MODEL TO DEPICT THE EFFECT OF PHYSICAL EXERCISE AT VARYING TEMPERATURE USING FINITE DIFFERENCE METHOD

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Abstract: The present paper investigates how human body responds to the temperature variations due to some physical activities. Here we develop a three dimensional explicit finite difference model to study the thermal variations at varying temperature in the human limbs in subcutaneous tissues. Here the biophysical parameters such as thermal conductivity, blood flow and metabolic activity assume distinct values in distinct layers of the subcutaneous tissues. The model is developed to give solution to set the thermal comfort limits for people who involve in physical activities under two cases i.e. a person does exercise and takes rest and a person repeats exercise and rest alternatively. Here we consider the discrete case using explicit finite difference method in difference equation. The simulation part is given through MATLAB.

Keywords: Explicit Finite Difference Model, Metabolic Reaction, Thermal conductivity, Varying temperature.

Introduction: Physical exercise has become one of the important part of life. Whether the key reason for physical exercise is either for physical fitness or for occupational purpose it is important to know the limit of thermal stress caused due to such physical activities. The skin in human body or animal body is boundary lamina, which plays an important role in temperature regulation. It is composed of three layers namely epidermis, dermis and subcutaneous tissues. There are no blood vessels in the epidermis, so there is no blood flow and metabolic activity in this outermost layer. The density of the blood vessels increases as we go down the dermis and becomes almost uniform in the layer below the dermis namely sub dermal tissues. The core of the main trunk of the human remains at almost uniform temperature $37°C$ by maintaining the balance between heat generation within the body and heat loss from the body, but in limbs the core temperature may vary under changing environmental conditions. The core temperature of the human limbs varies extensively as we move away from the body trunk. This is due to the fact that the arterial blood flows from the trunk at body core temperature to the outer shell, looses heat while moving towards the extremities and returns as a colder venous blood temperature. The core temperature also varies along the angular direction of the limbs. This is because the arteries carrying blood from the trunk to the limbs lie on one side of the limb, while the superficial veins responsible for carrying blood towards the heart, lie on the other side.

Earlier analysis and investigations were made by Pardasani and Adlakha [1, 2, 3], Pardasani and Saxena [12], Adlakha, Pardasani and Agrawal [4] to study temperature distribution in SST region under normal environmental and physiological conditions. Pardasani and Adlakha [11, 12], Agrawal [2, 3] to study problems involving abnormalities like tumors in SST regions of human body. Khanday and Saxena [6] developed a model to study the body fluid in different Skin and Subcutaneous Tissues layers and also to study the cold related problems associated with dermal layers in human body. Gisolfi and Mora [5] studied that the blood flow and metabolic values may vary according to the intense of physical activities. Kumari and adlakha [8, 9, 10] have the temperature variations in peripheral regions for one and two dimensional. Here three dimensional unsteady state has been discussed under two different cases of exercise under varying climatic conditions.

The paper is organized as follows: section 1 gives the brief introduction. In section 2, mathematical formulation for heat flow in body tissues using partial differential equation has been given and appropriate boundary conditions has been framed. Section 3 deals with the assumptions of values regarding thermal conductivity, blood mass and metabolic activity in three components of the peripheral region. Section 4 gives the formulation of equation for two cases of physical activities. In section 5 the explicit finite difference method has been formulated and the difference equations are obtained. Section 6 deals with numerical values and simulation. The conclusions are summarized in section 7.

Mathematical Formulation: The peripheral region of the limb has been divided into eight layers with radius $r_0, r_1, r_2, r_3, r_4, r_5, r_6, r_7$ and r_8. The outermost layer is epidermis. Below the epidermis are the three layers of dermis followed by four layers of sub-dermal tissues. The innermost part is the limb core consisting of bone, muscles, large blood vessels etc.

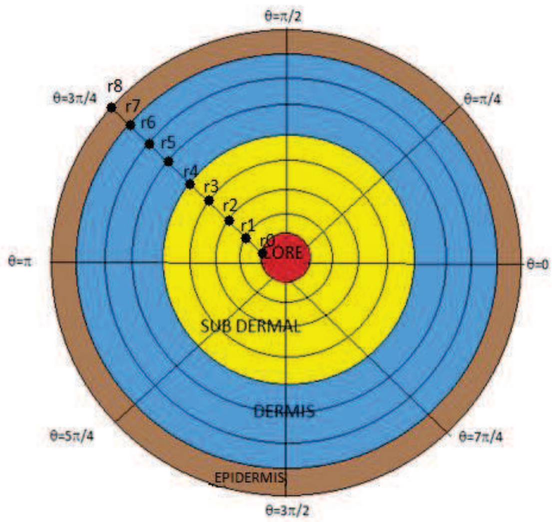

Figure: Cross section of the cylindrical shaped human limb

The following partial differential equation provides the heat flow in body tissues [13] for unsteady state

$$\rho\bar{c}\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}(k\frac{\partial T}{\partial x}) + m_b c_b(T_A - T) + M \qquad (2.1)$$

On the right hand side, the first, second and third term are obtained from Fick's law of diffusion, Fick's perfusion principal and rate of metabolic heat generation.

Where ρ- denotes density

$\bar{c}$ –denotes specific heat in tissues

K –denotes the thermal conductivity

M –denotes the rate of metabolic heat generation

m_b –blood mass flow rate

c_b –specific heat of the blood

T –temperature in body tissues

T_A –arterial blood temperature

Here specific heat of the blood and blood mass flow rates always comes as product. Thus we replace it by a single term B=$m_b c_b$. The mathematical model of equation (2.1) in polar cylindrical coordinates for three dimensional unsteady state is expressed as

$$\rho\bar{c}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} + \frac{K}{r^2}\frac{\partial^2 T}{\partial \theta^2}) + \left(K\frac{\partial^2 T}{\partial z^2}\right) + m_b c_b(T_A - T) + M$$

$$\rho\bar{c}\frac{\partial T}{\partial t} = K\frac{\partial^2 T}{\partial r^2} + \frac{K}{r}\frac{\partial T}{\partial r} + \left(\frac{K}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right) + \left(K\frac{\partial^2 T}{\partial z^2}\right)$$

$$+ m_b c_b(T_A - T) + M \qquad (2.2)$$

Heat loss from skin to environment takes place by conduction, convection, radiation and evaporation. And the outer surface of the skin is exposed to the environment. Therefore the outer boundary condition is given by

$$-k\frac{\partial T}{\partial x}\bigg|_{r=r9=5.8} = h(T - T_a) + LE \text{ at x=0, t>0} \qquad (2.3)$$

Where

h –denotes the heat transfer coefficient

T_a –is the atmospheric temperature

L –is the latent heat

E –is the rate of sweat evaporation

The blood flows from the trunk at body core temperature i.e., 37°C into the limbs. During this process the blood looses heat to the tissues while moving towards the extremities of the limbs. Therefore the blood attains lower temperature at extreme parts of the limbs. Then again the blood from the outer skin returns back to the trunk of the body through veins. So the inner core temperature of the limb is assumed to be at varying temperature along the axial direction of the limb. Hence we obtain the inner boundary condition as follows

$$T(t_i, r_0, \theta, z) = g_1 + g_2 e^{-\xi z} \qquad (2.4)$$

$$T_0(\theta, z) = T_{0a}(\theta) \text{ at z = a} \qquad (2.5)$$

$$T_0(\theta, z) = T_{0b}(\theta) \text{ at z = b} \qquad (2.6)$$

Here the constants g_1 and g_2 are obtained by the above conditions (2.5) and (2.6).

The two opposite sides of the inner core of the limb may be at different temperatures, so at the two ends of the limbs the following parabolic variations of the core temperature along angular direction [1, 4] has been taken

$$T_{0a}(\theta) = c_{1a} + c_{2a}\theta + c_{3a}\theta^2 \qquad (2.7)$$

$$T_{0b}(\theta) = c_{1b} + c_{2b}\theta + c_{3b}\theta^2 \qquad (2.8)$$

Such that

$T_{0a}(\theta) = T_{a0}\ at\ \theta = 0$

$T_{0a}(\theta) = T_{a\pi} at\ \theta = \pi$

$$T_{0a}(\theta) = T_{a0} at\ \theta = 2\pi \qquad (2.9)$$

And $T_{0b}(\theta) = T_{b0}\ at\ \theta = 0$

$T_{0b}(\theta) = T_{b\pi} at\ \theta = \pi$

$$T_{0b}(\theta) = T_{b0} at\ \theta = 2 \qquad (2.10)$$

The constantsc_{1a},c_{2a}, c_{3a}, c_{1b}, c_{2b} and c_{3b} in equation (2.7) and (2.8) are obtained by Using the above equations.

The major heat flow will take place along radial direction of the limb as the radial distances from the core of the limb to the outer surface of the limb are very small when compared to the axial distances. Therefore the temperature gradient along axial direction will be negligible as compared to the temperature along radial direction of the limb near the trunk. Therefore the following boundary condition is imposed

$$\frac{\partial T}{\partial z} = 0\ at\ z = a, 0 \le \theta \le 2\pi\,,$$

$$r_0 \le r \le r_8, t > 0 \qquad (2.11)$$

And the other extremity of the limb is assumed to be perfectly insulated and hence the following boundary condition is imposed.

$$\frac{\partial T}{\partial z} = 0 \quad at\ z = b, 0 \le \theta \le 2\pi\,,$$

$$r_0 \le r \le r_8, t > 0 \qquad (2.12)$$

At time=0, the limb is assumed to be insulated and hence the initial conditions is given by

$T(r, \theta, z, 0) = T_b \quad 0 \le \theta \le 2\pi\,,$

$r_0 \le r \le r_8,$

$$a \le z \le b \qquad (2.13)$$

Assumption of values regarding Thermal conductivity, Blood mass and metabolic activity:

Sub dermal tissues ($r_0 \le r < r_4$): The density of blood vessels is almost uniform in sub-dermal tissues. The blood flow, metabolic activity and thermal conductivity are found to be highest and almost constant in sub-dermal tissues of the peripheral region [8]. Therefore in this layer the value of K, B and M are taken to be constant as follows: $K_i = K_s = constant, B_i = b, M_i = m$ i=0(1)4

Dermis($r_4 \le r < r_7$): The density of blood vessels increases as we go down the dermis and it becomes almost uniform in sub-dermis. Thus the values of B, M and K will be high in the sub-dermal and minimum in epidermis. Hence the value of these biophysical parameters in dermis is average of that in epidermis and sub-dermal tissues. Therefore we get

$$K_i = \frac{(K_s + K_e)}{2},$$
$$B_i = \frac{b}{2},$$
$$M_i = \frac{m}{2} \qquad i=4(1)7$$

Epidermis ($r_7 \le r \le r_8$) : There are no blood vessels in epidermis. Hence there is no blood flow in epidermis and almost negligible metabolic activity in epidermis. The epidermis consists of mainly dead cells with lowest thermal conductivity than the other two. Therefore we take

$K_e = costant,$
$B_i = 0,$
$M_i = 0$ i=7,8

Discussion of the two cases:

Case(i): A person does physical exercise and stops at time t=0.

Here it is assumed that a person does physical activity and comes to rest at time t=0.Thus the blood flow and metabolic activity attains its maximum at t=0 and its value decreases as time t increases and will attain their minimum normal value as t tends to infinity. Therefore blood mass flow rate and metabolic activity in the peripheral region with respect to time is taken as negative exponential variation.

$$b(t) = b_{max} \quad for\ t = 0$$
$$b(t) = b_{min} \quad for\ t = \infty$$

Also

$$m(t) = m_{max} \quad for\ t = 0$$
$$m(t) = m_{min} \quad for\ t = \infty$$

Therefore

$$b(t) = C_{11} + C_{12}e^{-t} \qquad (4.1)$$
$$m(t) = C_{13} + C_{14}e^{-t} \qquad (4.2)$$
$$b_{max} = C_{11} + C_{12}, b_{min} = C_{11}$$

Here the constants $C_{11}, C_{12}, C_{13}\ and\ C_{14}$ are determined by the initial conditions defined at time t at zero and infinity.

Case(ii) : A person repeats exercise and rest alternatively.

Here it is assumed that the person does physical activity for some period of time and then rests for a period of time and again repeats the cycle of physical activity and rest in alternate periods. Thus we obtain the increasing and decreasing values alternatively for the blood flow and metabolic activity during the alternate period's .Hence a periodic variation of blood mass flow and of metabolic heat generation with respect to time has been assumed for a special case where the period of time the person does physical activity and rest is considered to be equal. Therefore here we get the following equations.

$$b(t) = b_{max} \quad for\ t = 0$$
$$b(t) = b_{min} \quad for\ t = \infty$$

Also

$$m(t) = m_{max} \quad for\ t = 0$$
$$m(t) = m_{min} \quad for\ t = \infty$$

Therefore

$$b(t) = C_{11}sin(\tfrac{\pi}{l})t + C_{12}cos(\tfrac{\pi}{l})t \qquad (4.3)$$
$$m(t) = C_{13}sin(\tfrac{\pi}{l})t + C_{13}cos(\tfrac{\pi}{l})t \qquad (4.4)$$
$$b_{max} = C_{11} + C_{12}, b_{min} = C_{11}$$

Here the constants $C_{11}, C_{12}, C_{13}\ and\ C_{14}$ are determined by the initial conditions defined at time t at zero and infinity.

Explicit Finite Difference Method: The heat equation evaluated at grid points (i , n) for one dimensional is given by

$$\left[\frac{\partial T}{\partial t}\right]_i^n = \alpha \left[\frac{\partial^2 T}{\partial x^2}\right]_i^n \qquad (5.1)$$

Consider the L.H.S:

$$\left[\frac{\partial T}{\partial t}\right]_i = \left[\frac{\partial T(x,t)}{\partial t}\right]_i = \frac{d}{dt}[T(x,t)_i] = \frac{d}{dt}T_i(t)$$

Therefore

$$\left[\frac{\partial T}{\partial t}\right]_i^n = \left[\frac{d}{dt}T_i(t)\right]^n$$

Consider the R.H.S:

In the Right hand side of (5.1) ordinary difference formula is applied

$$\alpha \left[\frac{\partial^2 T}{\partial x^2}\right]_i^n = \alpha \left[\frac{T_{i+1} - 2T_i + T_{i-1}}{(\Delta x)^2}\right]^n$$

Therefore equation (5.1) becomes:

$$\left[\frac{d}{dt}T_i(t)\right]^n = \alpha \left[\frac{T_{i+1} - 2T_i + T_{i-1}}{(\Delta x)^2}\right]^n$$

Using Explicit Method for (FTCS) Forward time centered space method in the equation (5.1) we get

$$\left[\frac{T_i^{n+1} - T_i^n}{\Delta t}\right] = \alpha \left[\frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{(\Delta x)^2}\right]$$

Mathematical Modeling Using Difference equations: Therefore using explicit Finite difference method in equation (2.2) for three dimension we get:

$$\overline{\rho c}\left[\frac{T_{i,j,k}^{n+1} - T_{i,j,k}^n}{\Delta t}\right] = K\left[\frac{T_{i-1,j,k}^n - 2T_{i,j,k}^n + T_{i+1,j,k}^n}{(\Delta r)^2}\right]$$

$$+\frac{K}{r}\left[\frac{T^n_{i+1,j,k}-T^n_{i,j,k}}{(\Delta r)}\right]+\frac{K}{r^2}\left[\frac{T^n_{i.j-1,k}-2T^n_{i,j,k}+T^n_{i,j-1,k}}{(\Delta\theta)^2}\right]$$
$$+K\left[\frac{T^n_{i,j,k-1}-2T^n_{i,j,k}+T^n_{i,j,k+1}}{(\Delta z)^2}\right]+B\left(T_b-T^n_{i,j,k}\right)+M \quad (5.2)$$

Simplifying (5.2) we have

$$T^{n+1}_{i,j,k} = (1-2A_1-A_2-2A_3-2A_4-A_5)T^n_{i,j,k} + (A_1+A_2)T^n_{i+1,j,k} + A_1T^n_{i-1,j,k} + A_3\left[T^n_{i.j-1,k}+T^n_{i,j-1,k}\right] + A_4\left[T^n_{i,j,k-1}+T^n_{i,j,k+1}\right] + A_5T_b + A_6 \quad (5.3)$$

Where

$A_1=\frac{K_i\Delta t}{\rho\bar{c}(\Delta r)^2}$, $A_2=\frac{K_i\Delta t}{\rho\bar{c}r(\Delta r)}$, $A_3=\frac{K_i\Delta t}{\rho\bar{c}r^2(\Delta\theta)^2}$, $A_4=\frac{K_i\Delta t}{\rho\bar{c}(\Delta z)^2}$, $A_5=\frac{b_i\Delta t}{\rho\bar{c}}$, $A_6=\frac{m_i\Delta t}{\rho\bar{c}}$

i=1,2,3....9 , j= 0...2л n=0,1,2 k=0,...10

Using Explicit Finite Difference method in equation (2.3) we get

$$T^{n+1}_{i,j,k} = UT^n_{i,j,k} + VT_a - WLE \quad (5.4)$$

Where $U=\left(1-\frac{h.\Delta t}{K}\right)$, $V=\frac{h.\Delta t}{K}$, $W=\frac{\Delta t}{K}$

Numerical Values and simulation: The computations have been performed for three different atmospheric conditions such as cold temperature, room temperature and hot temperature with respect degree Celsius values such as $T_a = 15\,^\circ C$, $T_a = 23\,^\circ C$ and $T_a = 33\,℃$ respectively. The biophysical parameters such as thermal conductivity, metabolic heat generation, blood mass flow rate may vary from person to person depending on age, gender, clothing and many. Here these values have been calculated for a person [5, 10] under all three atmospheric conditions.

Numerical Values for cold climatic conditions:

Atmospheric Temperature- $T_a(℃)$	$\bar{m}$ (Cal/cm^2-min)	$\bar{b}=m_bc_b$ $(Cal/cm^2-min\,℃)$	E $(\times 10^{-3}\,Kg.m^{-2}S^{-1})$
15	0.0357	0.003	0

The values of metabolic activity and blood flow for a particular case of physical exercise are taken as given below :

$m_{max}=10, m_{min}=\bar{m}, b_{max}=4, b_{min}=\bar{b}$.

Numerical Values for room temperature:

Atmospheric Temperature- $T_a(℃)$	$\bar{m}$ (Cal/cm^2-min)	$\bar{b}=m_bc_b$ $(Cal/cm^2-min\,℃)$	E $(\times 10^{-3}\,Kg.m^{-2}S^{-1})$
23	0.315	0.018	0

The values of metabolic activity and blood flow are taken as given below :

$m_{max}=11, m_{min}=\bar{m}, b_{max}=5, b_{min}=\bar{b}$.

Numerical Values for hot climatic conditions:

Atmospheric Temperature- $T_a(℃)$	$\bar{m}$ (Cal/cm^2-min)	$\bar{b}=m_bc_b$ $(Cal/cm^2-min\,℃)$	E $(\times 10^{-3}\,Kg.m^{-2}S^{-1})$
33	0.315	0.018	0

The values of metabolic activity and blood flow are taken as given below :

$m_{max}=15, m_{min}=\bar{m}, b_{max}=7, b_{min}=\bar{b}$.

Here $\bar{m}$ and $\bar{b}$ are rates of blood mass flow and metabolic activity in tissues when the person is at complete rest i.e. he/she stops doing the physical exercise.

The value of the other biophysical parameters takes the standard values as follows:

h=0.009 *cal*/*cm*^{2}-*min*,

L=579 *cal*/*gm*,

K_e=0.030 *cal*/*cm*-*min* ℃ for epidermis,

K_d=0.045 *cal*/*cm*-*min* ℃ for dermis,

K_s=0.060 *cal*/*cm*-*min* ℃ for sub-dermis,

ρ = 1.090 gms/cm^2,

$\bar{c}$ = 0.830 *cal*/*gm*- ℃,

T_{a0}=37°C, $T_{aл}$=36 °C, T_{b0}=34°C, $T_{bл}$=35°C.

Here for a specific case of thickness of peripheral layers of human limbs we have consider the following values

r_0=5 cm, r_1= 5.1 cm, r_2= 5.2 cm, r_3= 5.3 cm, r_4= 5.4 cm, r_5= 5.5 cm, r_6= 5.6 cm, r_7= 5.7 cm, r_8= 5.8 cm. z=0 to 10 cm, a=0, b=10 cm. Using matlab we simulate our result as follows

Simulations:

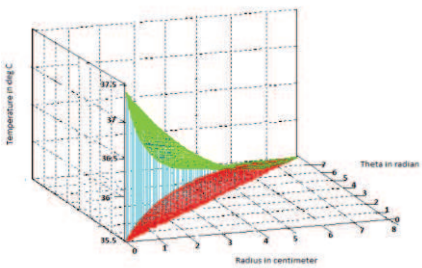

Figure: The temperature distribution for the first case under the atmospheric temperature $T_a = 23\,℃$ is plotted.

The graphs has been plotted for temperature T (in deg C) along radial direction r in 'cm' and theta θ (in 'radian') at $T_a = 23\,℃$. At time t=0 when he comes to rest after doing physical activity. The temperature distribution figure explains that the thermal disturbances are found to be decreasing slightly as we move along axial direction of human limb from the trunk (z=0) towards the extreme part of the limb (z=10 cm). Thus we observe that the maximum thermal stress due to physical activity takes place in the layers near the core of the limb and portions of the limb near the trunk. This may be due to the fact that the lower layers of peripheral regions are insulated by the upper layers which are exposed to the environment. The maximum thermal disturbances are observed in Figure in the sub-dermal tissues ($5.0 \leq r \leq 5.4$cm). From the Figure we can also observe that physical exercise as well as variation in core temperature of limb has significant effect on temperature profiles in different layers of human limbs at room temperature.

Conclusion: The information generated from this model will be useful for biomedical students f to set thermal comfort limits office work to ensure productivity and quality of work and also to protect the industrial workers from heat-related illness. These models can also be developed to understand the thermoregulatory responses in human body due to physical work.

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