SUPRA NEIGHBOURHOOD SYSTEM VIA LOCALLY CLOSED SETS IN GENERALIZED TOPOLOGY

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Abstract: We introduce a new type of closed sets called S*-Locally Closed sets and S_I*-Locally Closed sets. Finally, we establish the properties of them. We introduce the new class of closed sets are called S*-Locally Closed sets and S_{I^*}-Locally Closed sets and discussed the properties of such type of closed sets. Also we established the relations between S*-Locally Closed sets and S_I*-Locally Closed sets with other existing closed sets.

Key words: GNS, SNS, S*-open set, SI*-open set, S*-locally closed set, SI*-locally closed sets.

Introduction: The Supra Neighborhood Systems and Supra Neighborhood Spaces are introduced by Kim.Y.K and Min.W.K [3]. Also they investigated the properties of I*, Cl* and S_I*-continuity. The notion of Generalized Neighborhood System (briefly GNS) was introduced by Csaszar [1] and also introduced the notion of (ψ, ψ′)-continuity on generalized neighborhood systems ψ, ψ′. He [1] initiated a new notion called Generalized Topology and discussed its continuity. Supra Topological space initiated by A.S.Masshour et.al [6] and also they characterized its properties. Here, we introduce a new type of closed sets called S*-locally closed sets and S_I*-locally closed sets. Also we establish the properties of such closed sets and its relation between other existing closed sets.

Preliminaries:

Definition 2.1: [1] Let X be a non-empty set, exp(X) the power set of X and Ψ : X → exp(exp(X)) satisfy x ∈ V for V ∈ Ψ(x). Then V∈ Ψ(x) is called a generalized neighborhood of x ∈ X and Ψ is called a generalized neighborhood system (briefly GNS) on X.

Definition 2.2: [1] Let g be a collection of subsets of X. Then g is called a generalized topology on X iff $\emptyset \in g$ and $G_i \in g$ for $i \in I \neq \emptyset$ implies $G = \cup_{i\epsilon I}\ G_i \in g$. The elements of g are called g-open sets and the complements are g-closed sets.

Remark 2.3: [1, 2] For A⊆ X, we denote by i_gA the union of all g-open sets contained in A, i.e. the largest g-open set contained in A. Also we denote by c_gA the intersection of all g-closed sets containing A, i.e. the smallest g-closed set containing A. Any intersection of g-closed sets is g-closed.

Definition 2.4: [3] Let X be a non-empty set, exp(X) the power set of X and Ψ: X → exp(exp(X)). Then Ψ is called a supra neighborhood system on X if it satisfies the following:

1. For x $\in X$, Ψ(x) ≠ ∅
2. For V ∈ Ψ(x), x ∈ V

Then the pair (X,Ψ) is called supra neighborhood space (briefly SNS) on X. Then $V \in \Psi(x)$ is called supra neighborhood of $x \in X$.

Remark 2.5: [3] Let (X,Ψ) be an SNS on X and G ⊆ X. Then G is called an S*-open set if for each x∈ G, there is V ∈ Ψ(x) such that V ⊆ G. Let us denote S*O(X) the collection of all S*-open sets on an SNS (X,Ψ). The complements of S*-open sets are called S*-closed sets.

Definition 2.6: [6] Let X be a non-empty set. The sub family $\mu \subseteq$ P(X) where P(X) is the power set of X is said to be a Supra Topology on X if $X \in \mu$ and μ is closed under arbitrary unions. The pair (X,μ) is called supra topological space. The elements of μ are said to be supra open in (X,μ). The complements of supra open sets are called supra closed sets.

Remark 2.7: [3] The collection S*O(X) of all S*-open subsets of X is a supra topology on X.

Theorem 2.8: [3] Let(X,Ψ) be a SNS on X and A ⊆ X. Then

1. S*int(A)=∪ $\{G$: G ⊆ A and G ∈ $S^*O(X)\}$
2. S*cl(A)=∩ $\{F$: A ⊆ F and X − Fϵ $S^*C(X)\}$

Definition 2.9: [5] Let (X,Ψ) be a SNS on X and A ⊆ X. I*(A) ={x ∈ A: x ∈ Ψ(x)}.

Definition 2.10: [4] Let (X,Ψ) be a SNS on X and A ⊆ X. Cl*(A) = {x ∈ X: $X - A \notin \Psi$(x)}.

Theorem 2.11: [4] (X,Ψ) be a SNS on X and A ⊆ X. Then the following hold:

1. I*(A)= X − Cl* (X - A)
2. Cl*(A)=X−$I^*(X - A)$

Definition 2.12: [4] Let (X,Ψ) be an SNS on X and Ψ_{I^*}(X) = {∪ σ: $\sigma \subseteq$ B}, where B = {A ⊆ X : I*(A) = A}. For $G \subseteq X$, G is called an S_{I^*}-open set if $G \in \Psi_{I^*}$(X). The complements of S_{I^*}-open sets are called S_{I^*}-closed sets. We denote $S_{I^*}O(X)$ and $S_{I^*}C(X)$ respectively be the collection of all S_{I^*}-open and S_{I^*}-closed sets in X.

Definition 2.13: [5] Let (X,Ψ) be a SNS on X and A ⊆ X. The S_{I^*}-interior of A, denoted by S_{I^*}int(A), is the union of all G ⊆ A, G ∈ Ψ_{I^*}(X), and the S_{I^*}-closure of A, denoted by S_{I^*}cl(A), is the intersection of all S_{I^*}-closed sets containing A.

Theorem 2.14: [5] Let (X,Ψ) be an SNS on X and A ⊆ X. Then S_{I^*}int(A) ⊆ S*int(A) ⊆ A ⊆ S*cl(A) ⊆ S_{I^*}cl(A).

Remark 2.15: [5] Let (X, Ψ) be a SNS on X. Then clearly, $\Psi_{I^*}(X)$ is a generalized topology on X, but it need not be a supra topology.

S*-Locally Closed sets in Generalized Topology:

Definition 3.1: Let (X,Ψ) be a SNS on X and $A \subseteq X$ is said to be S*-Locally Closed set if $A = U \cap F$, where U is S*-open and F is S*-closed in X.

Remark 3.2: We denote S*LC(X) is the collection of all S*-Locally Closed Sets in X.

Example 3.3: Let X = {a, b, c}, define $\Psi: (X) \to$ exp(exp(X)) by Ψ(a) = {{a}, {a, c}}, Ψ(b)= {{b}, {b, c}}, Ψ(c)={X}.
Then S*O(X)={ ∅, {a}, {b}, {a,b}, X}}, S*C(X)={∅, {c},{a,c}, {b, c}, X}}, S*LC(X)={∅, {a}, {b} , {c}, {a,b}, {a,c}, {b, c}, X}.

Theorem 3.4: In a SNS (X, Ψ), every S*-open set is S*-Locally Closed set but converse need not be true.

Proof: Proof is trivially from the Definitions.

Example 3.5: Let X={a,b,c}, $\Psi: (X) \to$ exp(exp(X)) by Ψ(a)={{a}},Ψ(b)={{b,c}}, Ψ(c)={X},S*O(X)={ ∅, {a}, X}}, S*C(X)={∅, {b,c},X}},S*LC(X)={∅, {a},{b,c}, X}.
Here {b, c} is S*-Locally Closed Set but not S*-Open.

Theorem 3.6: If A is a S*-Locally Closed Set in a SNS (X, Ψ), if S*-closed set K in X such that $A \cap K = \emptyset$.

Proof: Let A be a S*-Locally Closed subset of X. Then $A = U \cap F$, where U is S*-open and F is S*-closed. Let $K = F \cap (X \backslash U)$. Then K is a S*-closed subset of X such that $A \cap K = \emptyset$.

Theorem 3.7: For a subset A of SNS (X, Ψ), the followings are equivalent

1. A is S*-Locally Closed
2. A = $U \cap S^*$cl (A) for some S^*-open set U.
3. S*cl (A) \A is S*-closed.
4. A ∪ (X\ S*$cl(A)$) is S*-open.
5. $A \subseteq S^*$int($A \cup (X \backslash S^*$cl(A))

Proof: (i) → (ii): Let A be a S*-Locally Closed subset of X. Then A = U ∩ F, where U is S*-open and F is S*-closed. Then A ⊆ F implies that S*cl(A) ⊆ F. So U ∩ S*cl(A) ⊆ U ∩ F = A. Again, A ⊆ U and A ⊆ S*cl(A) implies that A ⊆ U ∩ S*cl(A). Thus A= U∩S*cl(A).

(ii) → (iii): S*cl(A)\A = S*cl(A)\ [U ∩ S*cl(A)] for some S*-open set U. Then S*cl(A) ∩ [(X\$U$ ∩ S*cl(A)] =S*cl(A) ∩ [(X\$U$) ∪ ($X$\S*cl(A)] = S*cl(A) ∩ ($X$\ U), which is S*-closed.

(iii) → (iv): Since S*cl(A)\A is S*-closed, then X\[S*cl(A)\A] is S*-open and X\[S*cl(A)\A] = X\[S*cl(A) ∩ (X\A)] = A∪ [(X\S*cl(A)].

(iv) → (v): A ⊆ A ∪ [X\ S*cl(A)] = S*int[A ∪ (X\ S*cl(A))].

(v) → (i): A ⊆ S*int(A ∪ (X\ S*cl(A))). Thus A = A ⊆ S*int(A ∪ (X\ S*cl(A))) ∩ [S*cl(A)]
where S*int(A ∪ (X\ S*cl(A))) is S*-open and S*cl(A) is S*-closed . Hence A is S*-Locally Closed subset of X.

Theorem 3.8: Let (X, Ψ) be a SNS. If $A \subseteq B \subseteq X$ and B is S*-Locally Closed, then there exist S*-Locally Closed but C such that $A \subseteq C \subseteq B$.

Proof: Since B is S*-Locally Closed by Theorem 3.7, B = U∩S*cl(B) where U is S*-open. Then A ⊆ B ⊆ U. So A ⊆ U∩S*cl(A) = C(say). Then C is S*-Locally Closed and A ⊆ $C \subseteq B$.

S_{I^*}-Locally Closed Sets in Generalized Topology

Definition 4.1: Let (X,Ψ) be a SNS on X and $A \subseteq X$ is said to be S_{I^*}-Locally Closed set if $A = U \cap F$, where U is S_{I^*}-open and F is S_{I^*}-closed in X.

Remark 4.2: We denote S_{I^*}LC(X) is the collection of all S_{I^*}-Locally Closed Sets in X.

Example 4.3: Let X = {a, b, c}, define $\Psi: (X) \to$ exp(exp(X)) by Ψ(a) = {{a}}, Ψ(b)= {{b,c}}, Ψ(c)={{c},X}. Then S_{I^*}O(X)={∅, {a},{c}, {a,c}}, S_{I^*}C(X)={{b}, {a,b}, {b, c}, X}}, S_{I^*}LC(X)={∅ , {a}, {c}, {a,c}}.

Theorem 4.4: In a SNS (X, Ψ), every S_{I^*}-Locally Closed set is S*-Locally Closed but converse need not be true.

Proof: Proof is trivially from the Theorem 3.4 of [5] that S_{I^*}int (A) ⊆ S*int (A) ⊆ A ⊆ S*cl (A) ⊆ S_{I^*}cl (A).

Example 4.5: Let X={a,b,c}, $\Psi: (X) \to$ exp(exp(X)) by Ψ(a)={{a,c},{a,b}}, Ψ(b)={{b,c}}, Ψ(c)={{c}}, S*O(X)={ ∅, {c, {a,b}, {a,c}, {b,c},X}, S*C(X)={∅, {a}, {b}, {c}, {a,b}, X}}, S*LC(X)={∅, {a}, {b}, {c}, {a,b}, {a,c},{b,c},X} and S_{I^*}O(X)={∅,{c}}, S_{I^*}C(X)={{a,b},X}}, S_{I^*}LC(X)={∅ , {c}}. Here {b} is S*-Locally Closed set but not S_{I^*}-Locally Closed.

Theorem 4.6: In a SNS (X, Ψ), every S_{I^*}-open set is S_{I^*}-Locally Closed set.

Proof: Proof is trivially from the Definitions.

Theorem 4.7: If A is a S_{I^*}-Locally Closed Set in a SNS (X, Ψ), if S_{I^*}-closed set K in X such that $A \cap K = \emptyset$.

Proof: Let A be a S_{I^*}-Locally Closed subset of X. Then $A = U \cap F$, where U is S_{I^*}-open and F is S_{I^*}-closed. Let $K = F \cap (X \backslash U)$. Then K is a S_{I^*}-closed subset of X such that $A \cap K = \emptyset$.

Theorem 4.8: For a subset A of SNS (X, Ψ), the followings are equivalent

1. A is S_{I^*}-Locally Closed
2. A = $U \cap S_{I^*}$cl (A) for some S_{I^*}-open set U.
3. S_{I^*}cl (A) \A is S_{I^*}-closed.
4. A ∪ (X\ $S_{I^*}cl(A)$) is S_{I^*}-open.
5. $A \subseteq S_{I^*}$int($A \cup (X \backslash S_{I^*}$cl(A))

Proof: (i) → (ii): Let A be a S_{I^*}-Locally Closed subset of X. Then A = U ∩ F, where U is S_{I^*}-open and F is S_{I^*}-closed. Then A ⊆ F implies that S_{I^*}cl(A) ⊆ F. So U ∩ S_{I^*}cl(A) ⊆ U ∩ F = A. Again, A ⊆ U and A ⊆ S_{I^*}cl(A) implies that A ⊆ U ∩ S_{I^*}cl(A). Thus A= U∩ S_{I^*}cl(A).

(ii) → (iii): S_{I^*}cl(A)\A = S_{I^*}cl(A)\ [U ∩ S_{I^*}cl(A)] for some S_{I^*}-open set U. Then S_{I^*}cl(A) ∩ [(X\$U$ ∩ S_{I^*}cl(A)] = S_{I^*}cl(A) ∩ [(X\$U$) ∪ ($X$\$S_{I^*}$cl(A)] = S_{I^*}cl(A) ∩ (X\$U$), which is S_{I^*}-closed.

(iii) → (iv): Since S_{I^*}cl(A)\A is S_{I^*}-closed, then X\[S_{I^*}cl(A)\A] is S_{I^*}-open and X\[S_{I^*}cl(A)\A] = X\[S_{I^*}cl(A) ∩ $(X\backslash A)$] = A∪ [$(X\backslash S_{I^*}$cl(A)].

(iv) → (v): A ⊆ A ∪ [$X\backslash S_{I^*}$cl(A)] = S_{I^*}int[A ∪ ($X\backslash S_{I^*}$cl(A))].

(v) → (i): A ⊆ S_{I^*}int(A ∪ ($X\backslash S_{I^*}$cl(A))). Thus A = A ⊆ S_{I^*}int(A ∪ ($X\backslash S_{I^*}$cl(A))) ∩ [S_{I^*}cl(A)] where S_{I^*}int(A ∪ ($X\backslash S_{I^*}$cl(A))) is S_{I^*}-open and S_{I^*}cl(A) is S_{I^*}-closed . Hence A is S_{I^*}-Locally Closed subset of X.

Theorem 4.9: Let (X, Ψ) be a SNS. If $A \subseteq B \subseteq X$ and B is S_{I^*}-Locally Closed, then there exist S_{I^*}-Locally Closed but C such that $A \subseteq C \subseteq B$.

Proof: Since B is S_{I^*}-Locally Closed by Theorem 4.8, B = U∩ S_{I^*}cl(B) where U is S_{I^*}-open. Then A ⊆ B ⊆ U. So A ⊆ U∩ S_{I^*}cl(A) = C(say). Then C is S_{I^*}-Locally Closed and A ⊆ C ⊆ B.

Conclusion: Many Topologists intend their attention to researching generalizations of closed sets in various fields. This paper is developed new closed sets called S_{I^*}-Locally Closed and S*-Locally Closed sets and its properties. These kinds of closed sets can have considerable impact in various field such as digital topology, fuzzy topology etc.

Reference:

1. S.Sripriya,Ajeet Singh,B.L.Raina, Zeros of Polynomials With Extremecoefficients; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 3 Issue 1 (2014), Pg 432-435Csaszar.A, *Generalized Topology, Generalized Continuity*, Acta Math.Hungar., 96(2002), 351-357.
2. Kent.D.C and Min.W.K, N*eighborhood Spaces*, International Journal of Mathematical Sciences., 32(7)(2002),387-399.
3. Erra Rachana, Technology Begets Technology:A Stratum of Future; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 3 Issue 1 (2014), Pg 457-461
4. Kim.Y.K and Min.W.K, *On Supra-neighborhood Systems and Spaces*, Far East Journal of Mathematical Sciences., 80(2)(2013),261-272.
5. Kim.Y.K and Min.W.K, *Some Results on Supra-neighborhood Spaces*, International Journal of Mathematical Analysis., 8(47)(2014),2311-2317.
6. Kent.D.C and Min.W.K, *Remarks on Generalized Topologies induced by Supra Neighborhood Systems,* International Journal of Pure and Applied Mathematics., 98(3)(2015),323-332.
7. S. Pious Missier, J.Arul Jesti, Properties of S^*_G Functions in topological Spaces; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Issue 2 (2014), Pg 911
8. Mashhour.A.S, Allam.A.A, Mahmoud.F.S and Khadr.F.H, *On Supratopological Spaces*, Indian J. Pure Appl. Math., 14(4)(1983), 502-510.
9. M. Parimala, R. Perumal, Slightly αψ-Continuous Function; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Issue 2 (2014), Pg 899-902
10. Min.W.K, *Some Results on Generalized Topological Spaces and Generalized Systems,* Acta Math. Hungar., 108(1-2)(2005),171-181.
11. A.Senthilrajan, Grey Model Technique; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 4 Issue 1 (2015), Pg 216-219
12. Min.W.K, *On Weak Neighborhood Systems and Spaces,* Acta Math. Hungar., 121(3)(2008),283-292.
13. Vaddiparthi Yogeswara, Biswajit Rath, Ch.Ramasanyasi Rao, FS-Sets And Infinite Distributive Laws; Mathematical Sciences International Research Journal : ISSN 2278-8697Volume 4 Issue 2 (2015), Pg 251-256

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