ERYING-POWELL FLUID SLIP FLOW WITH TEMPERATURE AND SPACE DEPENDENT HEAT GENERATION

T.POORNIMA, P.SREENIVASULU, N.BHASKAR REDDY

Abstract: An analysis was made to study the effects of slip fluid flow and time dependent heat generation/absorption parameter on an incompressible Erying-Powell fluid. The boundary layer equations governed by the incompressible fluid flow is converted to dimensionless employing similarity transformations. The ultimate equations are solved using bvp4c method. The upshots of this analysis for various dimensionless material parameters on the flow field were elaborately discussed with the help of graphs and tables.

Keywords: Erying-Powell fluid, Non-Newtonian, Slip flow, heat generation/absorption, boundary value problem

Introduction: Non-Newtonian fluids have great importance because of its diversity in nature in terms of their viscous and elastic properties. A great attention was given by the researchers now-a-days on one of the non-Newtonian fluid called "Eyring–Powell fluid". This fluid model distinct itself from other non-Newtonian fluid models because it is a consequent from kinetic theory of liquids in spite of empirical relation. In addition, this fluid can be reduced to Newtonian behaviour for low and high shear rates. The Cauchy stress tensor, in as Eyring–Powell fluid [1] takes the following form:

$$\lambda_{ij} = \mu \frac{\partial u_i}{\partial x_j} + \frac{1}{\beta}\sinh^{-1}\left(\frac{1}{C}\frac{\partial u_i}{\partial x_j}\right) \qquad (1)$$

where μ is dynamic viscosity and β and C are the rheological fluid parameters of the Eyring–Powell fluid model.

Consider the second-order approximation of the *sinh1* function as follows:

$$\sinh^{-1}\left(\frac{1}{C}\frac{\partial u_i}{\partial x_j}\right) \cong \frac{1}{C}\frac{\partial u_i}{\partial x_j} - \frac{1}{6}\left(\frac{1}{C}\frac{\partial u_i}{\partial x_j}\right)^3, \quad \left|\frac{1}{C}\frac{\partial u_i}{\partial x_j}\right| \ll 1 \qquad (2)$$

The introduction of the appropriate terms into the flow model is considered next. The resulting boundary value problem is found to be well-posed and permits an excellent mechanism for the assessment of rheological characteristics on the flow behaviour. A rich research text on Erying –Powell fluid on different geometries and applications are available in literature [2]-[4]. Vittal et al. [5] analyzed the stagnation point flow of a MHD Powell-Eyring fluid over a nonlinearly stretching sheet in the presence of heat source/sink. Mahanthesh et al. [6] studied the effect of a nanoEyring-Powell fluid past a convectively heated stretching sheet in the presence of thermal radiation, viscous dissipation and Joule heating. El-Aziz et al. [7] investigated heat transfer effects with space- or-temperature-dependent heat source in a mixed convection flow.The space and temperature-dependent heat generation/absorption (non-uniform heat source/sink) q''' is defined as

$$q''' = \left(\frac{kU_w(x)}{x\upsilon}\right)\left(A^*\left(T_w - T_\infty\right)f' + B^*\left(T - T_\infty\right)\right) \qquad (3)$$

where A^* and B^* are parameters of the space and temperature dependent internal heat generation/absorption. The positive and negative values of A^* and B^* represents generation and absorption, respectively. Tsai et al. [8] investigated the flow and heat transfer over an unsteady stretching surface with non-uniform heat source. Khader and Megahed [9] studied the heat transfer of the Powell-Eyring fluid thin film over an unsteady stretching sheet with internal heat generation using the Chebyshev finite difference method. El-Aziz and Salem [7] studied the MHD-mixed convection and mass transfer from a vertical stretching sheet with diffusion of chemically reactive species and space- or temperature-dependent heat source. Abel et al. [10] analysed the heat transfer in MHD viscoelastic boundary layer flow over a stretching sheet with non-uniform heat source/sink.

On the other hand, in certain circumstances, the partial slip between the fluid and the moving surface may occur in situations when the fluid is particulate such as emulsions, suspensions, foam and polymer solutions. In these cases, the proper boundary condition is replaced by Navier's condition, where the amount of relative slip is proportional to local shear stress. Bhaskar Reddy et al. [11] analysed the influence of Variable thermal conductivity on MHD boundary layer Slip flow of Ethylene-Glycol based Cu nanofluids past a stretching sheet with convective boundary condition. Sreenivasulu et al. [12] studied the influence of Magnetic Field on a Nanofluid Past a non Linear Stretching Sheet with Uniform Heat Source. The earlier studies that took into account the slip boundary condition over a stretching sheet were conducted by Andersson [13]. He gave a closed form solution of a full Navier–Stokes equations for a magnetohydrodynamics flow over a stretching sheet. Following Andersson, Wang [14] found the closed form similarity solution of a full Navier–Stoke's equations for the flow due to a stretching sheet with partial slip.

In view of the above observations, an attempt is made to study the effects of Navier's slip on Erying-Powell fluid past a stretching surface with temperature-space dependent heat generation.

2. **Mathematical Formulation:** An incompressible and electrically conducting two dimensional boundary layer flow of an Erying-Powell fluid past a stretching sheet with partial slip and temperature dependent non-uniform heat generation/absorption is considered. An external magnetic field $B(x)$ is applied along the y direction. A non-uniform heat source/sink q''' is taken into account which depends upon space and temperature. There is a constant suction/injection velocity V_w normal to the sheet. The boundary layer equations that govern the present flow subject to the Boussinesq's approximations can be expressed as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{4}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(\upsilon+\frac{1}{\rho\tilde{\beta}C^{*}}\right)\frac{\partial^2 u}{\partial y^2}-\frac{1}{2\rho\tilde{\beta}C^{*3}}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \tag{5}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+q''' \tag{6}$$

The corresponding boundary conditions are

$$u=u_w+N\nu_f\frac{\partial u}{\partial y};\quad v=-V_w;\quad T=T_w(x)=T_\infty+bx \qquad at\quad y=0 \tag{7}$$
$$u=0;\quad T\to T_\infty \qquad as\quad y\to\infty$$

where$u$, v are the velocity components in the x and y directions, respectively, T - the temperature of the nanofluid, T_∞- the ambient fluid temperature, σ - the electric conductivity, B_o - the uniform magnetic field strength

Introducing the similarity transformations,

$$\eta=\sqrt{\frac{c}{\upsilon}}y,\ u=cxf'(\eta),\ v=-\sqrt{c\upsilon}f(\eta),\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{8}$$

$$M=\frac{\sigma B_0^2}{c\rho},\ \Gamma=\frac{x^2c^3}{2\upsilon C^{*2}},\ K=\frac{1}{\mu\beta C^{*}},\ V_w=\frac{v_0}{\sqrt{\upsilon c}},\ \mathrm{Re}=\frac{xU_w(x,t)}{\upsilon}$$

Equations (3)-(6) become

$$(1+K)f'''+ff''-f'^2-K\Gamma f''f'''-Mf'=0 \tag{9}$$

$$\theta''-\Pr(f'\theta-f\theta')+A^*f'+B^*\theta=0 \tag{10}$$

The associated boundary conditions

$$f'=1+\delta f'';f=S;\theta=1\ at\ \eta=0 \tag{11}$$
$$f'\to 0,\theta\to 0\ \ as\ \eta\to\infty$$

Equation (8) is subject to the constraint ΓK<<1 according to Javed et al. [3].

The quantities of practical interest in this study are the skin friction or the shear stress coefficient C_f and the local Nusselt number Nu, which are defined in dimensionless form as

$$\sqrt{\mathrm{Re}}\,C_f=-\left[(1+K)f''(0)-\frac{K\Gamma}{3}f''^3(0)\right],\quad \frac{Nu_L}{\sqrt{\mathrm{Re}}}=-\theta'(0) \tag{12}$$

RESULTS AND DISCUSSIONS

The governing boundary layer equations (9) and (10) subject to the boundary conditions (11) is solved numerically by employing bvp4c method. In order to get a physical insight into the problem, a representative set of numerical results are computed and depicted graphically in Figs.1-6. In order to test the accuracy of our results, we compared it with the existed literature (Mahapatra and Gupta [15] and Raju et al.[16]) , it is observed that the present results show an excellent correlation agreement with previous study(Table 1).

Figs. 1 -2 describe the effect of fluid parameter K on the velocity and temperature profiles. From Fig. 4, it can be seen that the velocity field and boundary layer thickness are increasing functions of K, whereas the temperature boundary thickness reduce thereby decreasing the fluid temperature(Fig.2).Figs. 3 and 4 illustrate the variation of the stream wise velocity and temperature for different values of velocity slip parameterλ. As slip parameter increases, the velocity of the flow at the surface decreases because under the slip condition, the pulling of the stretching sheet can be only partly transmitted to the fluid and converges swiftly (Fig.3). Physically, when slip occurs, the flow velocity near the sheet is no longer equal to the stretching velocity of the sheet and the differences between the wall and the fluid velocities near to the wall rises. As a result, the hydrodynamic boundary layer thickness and hence the skin friction decreases. $f'(0) \approx 1$, when there is no slip. Figs. 5-6 depict the effect of non-uniform heat generation/absorption parameters (A^*) and (B^*) on the temperature profiles of the flow. It is clear from the figures that an increase in the heat source/sink parameters enhances the temperature profiles of the flow. It is obvious to conclude that for lesser values of $A^*$and B^* there is all in temperature profiles, which reveals that negative values of $A^*$and B^* acts like heat absorption parameters and positive values as heat generators.

In the table 2, it is seen that the fluid parameter increases, the shear stress descends whereas the heat transfer rate increases. Shear stress decreases as an increase in the magnetic parameter. An enhancement of the heat source/sink parameter depreciates the heat transfer rate. Ascending slip parameter descends the skin friction coefficient, but increases the heat transfer rate.

Conclusions: The stagnation point flow of a MHD Powell-Eyring fluid over a nonlinearly stretching sheet in the presence of heat source/sink has been analysed. The following important observations can be derived from the numerical results:

The slip parameter influence depreciates the velocity.

As the fluid parameter KΓ(<<1) increases, the velocity profile increases whereas the temperature profile decreases. The skin friction coefficient also descends.

The thermal boundary layer thickness increases for heat generation A* and B*(< 0). The rate of heat transfer reduces as the temperature dependent heat generation enhances.

References:

1. M. Norouzi, M. Davoodi, O. Anwar, A.A. Joneidi (1944), Analysis of the effect of normal stress differences on heat transfer in creeping viscoelastic Dean ow, Int. J. Thermal Sciences 69 (2013) 61-69, http://dx.doi.org/10.1016/j.ijthermalsci.2013.02.002/.
2. Powell R. E., Eyring H, Nature, London, 427.
3. Javed T., et al. (2013), Flow of an Eyring-Powell Non-Newtonian Fluid over a Stretching Sheet, ChemEngComm200 pp. 327-336.
4. Hayat T., et al. (2009), Flow of an Eyring-Powell Fluid with Convective Boundary Conditions. J Mech29 pp. 217-224.
5. Ch.Vittal, M. Chenna Krishna Reddy and M. Monica (2016), Stagnation point flow of a MHD Powell-Eyring fluid over a nonlinearly stretching sheet in the presence of heat source/sink, Journal of Progressive Research in Mathematics(JPRM), Vol.8(2).
6. Kankeyanathan Kannan, Discrete Group With the Twisted Rapid Decay; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Issue 2 (2014), Pg 516-518
7. B. Mahanthesh, B.J. Gireesha and Rama Subba Reddy Gorla (2016),Unsteady three-dimensional MHD flow of a nanoEyring-Powell fluid past a convectively heated stretching sheet in the presence of thermal radiation, viscous dissipation and Joule heating, Journal of the Association of Arab Universities for Basic and Applied Sciences.
8. Mohamed Abd El-Aziz and Ahmed M. Salem (2007), MHD-mixed convection and mass transfer from a vertical stretching sheet with diffusion of chemically reactive species and space- or-temperature-dependent heat source ,Can. J. Phys., Vol.85,pp. 359–373 doi: 10.1139/P07-048.
9. R. Tsai', K.H. Huang, J.S. Huang (2008),Flow and heat transfer over an unsteady stretching surface with non-uniform heat source,International Communications in Heat and Mass Transfer,Vol.35(10), pp.1340–1343.
10. K.S. Dhounsi, Yasmeen, A Study On Generalized Hermite Polynomials; Mathematical Sciences International Research Journal : ISSN 2278-8697Volume 4 Issue 2 (2015), Pg 371-372
11. M. M. Khader, A. M. Megahed (2013),Numerical studies for flow and heat transfer of the Powell-Eyring fluid thin film over an unsteady stretching sheet with internal heat generation using the chebyshev finite difference method,Journal of Applied Mechanics and Technical Physics,Vol.54(3), pp. 440–450.
12. M. Subhas Abel, Mahantesh M. Nandeppanavar (2009),Heat transfer in MHD viscoelastic boundary layer flow over a stretching sheet with non-uniform heat source/sinkCommunications in Nonlinear Science and Numerical Simulation,Vol.14(5),pp. 2120–2131.
13. Preety, Stability Result for S-Iteration in Convex Metric; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 3 Issue 1 (2014), Pg 355-357
14. N. Bhaskar Reddy, T. Poornima and P. Sreenivasulu (2014),Influence of Variable thermal conductivity on MHD boundary layer Slip flow of Ethylene-Glycol based Cu nanofluids past a stretching sheet with convective boundary condition, International Journal of Engineering Mathematics,.
15. *T.N. Kavatha, A. Jayalakshmi,* Some Special Types of Near Idempotent Semi group; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Spl Issue (2014), Pg 956-960http://dx.doi.org/10.1155/2014/905158.
16. P. Sreenivasulu , T. Poornima and N. Bhaskar Reddy (2015), Influence of Magnetic Field on a Nanofluid Past a non Linear Stretching Sheet with Uniform Heat Source, AttidellaFondazione Giorgio Ronchi, Estratto da: AttidellaFondazione Giorgio Ronchi,Anno LXX, no. 2.
17. Andersson H (2002), Slip flow past a stretching surface. ActaMech, Vol.15(8), pp.121-125.
18. Wang C Y (2002), Flow due to a stretching boundary with partial slip-an exact solution of the Navier–Stokes equation. Chemical Engineering Sciences, Vol.57, pp.3745–3747.
19. T.R. Mahapatra, A.S. Gupta(2004), Stagnation-point flow of a viscoelastic fluid towards a stretching surface, Int. J. Non-Linear Mech.,Vol. 39,pp. 811–820.
20. N.Vithya, Dr. P. Thangavelu, on Generalised B#-Closed Sets; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Issue 2 (2014), Pg 561-563
21. C.S.K. Raju, N. Sandeep, S. Saleem (2016), Effects of induced magnetic field and homogeneous–heterogeneous reactions on stagnation flow of a Casson fluid,Engineering Science and Technology, an International Journal,Vol.19(2), pp. 875-887.
22. Akshaykumar Meshram, N.W.Khobragade, on the Security Analysis of Mor Public Key Cryptosystem for Camina Group; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 4 Issue 1 (2015), Pg 101-103

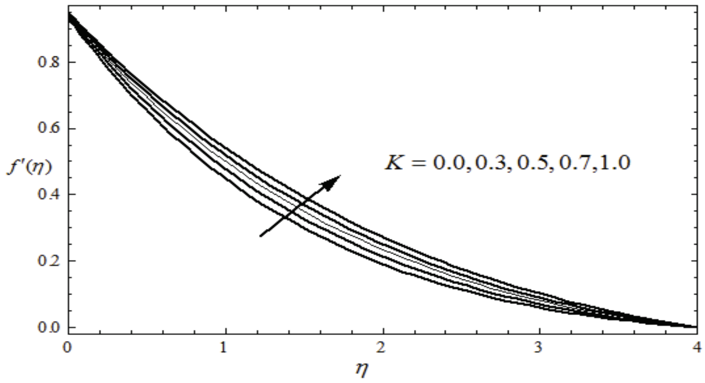

Fig.1 Velocity Profiles for different values of K

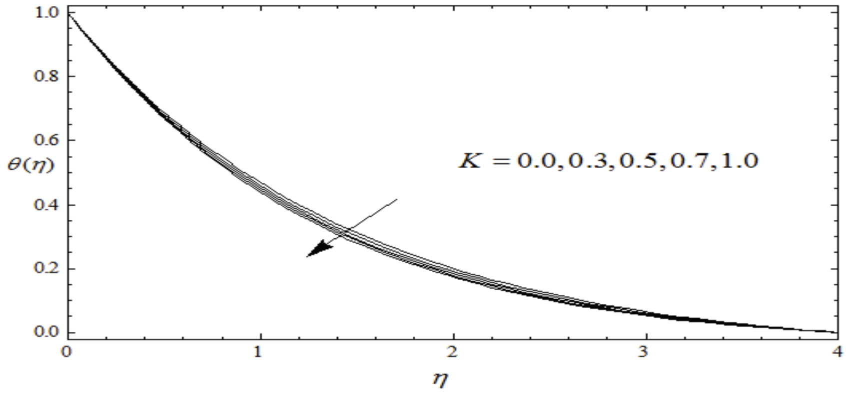

Fig.2 Temperature Profiles for different values of K

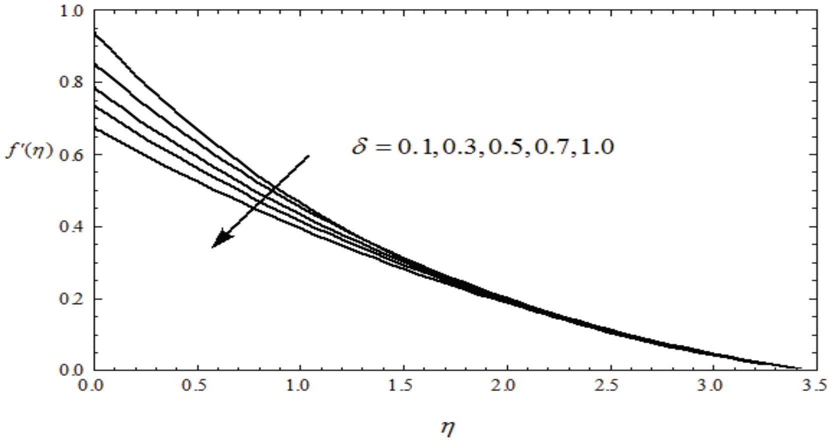

Fig.3 Velocity Profiles for different values of δ

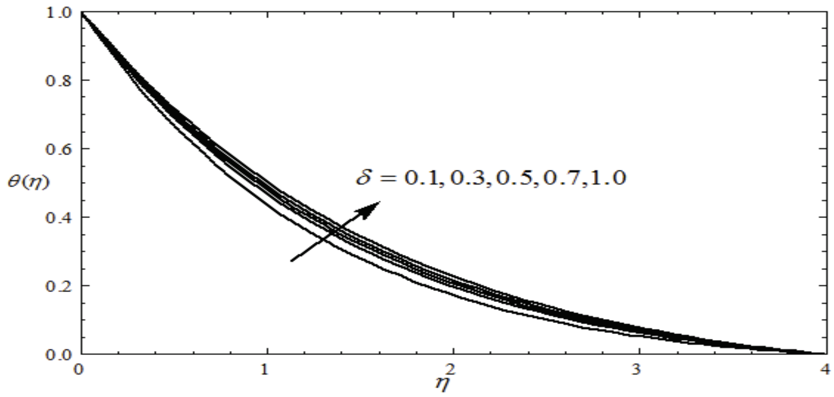

Fig.4 Temperature Profiles for different values of δ

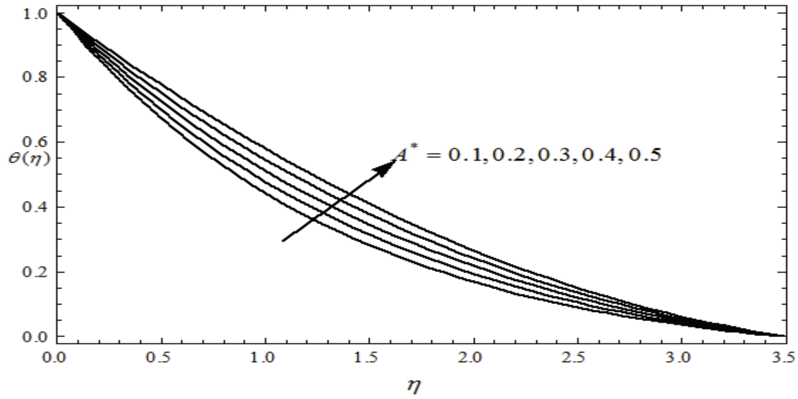

Fig.5 Temperature profiles for different values of A^*

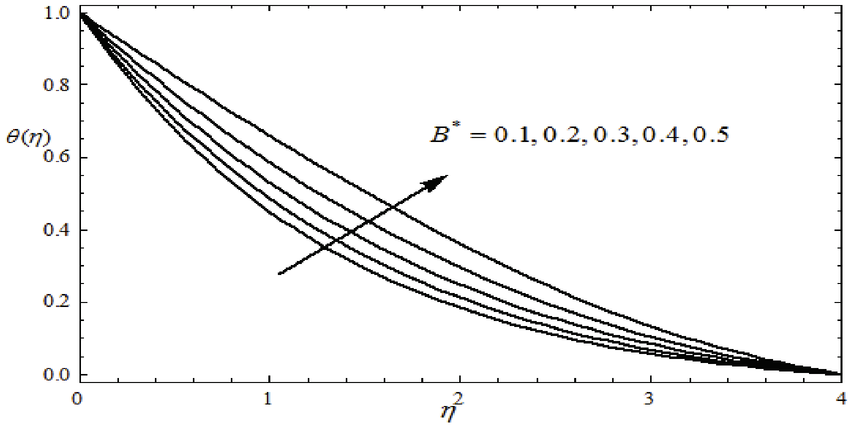

Fig.6 Temperature Profiles for different values of B^*

Table 1: Comparison of the rate of heat transfer when K=Γ= A*= B* = δ=0.

Pr	Mahapatra and Gupta [15]	Raju et al. [16]	Present Results
0.05	-0.081	-0.0811	-0.08162
0.5	-0.383	-0.3833	-0.38342
1	-0.603	-0.6031	-0.60310
1.5	-0.777	-0.7772	-0.77682

Table 2 Computation of Wall Shear stress and Nusselt number for various pertinent parameters.

K	Γ	A^*	B^*	δ	$-f''(0)$	$-\theta(0)$
0.1					0.841446	0.701558
0.5					0.731961	0.730463
1.0	0.1				0.646803	0.753311
	0.5				0.83437	0.702948
	1.0				0.825942	0.70463
		0.1			0.841446	2.70143
		0.3			0.841446	0.575588
		0.5	0.1		0.841446	0.449617
			0.3		0.841446	0.519352
			0.5	0.1	0.841446	0.266889
				0.5	0.57684	0.620299
				1.0	0.422456	0.561383

T. Poornima, Department of SAS,VIT University, Chennai, T.N., India.
P. Sreenivasulu, Department of Humanities and Sciences, SVCET, Chittoor, A.P., India
N. Bhaskar Reddy, Department of Mathematics, Sri Venkateswara University, Tirupati, A.P., India