SOME RESULTS ON CONTRA NANO $rg^{**}b$-CLOSED AND OPEN MAPS IN NANOTOPOLOGICAL SPACES

G. VASANTHAKANNAN, K. INDIRANI

Abstract: In this paper we introduce the concept of nano $rg^{**}b$-closed maps and Contra nano $rg^{**}b$-closed maps in nanotopological spaces. Also some characterizations and several properties concerning Contra nano $rg^{**}b$-closed maps and Contra strongly nano $rg^{**}b$-closed maps are derived.

1. **Introduction:** The concept of nanotopology was introduced by Lellis Thivagar [1] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He also established the week forms of nano open sets namely nano α-open sets, nano-semi open sets and nano pre-open sets in a nanotopological space.

In this paper we define a new class of maps, called nano $rg^{**}b$-closed maps,nano $rg^{**}b$-open maps,Contra nano $rg^{**}b$-closed maps,Contra nano $rg^{**}b$-open maps in nanotopological spaces. Also we study some of their characterizations and several properties are given.

2. **Preliminaries: Definition 2.1:**[2] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation, elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = \cup_{x\in U}\{R(x): R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.

ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = \cup_{x\in U}\{R(x): R(x) \cap X \neq \emptyset\}$ where $R(x)$ denotes the equivalence class determined by x.

iii) The boundary region of X with respect to R is set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$.That is $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2:[2]Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

1. U and$\emptyset \in \tau_R(X)$.
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on U called as the nanotopology on U with respect to XWe call $(U, \tau_R(X))$ as the Nanotopological space.

Definition 2.3:[2]Let $(U, \tau_R(X))$ be a nano topological space and$A \subseteq U$. Then A is said to be:

i) Nano semi open if $A \subseteq Ncl(Nint(A))$
ii) Nano pre-open if $A \subseteq Nint(Ncl(A))$
iii) Nano α-open if $A \subseteq Nint(Ncl(Nint(A)))$
iv) Nano semi pre-open if $A \subseteq Ncl(Nint(Ncl(A)))$

Let $(U, \tau_R(X))$ be a nano topological space and$A \subseteq U$. A is said to be nano semi closed, nano pre-closed, nano α-closed, nano semi pre-closed if its complement is respectively nano semi-open, nano pre-open, nano α- open, nano semi pre-open.

Definition 2.4: Let $(U, \tau_R(X))$ be a nano topological space and$A \subseteq U$. Then A is said to be:

(i) Nano regular generalized star star b-closed (briefly nano $rg^{**}b$-closed) [7] if $Nrg^{*}bcl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open in U.
(ii) Nano generalized closed (briefly nano g-closed) [8] if $Ncl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open in U.
(iii) Nano regular generalized star b-closed (briefly nano $rg^{*}b$-closed) [9] if $Ncl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano regular open in U.

Definition 2.5: Let$(U, \tau_R(X))$ and $(V, \tau_R(Y))$ be nanotopological spaces, then a map $f: (U, \tau_R(X)) \longrightarrow (V, \tau_R(Y))$ is said to be

(i) Nano continuous [3]if $f^{-1}(V)$ is nano closed in $(U, \tau_R(X))$ for each nano closed set Vin $(V, \tau_R(Y))$.
(ii) nano $rg^{**}b$-continuous[7] if $f^{-1}(V)$ is nano $rg^{**}b$-closed in $(U, \tau_R(X))$ for each nano closed set Vin $(V, \tau_R(Y))$.
(iii) nano $rg^{*}b$-continuous if $f^{-1}(V)$ is nano $rg^{*}b$- closed in $(U, \tau_R(X))$ for each nano closed set Vin $(V, \tau_R(Y))$.

3. Nano $rg^{**}b$- closed and open maps, Contra nano $rg^{**}b$- closed and open maps.

Definition 3.1:A map $f: (U, \tau_R(X)) \longrightarrow (V, \tau_R(Y))$ is said to be nano $rg^{**}b$-closed if the image of every nano closed set in $(U, \tau_R(X))$ is nano $rg^{**}b$-closed in $(V, \tau_R(Y))$.

Definition 3.2:A map $f: (U, \tau_R(X)) \longrightarrow (V, \tau_R(Y))$ is said to be nano $rg^{**}b$-open if $f(A)$ is nano $rg^{**}b$-openfor each nano open set A in $(U, \tau_R(X))$.

Definition 3.3:A map $f: (U, \tau_R(X)) \longrightarrow (V, \tau_R(Y))$ is said to be strongly nano $rg^{**}b$-closed if the image

$f(A)$ is nano $rg^{**}b$-closed in $(V, \tau_R(Y))$ for each nano $rg^{**}b$-closed in $(U, \tau_R(X))$.

Definition 3.4: A map $f: (U, \tau_R(X)) \longrightarrow (V, \tau_R(Y))$ is said to be strongly nano $rg^{**}b$-open if the image $f(A)$ is nano $rg^{**}b$-open in $(V, \tau_R(Y))$ for each nano $rg^{**}b$-open in $(U, \tau_R(X))$.

Theorem 3.5: Every strongly nano $rg^{**}b$-closed map is nano $rg^{**}b$-closed but not conversely.

Proof: Given that $f: (U, \tau_R(X)) \longrightarrow (V, \tau_R(Y))$ is strongly nano $rg^{**}b$-closed map. Let A be a nano closed set in U. Since every nano closed set is nano $rg^{**}b$-closed set. Therefore A is nano $rg^{**}b$-closed set. But f is strongly nano $rg^{**}b$-closed. Therefore $f(A)$ is nano $rg^{**}b$-closed in V. Hence f is nano $rg^{**}b$-closed map.

Example 3.6: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c\}$ then $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Let $V = \{a, b, c, d\}$ with $V/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $Y = \{a, b\}$ then o$\tau_R(Y) = \{V, \phi, \{a\}, \{a, b, c\}, \{b, c\}\}$. Define $f: U \to V$ as $f(a) = a, f(b) = b, f(c) = c, f(d) = d$. Then f is nano $rg^{**}b$-closed maps but not strongly nano $rg^{**}b$-closed. Since for the nano $rg^{**}b$-closed set $\{a, c\}, f(\{a, c\}) = \{a, c\}$ which is not nano $rg^{**}b$-closed set in V.

Theorem 3.7: A nano continuous, nano $rg^{*}b$-closed function maps, nano $rg^{**}b$-closed sets into nano $rg^{**}b$-closed sets.

Proof: Let $f: (U, \tau_R(X)) \longrightarrow (V, \tau_R(Y))$ be nano continuous and nano $rg^{*}b$-closed and let $A \subseteq U$ be nano $rg^{**}b$-closed. Let G be nano open in V such that $f(A) \subseteq G$. Then $A \subseteq f^{-1}(G)$ which is nano open in U, since f is nano continuous. Then $A = Nrg^{*}bcl(A) \subseteq f^{-1}(G)$, since A is nano $rg^{*}b$-closed in U. Then $f(Nrg^{*}bcl(A)) \subseteq G$. Since f is nano $rg^{*}b$-closed in U and $Nrg^{*}bcl(A)$ is nano $rg^{*}b$-closed in $U, f(Nrg^{*}bcl(A))$ is nano $rg^{*}b$-closed in V. Therefore $f(Nrg^{*}bcl(A)) = Nrg^{*}bcl\big(f(Nrg^{*}bcl(A)\big) \subseteq G$ and hence $Nrg^{*}bcl\big(f(Nrg^{*}bcl(A)\big) \subseteq G$. But $Nrg^{*}bcl\, f(A) \subseteq Nrg^{*}bcl\big(f(Nrg^{*}bcl(A)\big) \subseteq G$. Therefore $Nrg^{*}bcl\,\big(f(A)\big) \subseteq G$ whenever G is nano open in V and $f(A) \subseteq G$. Thus $f(A)$ is nano $rg^{**}b$-closed in V. Hence f maps nano $rg^{**}b$-closed sets into nano $rg^{**}b$-closed sets.

Remark 3.8: A nano continuous, nano $rg^{*}b$-closed function map does not map nano $rg^{**}b$-open sets into nano $rg^{**}b$-open sets.

Definition 3.9: A map $f: (U, \tau_R(X)) \longrightarrow (V, \tau_R(Y))$ is said to be Contra nano $rg^{**}b$-closed map (resp. Contra nano $rg^{**}b$-open) if the image of every nano closed (resp. nano open) set in $(U, \tau_R(X))$ is nano $rg^{**}b$-open (resp.nano $rg^{**}b$-closed) set in $(V, \tau_R(Y))$

Remark 3.10: Contra nano $rg^{**}b$-closedness and nano $rg^{**}b$-closedness are independent notions as shown in the following example.

Example 3.11: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c\}$ then $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Let $V = \{a, b, c, d\}$ with $V/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $Y = \{a, b\}$ then $\tau_R(Y) = \{V, \phi, \{a\}, \{a, b, c\}, \{b, c\}\}$. Define $f: U \to V$ as $f(a) = a, f(b) = c, f(c) = c, f(d) = d$. Then f is Contra nano $rg^{**}b$-closed but not nano $rg^{**}b$-closed because $\{a, b\}$ is nano closed in U but $f\{a, b\} = \{a, c\}$ is not nano $rg^{**}b$-closed in V.

Example 3.12: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c\}$ then $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Let $V = \{a, b, c, d\}$ with $V/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $Y = \{a, b\}$ then $\tau_R(Y) = \{V, \phi, \{a\}, \{a, b, c\}, \{b, c\}\}$. Define $f: U \to V$ as $f(a) = a, f(b) = b, f(c) = c, f(d) = d$. Then f is nano $rg^{**}b$-closed in U but not Contra nano $rg^{**}b$-closed because $\{b\}$ is nano closed in U but $f\{b\} = \{b\}$ is not nano $rg^{**}b$-open in V.

Similarly, Contra nano $rg^{**}b$-openness and nano $rg^{**}b$-openness are independent notions.

Theorem 3.13: Let $(U, \tau_R(X)), (V, \tau_R(Y))$ are nanotopological spaces. Then if

i) $f: (U, \tau_R(X)) \longrightarrow (V, \tau_R(Y))$ is Contra nano $rg^{**}b$-closed and A is nano closed subset of $(U, \tau_R(X))$, then $f_A: (A, U_A) \to (V, \tau_R(Y))$ is Contra nano $rg^{**}b$-closed.

ii) $f: (U, \tau_R(X)) \longrightarrow (V, \tau_R(Y))$ is Contra nano $rg^{**}b$-open and A is nano open subset of $(U, \tau_R(X))$, then $f_A: (A, U_A) \to (V, \tau_R(Y))$ is Contra nano $rg^{**}b$-open.

iii) $f: (U, \tau_R(X)) \longrightarrow (V, \tau_R(Y))$ is Contra nano $rg^{**}b$-closed bijective function and $A = f^{-1}(B)$ for some open subset B of $(U, \tau_R(X))$, then $f_A: (A, U_A) \to (V, \tau_R(Y))$ is Contra nano $rg^{**}b$-closed.

Proof: (i) Let B be a nano closed set of A. Then $B = A \cap S$ for some nano closed set S of $(U, \tau_R(X))$ and so B is nano closed in $(U, \tau_R(X))$. By hypothesis $f(B)$ is nano $rg^{**}b$-open. But $f(B) = f_A(B)$ and therefore, f_A is Contra nano $rg^{**}b$-closed.

(ii) Let B be a nano open set of A. Then $B = A \cap G$ for some nano open set G of $(U, \tau_R(X))$ and so B is nano open in $(U, \tau_R(X))$. By hypothesis, $f(B)$ is nano $rg^{**}b$-closed. But $f(B) = f_A(B)$ and therefore, f_A is Contra nano $rg^{**}b$-open.

(iii) Let D be a nano closed set of A. Then $D = A \cap H$ for some nano closed set H of $(U, \tau_R(X))$. But $f(D) = f_A(D) = f(A \cap H) = f(f^{-1}(B) \cap H) = B \cap f(H)$. Since f is Contra nano $rg^{**}b$-closed, $f(H)$ is nano $rg^{**}b$-open in $(V, \tau_R(Y))$. Hence f is Contra nano $rg^{**}b$-closed.

Theorem 3.14: Let $f: (U, \tau_R(X)) \longrightarrow (V, \tau_R(Y))$ and $g: (V, \tau_R(Y)) \longrightarrow (W, \tau_R(Z))$ be any two maps such that $g \circ f: (U, \tau_R(X)) \longrightarrow (W, \tau_R(Z))$ is Contra nano

$rg^{**}b$-open map. If f is Contra nano continuous and surjective, then g is nano $rg^{**}b$-closed.

Proof: Let A be nano closed set in $(V, \tau_R(Y))$. Since f is Contra nano continuous $f^{-1}(A)$ is nano open in $(U, \tau_R(X))$. Then $(g \circ f)f^{-1}(A)$ is nano $rg^{**}b$-closed in $(W, \tau_R(Z))$. That is $g(A)$ is nano $rg^{**}b$-closed in $(W, \tau_R(Z))$. Therefore, g is nano $rg^{**}b$-closed map.

Definition3.15:A map $f: (U, \tau_R(X)) \longrightarrow (V, \tau_R(Y))$ is said to be Contra strongly nano $rg^{**}b$-closed map (resp. Contra strongly nano $rg^{**}b$-open) if the image of every nano $rg^{**}b$- closed (resp. nano $rg^{**}b$ -open) set in $(U, \tau_R(X))$ is nano $rg^{**}b$-open (resp.nano $rg^{**}b$-closed) set in $(V, \tau_R(Y))$.

Theorem 3.16:(i) Every Contra strongly nano $rg^{**}b$-closed map is Contra nano $rg^{**}b$-closed.

(ii) Every Contra strongly nano $rg^{**}b$-open map is Contra nano $rg^{**}b$-open.

Proof:(i)Let f be Contra strongly nano $rg^{**}b$-closed map and A be nano closed set in $(U, \tau_R(X))$.Since every nano closed set is nano $rg^{**}b$-closed, A is nano $rg^{**}b$-closed and $f(A)$ is nano $rg^{**}b$-open in $(V, \tau_R(Y))$. Hence f is Contra nano $rg^{**}b$-closed.

(ii) Proof is similar to the proof of (i).

Remark3.17: The implications in above theorem are not reversible in general as seen from the following examples.

Example 3.18: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c\}$ then $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Let $V = \{a, b, c, d\}$ with $V/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $Y = \{a, b\}$ then $\tau_R(Y) = \{V, \phi, \{a\}, \{a, b, c\}, \{b, c\}\}$. Define $f: U \to V$ as $f(a) = a, f(b) = c, f(c) = d, f(d) = b$. Then f is Contra nano $rg^{**}b$-closed but not Contra strongly nano $rg^{**}b$-closed because $\{c, d\}$ is nano $rg^{**}b$-closed in U but $f\{c, d\} = \{b, d\}$ is not nano $rg^{**}b$-open in V.

Example 3.19: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c\}$ then $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Let $V = \{a, b, c, d\}$ with $V/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $Y = \{a, b\}$ then $\tau_R(Y) = \{V, \phi, \{a\}, \{a, b, c\}, \{b, c\}\}$. Define $f: U \to V$ as $f(a) = a, f(b) = c, f(c) = b, f(d) = d$. Then f is Contra nano $rg^{**}b$-open but not Contra strongly nano $rg^{**}b$-open because $\{a, d\}$ is nano $rg^{**}b$-open in U but $f\{a, d\} = \{a, d\}$ is not nano $rg^{**}b$-closed in V.

Theorem 3.20: For a map$f: (U, \tau_R(X)) \longrightarrow (V, \tau_R(Y))$ the following are equivalent if the arbitrary union of class of nano $rg^{**}b$-open sets in $(V, \tau_R(Y))$ is also nano $rg^{**}b$-open.

(i) f is contra stronglynano $rg^{**}b$-open

(ii) For every subset B of $(V, \tau_R(Y))$ and for every nano $rg^{**}b$-closed subset F of $(U, \tau_R(X))$ with $f^{-1}(B) \subseteq F$, there exists a nano $rg^{**}b$-open subset G of $(V, \tau_R(Y))$ with $B \subseteq G$ and $f^{-1}(G) \subseteq F$.

(iii) For every $v \in V$ and for every nano $rg^{**}b$-closed subset F of U with $f^{-1}(B) \subseteq F$, there exist a nano $rg^{**}b$-open subset G of V with $v \in G$ and $f^{-1}(G) \subseteq F$.

Proof:$(i) \Rightarrow (ii)$Let B be a subset of V and f be a nano $rg^{**}b$-closed subset of U with $f^{-1}(B) \subseteq F$. Since f is Contra strongly nano $rg^{**}b$-open and F^c is nano $rg^{**}b$-open subset of U, $f(F^c)$ is nano $rg^{**}b$-closed subset of V. Put $G = \left(f(f^c)\right)^c$ then G is nano $rg^{**}b$-open subset of V and since $f^{-1}(B) \subseteq F$ we have $f(f^c) \subseteq B^c$and hence $B \subseteq G$ moreover $f^{-1}(G) \subseteq f^{-1}\left(f(f^c)\right)^c \subseteq F$.

$(ii) \Rightarrow (iii)$ It is sufficient to put $B = \{V\}$.

$(iii) \Rightarrow (i)$ Let A be nano $rg^{**}b$-open subset of U and $v \in (f(A))^c$ and $F = A^c$. Then F is nano $rg^{**}b$-closed subset of U.By(iii) there exist a nano $rg^{**}b$-open subset G_v of V with $v \in G_v$ and $f^{-1}(G_v) \subseteq F$. Then $v \in G_v \subseteq (f(A))^c$. Hence $(f(A))^c = \cup\{G_v/V \in (f(A))^c\}$ is nano $rg^{**}b$-open. Therefore $f(A)$ is nano $rg^{**}b$-closed subset of V. Hence f is Contra strongly nano $rg^{**}b$-open.

Theorem 3.21: If the map is bijective, then Contra strongly nano $rg^{**}b$-closedness and Contra strongly nano $rg^{**}b$-openess are equivalent.

Proof: Let f be Contra strongly nano $rg^{**}b$-closedness and G be a nano $rg^{**}b$-open set in $(U, \tau_R(X))$, then G^c is nano $rg^{**}b$-closed and $f(G^c)$ is nano $rg^{**}b$-open in $(V, \tau_R(Y))$. That is $f(G)$ is nano $rg^{**}b$-closed in $(V, \tau_R(Y))$. Hence f is Contra strongly nano $rg^{**}b$-open.

Similarly the other part can be proved.

Remark 3.22: Contra strongly nano $rg^{**}b$-closedness and strongly nano $rg^{**}b$-closedness are independent notions as shown in the following example.

Example 3.23: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c\}$ then $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Let $V = \{a, b, c, d\}$ with $V/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $Y = \{a, b\}$ then $\tau_R(Y) = \{V, \phi, \{a\}, \{a, b, c\}, \{b, c\}\}$. Define $f: U \to V$ as $f(a) = a, f(b) = c, f(c) = c, f(d) = d$. Then f is Contra strongly nano $rg^{**}b$-open but not strongly nano $rg^{**}b$-closed because $\{a, b, d\}$ is nano $rg^{**}b$ -closed in U but $f\{a, b, c\} = \{a, c, d\}$ is not nano $rg^{**}b$-closed in V.

Example 3.24: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c\}$ then $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Let $V = \{a, b, c, d\}$ with $V/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $Y = \{a, b\}$ then $\tau_R(Y) = \{V, \phi, \{a\}, \{a, b, c\}, \{b, c\}\}$. Define $f: U \to V$ as $f(a) = a, f(b) = b, f(c) = c, f(d) = d$. Then f is strongly nano $rg^{**}b$-closed in U but not Contra strongly nano $rg^{**}b$-closed because $\{b\}$ is nano closed in U but $f\{b\} = \{b\}$ is not nano $rg^{**}b$-open in V.

Similarly, Contra strongly nano $rg^{**}b$-openness and strongly nano $rg^{**}b$-openness are independent notions.

Remark 3.25: The composition of two Contra strongly nano $rg^{**}b$-closed maps need not be Contra strongly nano $rg^{**}b$-closed as shown in the following example.

Example 3.26: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c\}$ then $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Let $V = \{a, b, c, d\}$ with $V/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $Y = \{a, b\}$ then $\tau_R(Y) = \{V, \phi, \{a\}, \{a, b, c\}, \{b, c\}\}$. Define $f: U \to V$ as $f(a) = a, f(b) = c, f(c) = c, f(d) = d$ and $g: V \to W$ as $g(a) = a, g(b) = c, g(c) = d, g(d) = b$.Then f and g are Contra strongly nano $rg^{**}b$-closed maps but $g \circ f: (U, \tau_R(X)) \to (W, \tau_R(Z))$ is not Contra strongly nano $rg^{**}b$-closed maps because $\{g \circ f\{d\} = \{b\}$ is not nano $rg^{**}b$-open in $(W, \tau_R(Z))$.

Remark 3.27: The composition of two Contra strongly nano $rg^{**}b$-open maps need not be Contra strongly nano $rg^{**}b$-open as shown in the following example.

Example 3.28: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, c\}$ then $\tau_R(X) = \{U, \phi, \{a\}, \{a, c, d\}, \{c, d\}\}$. Let $V = \{a, b, c, d\}$ with $V/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $Y = \{a, b\}$ then $\tau_R(Y) = \{V, \phi, \{a\}, \{a, b, c\}, \{b, c\}\}$. Define $f: U \to V$ as $f(a) = d, f(b) = b, f(c) = a, f(d) = b$ and $g: V \to W$ as $g(a) = c, g(b) = b, g(c) = d, g(d) = d$.Then f and g are Contra strongly nano $rg^{**}b$-closed maps but $g \circ f: (U, \tau_R(X)) \to (W, \tau_R(Z))$ is not Contra strongly nano $rg^{**}b$-open maps because $\{g \circ f\{c, d\} = \{b, c\}$ is not nano $rg^{**}b$-open in $(W, \tau_R(Z))$.

Theorem 3.29: Let $f: (U, \tau_R(X)) \longrightarrow (V, \tau_R(Y))$ and $g: (V, \tau_R(Y)) \longrightarrow (W, \tau_R(Z))$ be any two maps such that $g \circ f: (U, \tau_R(X)) \longrightarrow (W, \tau_R(Z))$

(i) If f is strongly nano $rg^{**}b$-closed and g is Contra strongly nano $rg^{**}b$-closed then $g \circ f$ Contra strongly nano $rg^{**}b$-closed.

(ii) If f is Contra strongly nano $rg^{**}b$-closed and g is strongly nano $rg^{**}b$-open then $g \circ f$ Contra strongly nano $rg^{**}b$-closed.

Proof:(i) Let G be nano $rg^{**}b$-closed in $(U, \tau_R(X))$. Then $f(G)$ is nano $rg^{**}b$-closed in $(V, \tau_R(Y))$ and $g(f(G)$ is nano $rg^{**}b$-open in $(W, \tau_R(Z))$. Hence $g \circ f$ Contra strongly nano $rg^{**}b$-closed.

(ii) Let G be nano $rg^{**}b$-closed in $(U, \tau_R(X))$. Then $f(G)$ is nano $rg^{**}b$-open in $(V, \tau_R(Y))$ and $g(f(G)$ is nano $rg^{**}b$-open in $(W, \tau_R(Z))$. Hence $g \circ f$ Contra strongly nano $rg^{**}b$-closed.

Theorem 3.30: Let $f: (U, \tau_R(X)) \longrightarrow (V, \tau_R(Y))$ and $g: (V, \tau_R(Y)) \longrightarrow (W, \tau_R(Z))$ be any two maps such that $g \circ f: (U, \tau_R(X)) \longrightarrow (W, \tau_R(Z))$

(i) If f is strongly nano $rg^{**}b$-open and g is Contra strongly nano $rg^{**}b$-open then $g \circ f$ Contra strongly nano $rg^{**}b$-open.

(ii) If f is Contra strongly nano $rg^{**}b$-open and g is strongly nano $rg^{**}b$-closed then $g \circ f$ Contra strongly nano $rg^{**}b$-open.

Proof:(i) Let G be nano $rg^{**}b$-open in $(U, \tau_R(X))$. Then $f(G)$ is nano $rg^{**}b$-open in $(V, \tau_R(Y))$ and $g(f(G)$ is nano $rg^{**}b$-closed in $(W, \tau_R(Z))$. Hence $g \circ f$ Contra strongly nano $rg^{**}b$-open.

(iii) Let G be nano $rg^{**}b$-open in $(U, \tau_R(X))$. Then $f(G)$ is nano $rg^{**}b$-closed in $(V, \tau_R(Y))$ and $g(f(G)$ is nano $rg^{**}b$-closed in $(W, \tau_R(Z))$. Hence $g \circ f$ Contra strongly nano $rg^{**}b$-open.

References:

1. M. Lellis Thivagar and Carmel Richard, note on nanotopological spaces, communicated.
2. M. Lellis Thivagar and Carmel Richard, On nano forms of weakly open sets, International Journal of Mathematics and statistics Invention, (2013), 31-37.
3. M. Lellis Thivagar and Carmel Richard, "' On nano Continuity", Mathematical theory and modelling, (2013), 7, 32-37.
4. A.Dhanis Arul Mary and I. Arockiarani, Note on Contra nano gb-closed maps, Journal of Global Research in Mathematical Achieves, 2(10), (2014), 55-66.
5. J. Donntchev, Contra continuous functions and strongly-S closed spaces, Internat. J. Math. Sci. 19(1996), 303-310.
6. A.Dhanis Arul Mary and I. Arockiarani, Properties of nano gb-closed maps, IOSR Journal of Mathematics (IOSR-JM) e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 2 Ver. II (Mar-Apr. 2015), PP 21-24.
7. G. Vasanthakannan and K. Indirani, On nano regular generalized star star b-closed set in nanotopological spaces, IJMA, 7(10), 2016, 115-122.
8. Bhuvaneshwari and Mythili Gnanapriya K, Nano generalized closed sets in nanotopological spaces, International Journal of Sceintific and Research Publications, (2014).
9. M.G. Smitha and Indirani. K, On nano regular generalized star b-closed set in nanotopological spaces, International Journal of Physics and Mathematical Sciences, 5(3), (2015), 22-26.

G. Vasanthakannan, K. Indirani
Nirmala College for Women, Red fields, Coimbatore-India.