ON THE BOUNDARY LAYER FLOW OF CASSON DISSIPATING CONVECTIVE FLUID FLOW PAST A NON LINEAR STRETCHING SHEET WITH NON UNIFORM HEAT GENERATION/ABSORPTION

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Abstract: An analysis was made to study the effects of non-uniform heat source or sink and viscous dissipation on boundary layer flow of Casson fluid past a non-linear stretching sheet with convective boundary condition. The boundary layer equations governed by the incompressible fluid flow is converted to dimensionless by employing similarity transformations. The ultimate equations are solved using Runge-Kutta fourth order method along with shooting technique. The effects of various governing parameters on velocity, temperature, surface skin-friction and rate of heat transfer are elaborately discussed with the help of graphs and tables.

Keywords:Casson fluid, Non-Uniform heat Source or Sink, Viscous Dissipation, Non linear Stretching Sheet.

Introduction: The study of non-Newtonian fluids has keen interest among the researchers because of its variety of applications in engineering, chemical and petroleum industries such as molten plastics, polymeric liquids, blood, food stuff, and slurries. In the class of non-Newtonian fluids, Casson fluid has unique characteristics, which have wide application in food processing, in metallurgy, drilling operation and bio-engineering operations, etc.We can define a casson fluid as a shear thinning fluid which is assumed to have an infinite viscosity at zero rate of shear. Casson's constitutive equations are found to describe accurately the flow curves of suspensions of pigments in lithographic varnishes used for preparation of printing inks and silicon suspensions [1]. Various experiments performed on blood with varying haematocrits, anticoagulants, temperatures, and so forth strongly suggest the behavior of blood as a casson fluid [2-4]. In particular, casson fluid model describes the flow characteristics of blood more accurately at low shear rates and when it flows through small vessels [5].

In recent decades the scientists and engineers are interested to study the flow caused by steady or unsteady stretching sheets because of its rigorous applications in various engineering fields such as Polymer sheet extruded continuously from a dye, cooling of metallic plate in a cooling bath and heat-treated materials that travel between feed and wind-up rolls or on a conveyor belt, etc. The exact solution for the flow due to stretching of flat surface was first obtained by Crane [6]. The solution of the three-dimensional fluid motion caused by the stretching boundary was studied by Wang C.Y. [7]. Nadeem *et. al.* [8], studied the MHD flow of a Casson fluid over an exponentially shrinking sheet.

All the above studies are considered only for a linear stretching sheet, but it is true that not in all cases the stretching sheet is linear. It was first identified by Gupta and Gupta [9]. Vajravelu [10] studied the viscous flow over a nonlinearly stretching sheet. Mukhopadhyay [11] investigated the study of heat transfer in a viscous fluid over a non-linearly stretching sheet with Casson fluid flow and heat transfer over a nonlinearly stretching surface.

The study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions and those concerned with dissociating fluids. Possible heat generation effects may alter the temperature distribution consequently, the particle deposition rate in nuclear reactors, electronic chips and semiconductor wafers. Khedr*et al.* [12] studied the effects of suction/ injection and heat generation on MHD flow of a micropolar fluid past a stretched permeable surface. Sulochana*et. al.* [13] studied the Similarity solution of 3D Casson nanofluid flow over a stretching sheet with convective boundary conditions.

In this paper an attempt is made to study the effects of non-uniform heat source /sink and viscous dissipation on boundary layer flow of Casson fluid past a non-linearly stretching sheet.

Mathematical Modelling: Consider a steady tow dimensional laminar boundary layer incompressible flow of casson fluid past a non-linearly stretching sheet coinciding with the plane y = 0. The flow is confirmed to *y*> 0 region. Keeping the origin fixed, two equal and opposite forces along the x - axis is applied so that the sheet is then stretched with velocity $u_w(x) = a\ x^n$, where a is constant. We also assume the rheological equation of Casson fluid reported by Mukopadyay [11].

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + p_y / \sqrt{2\pi}\right)e_{ij}, & \pi > \pi_c \\ 2\left(\mu_B + p_y / \sqrt{2\pi_c}\right)e_{ij}, & \pi < \pi_c \end{cases}$$

Where π is the product of the component of deformation rate with itself, $\pi = e_{ij}e_{ij}$, e_{ij} is the (i, j) component of the deformation rate, $\pi_c$is a critical value of this product based on the non-Newtonian model, μ_B is the

plastic dynamic viscosity of the non-Newtonian fluid and P_y is the yield stress of the fluid.
Under the usual assumptions, the governing boundary layer equations are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu_{nf}\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho Cp}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho Cp}\left(\frac{\partial u}{\partial y}\right)^2+q''' \quad (3)$$

The boundary conditions for the velocity and temperature fields are

$$u=u_w=ax^n,\quad v=0,\quad T=T_w \quad at \quad y=0$$
$$u\to 0,\quad T\to T_\infty \quad as \quad y\to\infty \quad (4)$$

where *u, v* are the velocity components in the *x* and *y* directions, respectively, *T* - the temperature of the nanofluid, T_∞- the ambient fluid temperature, $\beta=\mu_B\sqrt{2\pi_c}\,p_y$ is the parameter of the Casson fluid, *k* is the thermal conductivity of the fluid, and T_w is the convective fluid temperature at the surface of the sheet. *Cp* is the specific heat at constant pressure, ρ is the density of the fluid.
The non-uniform heat generation or absorption q''' has been taken as

$$q'''=\frac{Ku_w}{x\nu}\left(Q_0\left(T_w-T_\infty\right)f'(\eta)+Q_1\left(T-T_\infty\right)\right) \quad (5)$$

Where Q_0andQ_1 are the coefficients of space and temperature dependent heat generation or absorption respectively. The case Q_0> 0 corresponds to internal heat generation while Q_0< 0 and Q_1< 0 corresponds to internal heat absorption.
To get similarity solutions of equations (1) - (3) subject to the boundary conditions (4), we introduce the following similarity transformations.

$$\eta=y\sqrt{\frac{a(n+1)}{2\nu}}x^{\frac{n-1}{2}},\quad u=ax^n f'(\eta),$$
$$v=-\sqrt{a\nu\frac{(n+1)}{2}}x^{\frac{n-1}{2}}\left\{f(\eta)+\frac{n-1}{n+1}\eta f'(\eta)\right\}, \quad (6)$$
$$\theta=\frac{T-T_\infty}{T_w-T_\infty},\quad \Pr=\frac{\nu}{\alpha},\quad Ec=\frac{u_w}{Cp(T_w-T_\infty)},$$
$$A=\frac{2Q_0}{n+1},\quad B=\frac{2Q_1}{n+1},\quad \gamma=-\frac{h}{k}\sqrt{\frac{2}{n+1}}\,\mathrm{Re}_x^{-1/2}$$

where ν is the kinematic viscosity of the base fluid.
Equations (2),(3) and (5) take the following dimensionless form.

$$\left(1+\frac{1}{\beta}\right)f'''+ff''-\frac{2n}{n+1}f'^2=0 \quad (6)$$

$$\theta''+\Pr\left[f\theta'+Ec\left(1+\frac{1}{\beta}\right)f''^2\right]+Af'+B\theta=0 \quad (7)$$

The corresponding boundary conditions are

$$f'(0)=1,\quad f(0)=0,\quad \theta(0)=1$$
$$f'(\infty)\to 0,\quad \theta(\infty)\to 0 \quad (8)$$

where prime denotes the differentiation with respect to *η*.βis the Casson fluid parameter, *n* is the non-linear stretching parameter, *Pr* is the Prandtl number and *Ec* is the Eckert number,
The most important physical quantities of practical interest in this study are the skin friction or the shear stress coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f=\frac{\mu}{\rho u_w^2}\left(1+\frac{1}{\beta}\right)\left(\frac{\partial u}{\partial y}\right)_{y=0},\quad Nu_x=\frac{xk}{k(T_w-T_\infty)}\left(-\frac{\partial T}{\partial y}\right)_{y=0} \quad (9)$$

Using equation (6), the skin friction coefficient and local Nusselt number can be expressed as

$$\sqrt{\mathrm{Re}_x}C_f=\sqrt{\frac{n+1}{2}}\left(1+\frac{1}{\beta}\right)f''(0),\quad \frac{Nu_x}{\sqrt{\mathrm{Re}_x}}=-\sqrt{\frac{n+1}{2}}\theta'(0) \quad (10)$$

where $\mathrm{Re}_x=\frac{u_w x}{\nu}$ is the Reynolds number.

Result and Discussion: The governing boundary layer equations (6) and (7) subject to the boundary conditions (8) is solved numerically by employing Runge-Kutta fourth order technique along with shooting method. In order to get a physical insight into the problem, a representative set of numerical results are depicted graphically in Figs.1-8. The present results are compared with that of Mukhopadyay [11] and found an excellent agreement for the reduced cases as displayed in Table 2.
The influence of non-linear stretching parameter *n* on velocity and temperature are depicted in Figs. (1) and (2), respectively. It is found that the fluid velocity is decreases with an increase in *n* while the temperature of the fluid increases with an increase in *n*. The effect of non-linear stretching parameter is significant only when *n* islow. But the decrease of the velocity profile is negligible for large *n* because the coefficient 2n/(n+1) approaches to 2 as $n\to\infty$. The effect of Casson parameter β on velocity and temperature are shown in Figs. (3) and (4) for linear and non-linear stretching sheets, respectively. It is clear that the fluid velocity is decreases with an increase in Casson parameter β for both the cases whereas the temperature of the fluid increases with increasing β. The fluid velocity in the case of non-linearly stretching sheet (*n* = 10) in much more conquered than that of linearly stretching sheet (*n* = 1) (Fig.(3)). That is at a fixed point of the sheet, the velocity in the linearly stretching sheet is higher than that in the non-linearly stretching sheet. On the other hand, the increase of the temperature field in the case of non-linearly stretching sheet (*n* = 10) in much more articulated than that of linearly

stretching sheet (n = 1) (Fig.(4)). That is at a fixed point of the sheet, the temperature in the non-linearly stretching sheet is higher than that in the linearly stretching sheet.

The influence of the Prandtl number *Pr* on temperature for linearly and non-linearly stretching sheets is presented in Fig. (5). It is evident that the temperature of the fluid decreases with an increase in *Pr* for both the cases of linearly and non-linearly stretching sheets. Also it is observed that the temperature in the non-linearly stretching sheet is higher than that in the linearly stretching sheet. This is due to fact that, the prandtl number defines that the ratio of momentum diffusivity to thermal diffusivity.

The effect of Eckert number *Ec* on temperature for linearly and non-linearly stretching sheet is shown in Fig.(6). From this it is clear that the temperature of the fluid increases with an increase in Eckert number *Ec*. The Eckert number *Ec* defines the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conservation of kinetic energy into internal energy by work done against the viscous fluid stress. The positive Eckert number implies cooling of the sheet i.e., loss of heat from the sheet to the fluid. Also it is observe that the temperature of the fluid is higher in case of non-linearly stretching sheet (n = 10) than that of linearly stretching sheet (n = 1).

The influence of heat source or sink parameters A and B on the temperature are shown in Figs. (7) and (8). From this it is observed that the temperature of the increases with an increase in both the parameters A and B. Also it is observe that the temperature of the fluid is higher in case of non-linearly stretching sheet (n = 10) than that of linearly stretching sheet (n = 1). It is fact that the larger heat source corresponding to A> 0 and B > 0 rises the fluid temperature, while the non-uniform heat sink corresponds to A < 0 and B < 0 can contribute in quenching the heat from stretching sheet effectively.

Conclusions: The fluid velocity decreases as the Casson parameter increases. The temperature enhances for an increase in *Ec*or heat generation/absorption enhances. The temperature pronounces more when the sheet stretches highly non-linear. As Pr increases, the temperature decreases.

References:

1. R. Subba Rao, A. Naga Durgamamba, R.R.L. Kantam, Group Acceptance Sampling Plan – Size Biased Lomax Model; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Issue 2 (2014), Pg 552-557
2. W. P. Walawender, T. Y. Chen, and D. F. Cala, "An approximate casson fluid model for tube flow of blood," *Biorheology*, vol. 12, no. 2, pp. 111–124, 1975.
3. G. W. S. Blair, "An equation for the flow of blood, plasma and serum through glass capillaries," *Nature*, vol. 183, no. 4661, pp. 613–614, 1959.
4. Huldah Samuel, V. Rajkumar Dare, Generalized Parikh Vectors and Partial Line Languages; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Issue 2 (2014), Pg 558-560
5. S. Charm and G. Kurland, "Viscometry of human blood for shear rates of 0–100,000 *sec*−1," *Nature*, vol. 206, no. 4984, pp. 617–618, 1965.
6. E.W.Merrill,A.M. Benis, E. R.Gilliland, T.K. Sherwood, and E. W. Salzman, "Pressure-flow relations of human blood in hollow fibers at low flow rates," Journal of Applied Physiology, vol. 20, no. 5, pp. 954–967, 1965.
7. D. A. Mc Donald, Blood flows in arteries, chapter 2, Arnold, London, UK, 2nd edition, 1974.
8. Shashi Bhushan, Abhaypratappandey, Class of Estimators for Estimating Ratio of Two Population Means Using Two Auxiliary Variables Under Measurement Errors; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 4 Issue 1 (2015), Pg 126-131
9. Crane, L.J. (1970) Flow past a Stretching Plate. Zeitschriftfür Angewandte Mathematik und Physik, 21, 645-647.
10. Wang, C.Y. (1984) The Three-Dimensional Flow Due to a Stretching Flat Surface. Physics of Fluids, **27**, 1915-1917.
11. Sheela Suthar , Om Prakash, A Note on Zero Divisor Graph of Polynomial Ring Over Z; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 4 Spl Issue (2015), Pg 14-19
12. S. Nadeem, RizwanUlHaq, and C. Lee, MHD flow of a casson fluid over an exponentially shrinking sheet, Scientia Iranica B (2012) 19 (6), 1550-1553.
13. Gupta, P.S. and Gupta, A.S. (1977),Heat and mass transfer on a stretching sheet with suction or blowing.The Canadian Journal of Chemical Engineering, **55**, 744-746.
14. Srinivasa Reddy B, Tulasi Lakshmi Devi B, Alfunsa Prathiba, Convective Heat Transfer Of A Micro Polar Fluid Through A Porous Medium In A Cylindrical Annulus With Viscous Dissipation; Mathematical Sciences International Research Journal : ISSN 2278-8697Volume 4 Issue 2 (2015), Pg 378-383
15. Vajravelu, K. (2001) Viscous flow over a nonlinearly stretching sheet. Applied Mathematics and Computation, 124, 281-288.

16. Swati Mukhopadhyay, The study of heat transfer in a viscous fluid over a non-linearly stretching sheet with Casson fluid flow and heat transfer over a nonlinearly stretching surface.Chin. Phys. B Vol. 22, No. 7 (2013) 074701.
17. A.Praveenprakash,N.Murugammal,C.John Paul Raja, A Study on the Impact of Media on Adolescents; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 3 Issue 1 (2014), Pg 351-354
18. M.E.M. Khedr, A.J. Chamkha, M. Bayomi, MHD flow of a micropolar fluid past a stretched permeable surface with heat generation or absorption. Nonlinear Analysis Modelling and Control, vol.14(2009), No. 1, 27-40.
19. *Biswapati Jana,* Survey on Cheating and Prevention Techniques in Visual Cryptography; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Spl Issue (2014), Pg 961-976
20. C. Sulochana, G.P. Ashwinkumar and N. Sandeep, Similarity solution of 3D Casson nanofluid flow over a stretching sheet with convective boundary conditions, Journal of the Nigerian Mathematical Society 35 (2016) 128–141.

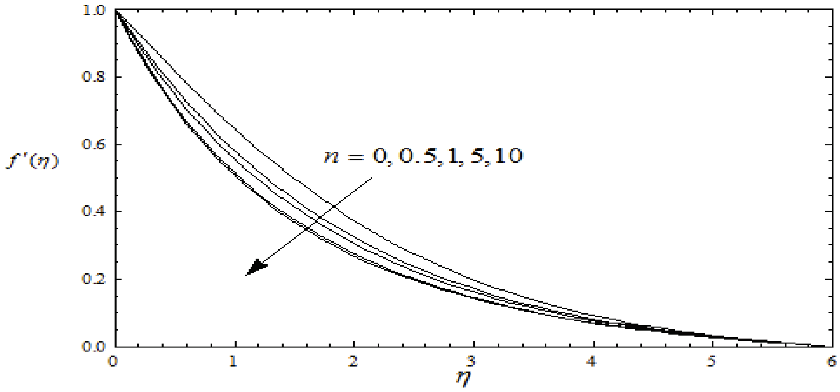

Fig.1 Velocity profiles for different values of n

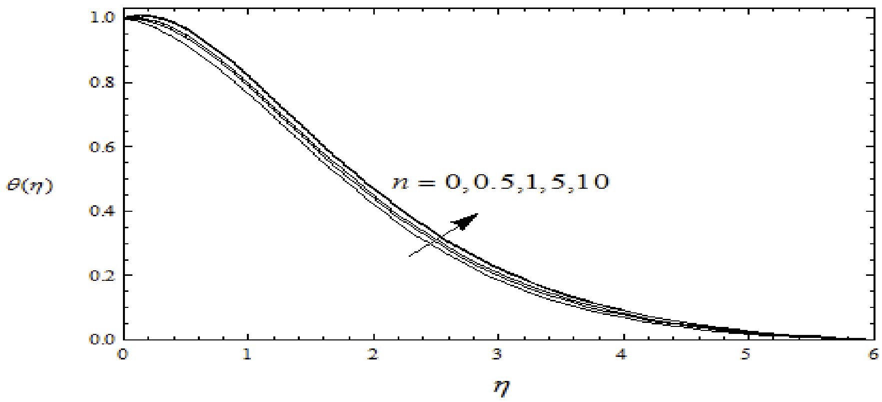

Fig.2 Temperature profiles for different values of n

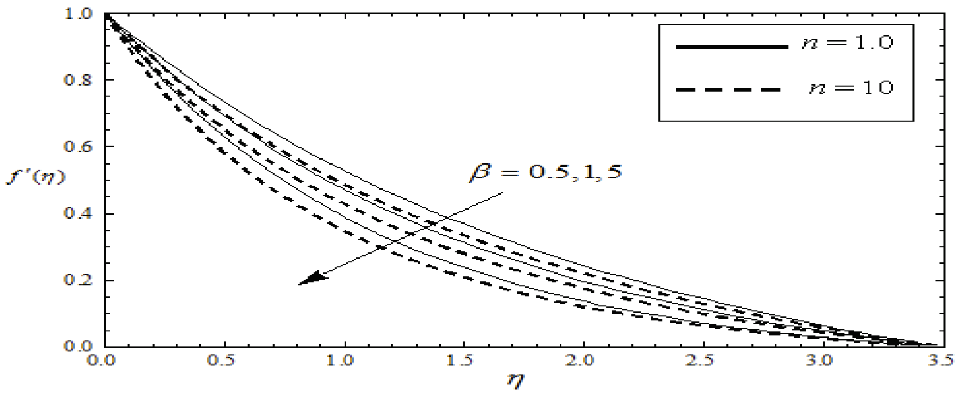

Fig.3 Velocity profiles for different values of β

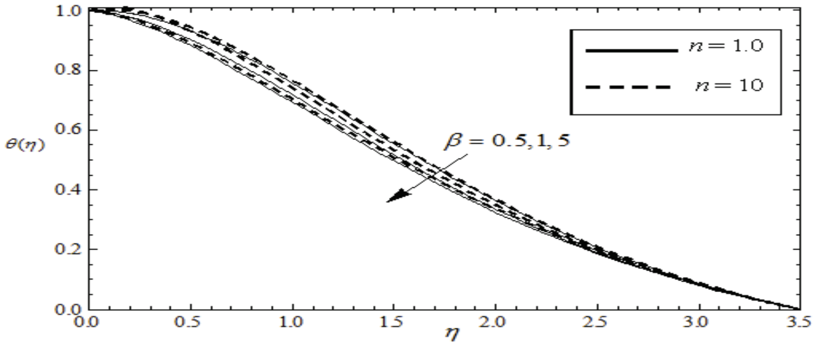

Fig.4 Temperature profiles for different values of β

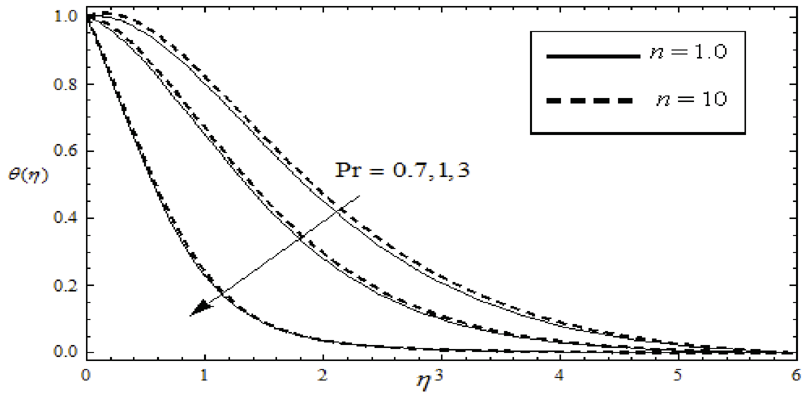

Fig.5 Temperature profiles for different values of Pr

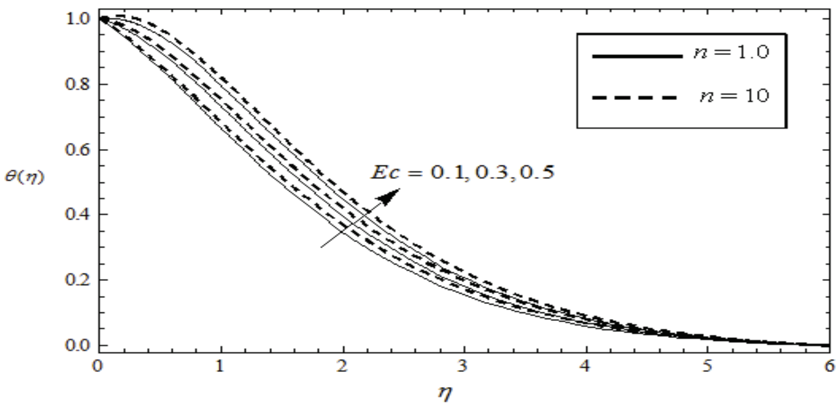

Fig.6 Temperature profiles for different values of Ec

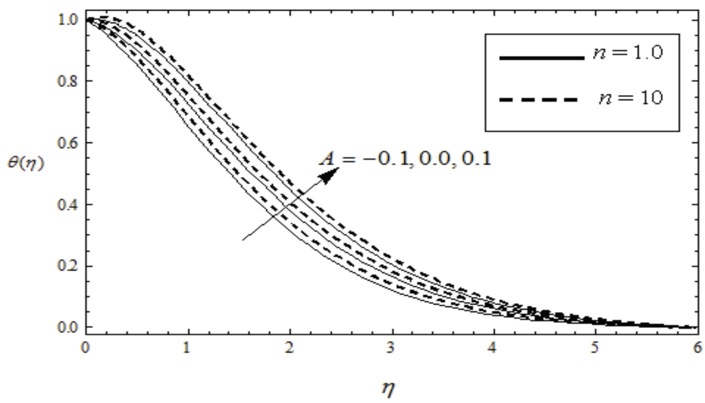

Fig.7 Temperature profiles for different values of A

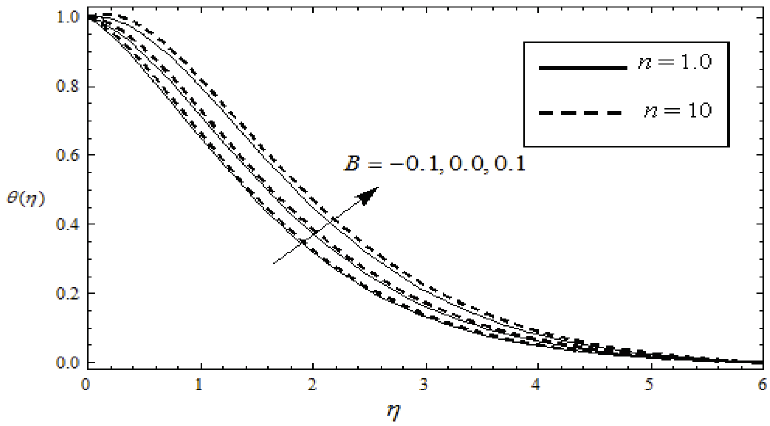

Fig.8 Temperature profiles for different values of A

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