition 3.2:A set A in a topological space (X, τ) is called $\hat{\mu}\beta$ – neighborhood(briefly $\hat{\mu}\beta$–nbd) of a point x ∈ X if there exist a $\hat{\mu}\beta$ – open set B such that x ∈ B ⊆ A.

Definition 3.3: Let (X, τ) be a topological space. A point x is said to be:

(i). $\hat{\mu}\beta$ – adherent point of A ⊆ X if and only if for each U ∈ $\hat{\mu}\beta$ – O(X),x ∈ U A∩U\ {x}≠ φ.

(ii).$\hat{\mu}\beta$ – kernelled point of A⊆X(briefly x∈ $\hat{\mu}\beta$ – kerd(A) if and only if for each , $\hat{\mu}\beta$ – closed set F containing x, F∩A≠ φ.

(iii). Derived $\hat{\mu}\beta$ –kernelled point of A(briefly x ∈ $\hat{\mu}\beta$ – $kerd_{dr}$(A)) if and only if for each , $\hat{\mu}\beta$ – closed set F containing x A∩F/{x} ≠ φ.

(iv) Boundary $\hat{\mu}\beta$ – kernelled point of A (briefly x ∈ $kerd_{bd}$(A)) if and only if for each $\hat{\mu}\beta$ – closed set F containing x F∩A≠ φ and F∩A^c≠ φ.

The following results can be easily proved.

Remark 3.4: Let (X, τ) be a topological space, then y ∈ $\hat{\mu}\beta$–ker(x) if and only if x ∈ $\hat{\mu}\beta$- cl({y}) for each x≠y∈ X.

Remark 3.5: Let (X, τ) be a topological space and x ≠ y∈ X. Then x is $\hat{\mu}\beta$ – kernelled point of {y} if and only if y is a $\hat{\mu}\beta$ – adherent point of {x}.

Lemma 3.5: Let (X, τ) be a topological space and A be a subset of X. Then, $\hat{\mu}\beta$ –ker(A) = {x ∈ X : $\hat{\mu}\beta$ –cl (x) ∩ A ≠ φ}.

Proof: Let x ∈ $\hat{\mu}\beta$ –ker(A) and suppose that $\hat{\mu}\beta$ – cl({x}) ∩ A = φ. Now x ∉ X - $\hat{\mu}\beta$ – cl({x}), which is $\hat{\mu}\beta$ – open set containing A. This is impossible, since x ∈ $\hat{\mu}\beta$ –ker(A). Consequently, $\hat{\mu}\beta$ –cl({x}) ∩A ≠ φ.Conversely let x ∈X such that $\hat{\mu}\beta$ –cl({x}) ∩A ≠ φ and suppose that x ∉ $\hat{\mu}\beta$ –ker(A). Then there exist $\hat{\mu}\beta$ – open set U containing A and x ∉ U. Let y ∈ $\hat{\mu}\beta$ – cl({x}) ∩A. Hence U is $\hat{\mu}\beta$ –nbd of y which does not contain x. By this contradiction it is concluded that x ∈ $\hat{\mu}\beta$ –ker(A). Hence $\hat{\mu}\beta$ –ker(A) = {x ∈ X : $\hat{\mu}\beta$ –cl (x) ∩ A ≠ φ}.

Theorem 3.6: Let (X, τ) be a topological space and A⊆X, then A ∪ $\hat{\mu}\beta$-$kerd_{bd}$(A)=A∪ $\hat{\mu}\beta$-$kerd_{dr}$(A).

Proof: Let x ∈ A∪ $\hat{\mu}\beta$-$kerd_{dr}$(A) and if x∈ $\hat{\mu}\beta$-$kerd_{dr}$(A) then every $\hat{\mu}\beta$ closed set F intersects A(in a point different from x). Therefore, x∈ $\hat{\mu}\beta$-kerd({x}). Hence $\hat{\mu}\beta$-$kerd_{dr}$(A) ⊆ $\hat{\mu}\beta$-kerd(A), it follows that A∪ $\hat{\mu}\beta$-$kerd_{dr}$(A) ⊆ $\hat{\mu}\beta$-kerd(A). Conversely, let x be a point of $\hat{\mu}\beta$-kerd(A). If x∈A, then x ∈A∪ $\hat{\mu}\beta$-$kerd_{dr}$(A).Suppose that x∉A. Since x∈ $\hat{\mu}\beta$-kerd(A), Then x ∈ $\hat{\mu}\beta$-$kerd_{dr}$(A), so that x ∈A ∪ $\hat{\mu}\beta$-$kerd_{dr}$(A). Hence, $\hat{\mu}\beta$-kerd(A) ⊆A ∪ $\hat{\mu}\beta$-$kerd_{dr}$(A) . Thus $\hat{\mu}\beta$-kerd(A)= A ∪ $\hat{\mu}\beta$-ker_{dr}(A).Let x ∈ A ∪ $\hat{\mu}\beta$-$kerd_{bd}$(A) and if x ∈ $\hat{\mu}\beta$-$kerd_{bd}$(A) then every $\hat{\mu}\beta$ closed set F intersects A, therefore $x \in \hat{\mu}\beta$ – kerd(cl{x}). Hence $\hat{\mu}\beta$-$kerd_{bd}$(A) ⊆ $\hat{\mu}\beta$-kerd(A), it follows that A ∪ $\hat{\mu}\beta$-$kerd_{bd}$(A) ⊆ $\hat{\mu}\beta$-kerd(A). Conversely, let x be a point of $\hat{\mu}\beta$-kerd(A). If x∈A, then x∈A∪ $\hat{\mu}\beta$-$kerd_{bd}$(A), suppose that x∉A, then $x \in A^c$. Since $x \in \hat{\mu}\beta$-kerd(A),then every $\hat{\mu}\beta$ closed set F containing x intersects A. Then $x \in \hat{\mu}\beta$-$kerd_{bd}$(A), so that x∈A∪ $\hat{\mu}\beta$-$kerd_{bd}$(A). Hence, $\hat{\mu}\beta$-kerd(A) ⊆A∪ $\hat{\mu}\beta$-$kerd_{bd}$(A). Thus $\hat{\mu}\beta$-kerd(A) = A∪ $\hat{\mu}\beta$-$kerd_{bd}$(A) . Therefore A ∪ $\hat{\mu}\beta$-$kerd_{bd}$(A)=A∪ $\hat{\mu}\beta$-$kerd_{dr}$(A).

Definition 3.7: A topological space (X, τ) is called $\hat{\mu}\beta$-R_o space if $\hat{\mu}\beta$-cl({x}) ⊆ U, for every $\hat{\mu}\beta$ – open set U containing x.

Definition 3.8: A topological space (X, τ) is called $\hat{\mu}\beta$ –R_1 space if for each two distinct points x and y of X with $\hat{\mu}\beta$ –cl({x}) ≠ $\hat{\mu}\beta$ –cl({y}), there exist disjoint $\hat{\mu}\beta$ – open sets U,V such that $\hat{\mu}\beta$ –cl({x}) ⊆U and $\hat{\mu}\beta$ – cl({y}) ⊆V.

Theorem 3.9: A topological space (X, τ) is a $\hat{\mu}\beta$-R_o space if and only if $\hat{\mu}\beta$ –cl(x) = $\hat{\mu}\beta$-ker({x}), for each x∈X.

Proof: Let (X, τ) be a $\hat{\mu}\beta$-R_o space. If $\hat{\mu}\beta$ – cl({x}) ≠ $\hat{\mu}\beta$-ker({x}), x∈X, then their exist another point y≠x such that y∈ $\hat{\mu}\beta$-cl({x}) and y∉ $\hat{\mu}\beta$-ker({x}) that is there exist a $\hat{\mu}\beta$ –open set U_x, y∉U_x implies $\hat{\mu}\beta$ – cl({x}) ⊈ U_x it is a contradiction, since x is a $\hat{\mu}\beta$-R_o space. Hence $\hat{\mu}\beta$ – cl({x}) = $\hat{\mu}\beta$-ker({x}). Conversely, let $\hat{\mu}\beta$ – cl({x}) = $\hat{\mu}\beta$-ker({x}), for each $\hat{\mu}\beta$ –open set U, x∈U then $\hat{\mu}\beta$-ker({x}) = $\hat{\mu}\beta$ –cl({x}) ⊆ U. Hence by definition 3.7, (X, τ) is $\hat{\mu}\beta$-R_o space.

Theorem 3.10: A topological space (X, τ) is $\hat{\mu}\beta$-R_o space if and only if for each , $\hat{\mu}\beta$ closed set F containing x, $\hat{\mu}\beta$-ker({x}) ⊆ F.

Proof: Let for each, $\hat{\mu}\beta$ closed set F containing x, $\hat{\mu}\beta$-ker({x}) ⊆ F and let U be $\hat{\mu}\beta$-open set, and x∈U then for each y∉U implies $y \in U^c$ is $\hat{\mu}\beta$ closed set implies $\hat{\mu}\beta$–ker({y}) ⊆ U^c (by assumption). Therefore x ∉ $\hat{\mu}\beta$-ker({y}) which implies y∉ $\hat{\mu}\beta$ –cl({x}) (by Remark 3.4). So $\hat{\mu}\beta$ –cl({x}) ⊆U. Thus (X, τ) is $\hat{\mu}\beta$-R_o space. Conversely, let (X, τ) be a $\hat{\mu}\beta$-R_o space and F be $\hat{\mu}\beta$ closed set and x∈F. Then for each y ∉ F implies $y \in F^c$ is $\hat{\mu}\beta$–open set, then $\hat{\mu}\beta$–cl({y}) ⊆ F^c So $\hat{\mu}\beta$-ker({x}) = $\hat{\mu}\beta$ –cl({x}). Thus, $\hat{\mu}\beta$-ker({x}) ⊆F.

Corollary 3.11: A topological space (X, τ) is $\hat{\mu}\beta$-R_o space if and only if for each , $\hat{\mu}\beta$- open set U and x∈U, $\hat{\mu}\beta$-cl($\hat{\mu}\beta$-ker({x})) ⊆ U.

Remark 3.12: Every $\hat{\mu}\beta$-R_1 space is $\hat{\mu}\beta$-R_o space.

Theorem 3.13: A topological space (X, τ) is $\hat{\mu}\beta$-R_1 space if and only if for each x≠y ∈ X with $\hat{\mu}\beta$-ker({x}) ≠ $\hat{\mu}\beta$-ker({y}), there exist $\hat{\mu}\beta$ clopen sets G_1, G_2 such that $\hat{\mu}\beta$-ker({x}) ⊆ G_1, $\hat{\mu}\beta$-ker({x}) ∩ G_2 = φ and $\hat{\mu}\beta$-ker({y}) ⊆ G_2, $\hat{\mu}\beta$-ker({y}) ∩ G_1 = φ and $G_1 \cup G_2$ = X.

Proof: Let (X, τ) be a $\hat{\mu}\beta$-R_1 space. Then for each x≠y ∈ X with $\hat{\mu}\beta$-ker({x}) ≠ $\hat{\mu}\beta$-ker({y}). As every $\hat{\mu}\beta$-R_1 space is $\hat{\mu}\beta$-R_o space and $\hat{\mu}\beta$ –cl(x) = $\hat{\mu}\beta$-ker({x}). Therefore $\hat{\mu}\beta$ –cl({x}) ≠ $\hat{\mu}\beta$ –cl({y}), there exist two disjoint $\hat{\mu}\beta$ –open sets U_1, U_2 such that $\hat{\mu}\beta$ –cl({x}) ⊆ U_1 and $\hat{\mu}\beta$ –cl({y}) ⊆ U_2 then U_1^c and U_2^c are $\hat{\mu}\beta$ closed sets such that $U_1^c \cup U_2^c$ = X. Put $G_1 = U_1^c$ and $G_2 = U_2^c$. Thus x ∈ U_1 ⊆ G_2 and y ∈ U_2 ⊆ G_1, so that $\hat{\mu}\beta$-ker({x}) ⊆ U_1 ⊆ G_2 and $\hat{\mu}\beta$-ker({y}) ⊆ U_2 ⊆ G_1. Conversely, let for each x≠y ∈ X with $\hat{\mu}\beta$-ker({x}) ≠ $\hat{\mu}\beta$-ker({y}), there exist $\hat{\mu}\beta$ closed sets G_1 and G_2 such that $\hat{\mu}\beta$-ker({x}) ⊆ G_1, $\hat{\mu}\beta$-ker({x}) ∩ G_2 = φ and $\hat{\mu}\beta$-ker({y}) ⊆ G_2, $\hat{\mu}\beta$-ker({y}) ∩ G_1 = φ and $G_1 \cup G_2$ = X, then G_1^c and G_2^c are $\hat{\mu}\beta$ open sets such that $G_1^c \cap G_2^c = \varphi$. Put $G_1^c = U_2$ and $G_2^c = U_1$. Thus $\hat{\mu}\beta$-ker({x}) ⊆ U_1 and $\hat{\mu}\beta$-ker({y}) ⊆ U_2 and $U_1 \cap U_2 = \varphi$, so that x ∈ U_1 and y ∈ U_2 implies x ∉ $\hat{\mu}\beta$-cl({y}) and y ∉ $\hat{\mu}\beta$-cl({x}) , then $\hat{\mu}\beta$ –cl({x}) ⊆ U_1 and $\hat{\mu}\beta$ – cl({y}) ⊆ U_2. Thus (X, τ) is $\hat{\mu}\beta$-R_1 space.

Corollary 3.13: A topological space (X, τ) is $\hat{\mu}\beta$-R_1 space if and only if for each x ≠ y∈ X with $\hat{\mu}\beta$ –cl({x}) ≠ $\hat{\mu}\beta$ –cl({y}) there exist disjoint $\hat{\mu}\beta$-open sets U, V such that $\hat{\mu}\beta$ –cl($\hat{\mu}\beta$-ker({x})) ⊆ U and that $\hat{\mu}\beta$ – cl($\hat{\mu}\beta$-ker({y})) ⊆ V.

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J. Subashini

Department of Mathematics, College of Arts & Science, Sulur, Coimbatore -, India

K. Indirani, Nirmala College for women, Red fields, Coimbatore-,India.

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