

## ON INTUITIONISTIC $\beta$ -OPEN SETS

A.SINGARAVELAN

**Abstract:** In this paper the properties of Intuitionistic  $\beta$ -open set (for short  $I\beta$ -open sets) are studied. Its relation with other existing forms of intuitionistic weak open sets is established. Also intuitionistic  $\beta$ -closure (for short  $I\beta cl$ ) and intuitionistic  $\beta$ -interior (for short  $I\beta int$ ) of an intuitionistic set are defined and their properties are investigated. Further the theorems which exhibit the characterizations of  $I\beta$ -open set in intuitionistic topological spaces are proved and some of the interesting properties of  $I\beta$ -open sets are obtained.

**Keywords:**  $I\beta$ -open sets,  $\beta$ -closed sets,  $I\beta$ - closure,  $I\beta$ - interior.

**Introduction:** In 1986, Atanassov[4] introduced the concept of Intuitionistic fuzzy sets as a generalization of fuzzy sets. Later in 1996, Coker [5] introduced the concept of Intuitionistic set and Intuitionistic points. This is a discrete form of intuitionistic fuzzy sets where all the sets are crisp set. In 2000, Coker [7] also introduced the concept of “intuitionistic topological space” and investigated basic properties of continuous functions and compactness. N. Levine [10] introduced semi open sets and semi continuity in topological space and M.E. Abd El. Monsef et.al [1] introduced “ $\beta$ -open sets and  $\beta$ -continuous mapping” and discussed some basic properties.

D. Andrijevic[3] introduced and discussed some more properties of semi preopen set in topological space. Gnanambal Ilango and Selvanayaki [9], introduced generalized pre regular closed sets in intuitionistic topological spaces.

In this paper intuitionistic  $\beta$ -open sets is introduced and intuitionistic  $\beta$ -interior and intuitionistic  $\beta$ -closure are defined and some basic properties are investigated.

**Preliminaries:** Let us recall some basic definitions and results which are useful for this sequel. Throughout the present study, a space  $X$  means an intuitionistic topological space.

**Definition 2.1 [5]:** Let  $X$  is a non empty set. An intuitionistic set (IS for short)  $A$  is an object having the form  $A = \langle X, A_1, A_2 \rangle$ , where  $A_1$  and  $A_2$  are subsets of  $X$  satisfying  $A_1 \cap A_2 = \emptyset$ . The set  $A_1$  is called the set of members of  $A$ , while  $A_2$  is called the set of non-members of  $A$ .

**Definition 2.2 [5]:** Let  $X$  be a non empty set and let  $A, B$  are intuitionistic sets in the form

$A = \langle X, A_1, A_2 \rangle, B = \langle X, B_1, B_2 \rangle$  respectively. Then

1.  $A \subseteq B$  iff  $A_1 \subseteq B_1$  and  $B_2 \subseteq A_2$
2.  $A = B$  iff  $A \subseteq B$  and  $B \subseteq A$
3.  $A^c = \langle X, A_2, A_1 \rangle$
4.  $[ ] A = \langle X, A_1, (A_1)^c \rangle$
5.  $A - B = A \cap B^c$ .
6.  $\phi_\sim = \langle X, \phi, X \rangle, X_\sim = \langle X, X, \phi \rangle$
7.  $A \cup B = \langle X, A_1 \cup B_1, A_2 \cap B_2 \rangle$ .
8.  $A \cap B = \langle X, A_1 \cap B_1, A_2 \cup B_2 \rangle$ .

Furthermore, let  $\{A_\alpha: \alpha \in J\}$  be an arbitrary family of intuitionistic sets in  $X$ , where  $A_\alpha = \langle X, A_\alpha^{(1)}, A_\alpha^{(2)} \rangle$ . Then

- (i)  $\cap A_\alpha = \langle X, \cap A_\alpha^{(1)}, \cup A_\alpha^{(2)} \rangle$ .
- (j)  $\cup A_\alpha = \langle X, \cup A_\alpha^{(1)}, \cap A_\alpha^{(2)} \rangle$ .

**Definition 2.3 [6]:** An intuitionistic topology (for short IT) on a non empty set  $X$  is a family of IS's in  $X$  satisfying the following axioms.

1.  $\phi_\sim, X_\sim \in \tau$
2.  $G_1 \cap G_2 \in \tau$  for any  $G_1, G_2 \in \tau$ .
3.  $\cup G_\alpha$  for any arbitrary family  $\{G_\alpha/\alpha \in J\} \subseteq \tau$  where  $(X, \tau)$  is called an intuitionistic topological space (for short ITS( $X$ )) and any intuitionistic set in  $X$  is called an intuitionistic open set (for short IOS) in  $X$ . The complement  $A^c$  of an IOS  $A$  is called an intuitionistic closed set (for short ICS) in  $X$ .

**Definition 2.4[7]:** Let  $(X, \tau)$  be an intuitionistic topological space (for short ITS( $X$ )) and  $A = \langle X, A_1, A_2 \rangle$  be an IS in  $X$ . Then the interior and closure of  $A$  are defined by  $Icl(A) = \cap \{K : K \text{ is an ICS in } X \text{ and } A \subseteq K\}$ ,  $lint(A) = \cup \{G : G \text{ is an IOS in } X \text{ and } G \subseteq A\}$ . It can be shown that  $Icl(A)$  is an ICS and  $lint(A)$  is an IOS in  $X$  and  $A$  is an ICS in  $X$  iff  $Icl(A) = A$  and is an IOS in  $X$  iff  $lint(A) = A$ .

**Definition 2.5[5]:** Let  $X$  be a non empty set and  $P \in X$ . Then the IS  $P$  defined by  $P = \langle X, \{p\}, \{p\}^c \rangle$  is called an intuitionistic point (IP for short) in  $X$ . The intuitionistic point  $P$  is said to be contained in  $A = \langle X, A_1, A_2 \rangle$  ( i.e  $p \in A$ ) if and only if  $p \in A_1$ .

**Definition 2.6[12]:** Let  $(X, \tau)$  be an ITS( $X$ ). An intuitionistic set  $A$  of  $X$  is said to be

1. Intuitionistic semiopen if  $A \subseteq Icl(lint(A))$ .
2. Intuitionistic preopen if  $A \subseteq lint(Icl(A))$ .
3. Intuitionistic regular open if  $A = lint(Icl(A))$ .
4. Intuitionistic  $\alpha$ -open if  $A \subseteq lint(Icl(lint(A)))$ .

The family of all intuitionistic pre-open, intuitionistic regular open and intuitionistic  $\alpha$ -open sets of  $(X, \tau)$  are denoted by IPOS, IROS and  $I\alpha$ OS respectively.

**Definition 2.7 [9]:** Let  $(X, \tau)$  be an ITS( $X$ ) and let  $A = \langle X, A_1, A_2 \rangle$  be IS in  $X$ . Then  $A$  is said to be

1. Intuitionistic generalized closed (Ig-closed) if  $Icl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is intuitionistic open in  $X$ .
2. Intuitionistic generalized preclosed (Igp-closed) if  $Ipcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is intuitionistic open in  $X$ .
3. Intuitionistic regular generalized closed (Irg-closed) if  $Icl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is intuitionistic regular open in  $X$ .
4. Intuitionistic generalized  $\alpha$ -closed (Ig $\alpha$ -closed) if  $I\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is intuitionistic open in  $X$ .

**On Intuitionistic  $\beta$ -Open Sets:** Here intuitionistic  $\beta$ -open set is defined and its relation with existing intuitionistic sets is seen and its basic properties are studied.

**Definition 3.1:** A subset  $A$  of an intuitionistic topological space  $X$  is intuitionistic  $\beta$ -open set, if there exists a intuitionistic preopen set  $U$  in  $X$ , such that  $U \subseteq A \subseteq Icl(U)$ . The family of all intuitionistic  $\beta$ -open sets in  $X$  will be denoted by  $I\beta OS(X)$ .

**Theorem 3.2:** A subset  $A = \langle X, A_1, A_2 \rangle$  of an  $ITS(X)$  is intuitionistic  $\beta$ -open set if and only if  $A \subseteq Icl(\text{Iint}(Icl(A)))$ .

**Proof: Necessity:** Let  $A = \langle X, A_1, A_2 \rangle$  be a intuitionistic  $\beta$ -open set, then there exist an intuitionistic pre-open set  $B$ , such that  $B \subseteq A \subseteq Icl(B)$  which implies  $Icl(A) = Icl(B)$  and hence  $Icl(\text{Iint}(Icl(A))) = Icl(\text{Iint}(Icl(B)))$ . Since  $B$  is intuitionistic pre-open we have  $IB \subseteq \text{Iint}(Icl(B))$  and hence  $A \subseteq Icl(B) \subseteq Icl(\text{Iint}(Icl(A)))$  which implies  $A \subseteq Icl(\text{Iint}(Icl(A)))$ . Hence  $A$  is intuitionistic  $\beta$ -open set.

**Sufficiency:** Let  $A \subseteq Icl(\text{Iint}(Icl(A)))$ , then we have  $\text{Iint}(Icl(A)) \subseteq A$ , so  $\text{Iint}(Icl(A)) \subseteq A \subseteq Icl(\text{Iint}(Icl(A)))$ . Let  $B = \text{Iint}(Icl(A))$ , which implies  $IB \subseteq A \subseteq Icl(B)$ . Hence  $A$  is intuitionistic  $\beta$ -open in  $ITS(X)$ .

**Remark 3.3:** The complement of an intuitionistic  $\beta$ -open set is called intuitionistic  $\beta$ -closed.

**Theorem 3.4:** Every intuitionistic open set is intuitionistic  $\beta$ -open set.

**Proof:** Let  $A$  be a intuitionistic open set in intuitionistic topological space, the  $\text{Iint}(A) = A$  implies  $IU \subseteq \text{Iint}(A) = A \subseteq Icl(U)$ , implies  $IU \subseteq \text{Iint}(A) \subseteq Icl(U)$ . Hence  $A$  is intuitionistic  $\beta$ -open set ( $I\beta O(X)$  for short).

The converse of the above theorem need not be true as seen from the following example.

**Example 3.5:** Let  $X = \{a, b\}$  with  $\tau = \{X, \phi, A, B, C, D\}$  where  $A = \langle X, \{a\}, \{\phi\} \rangle$ ,  $B = \langle X, \{b\}, \phi \rangle$  and  $C = \langle X, \phi, \{b\} \rangle$ ,  $D = \langle X, \{\phi\}, \{\phi\} \rangle$ .

Then the intuitionistic set  $\langle X, \{a\}, \{b\} \rangle$  belongs to  $I\beta O(X)$  but not intuitionistic open sets in  $(X, \tau)$ .

**Theorem 3.6:** Every intuitionistic semi open set is intuitionistic  $\beta$ -open set.

**Proof:** Let  $A$  be a intuitionistic semi open set in intuitionistic topological space, then we have  $\text{Iint}(A) = \text{Iint}(cl(A))$ , therefore  $A \subseteq Icl(\text{Iint}(cl(A)))$ . Hence  $A$  is intuitionistic  $\beta$ -open set.

The converse of the above theorem need not be true as seen from the following example.

**Example 3.7:** Let  $X = \{a, b, c\}$  with  $\tau = \{X, \phi, A, B, C, D, E, F\}$  where  $A = \langle X, \phi, \{a, b\} \rangle$ ,  $B = \langle X, \{c\}, \{a, b\} \rangle$  and  $C = \langle X, \phi, \{b, c\} \rangle$ ,  $D = \langle X, \{c\}, \{b\} \rangle$ ,  $E = \langle X, \{a, c\}, \{b\} \rangle$  and  $F = \langle X, \phi, \{b\} \rangle$ .

Then the intuitionistic set  $U = \langle X, \{a\}, \{b\} \rangle$  is a intuitionistic  $\beta$ -open set in  $ITS(X)$ , but not intuitionistic semi open in  $(X, \tau)$ .

**Theorem 3.8:** Every intuitionistic pre-open set is a intuitionistic  $\beta$ -open set.

**Proof:** Let  $A$  be intuitionistic pre-open set in  $ITS(X)$ , then  $A \subseteq \text{Iint}(Icl(A))$  implies  $\text{Iint}(A) \subseteq \text{Iint}(Icl(A))$  implies  $(\text{Iint}(Icl(A))) \subseteq Icl(\text{Iint}(Icl(A)))$  implies  $\text{Iint}(Icl(A)) \subseteq A \subseteq Icl(\text{Iint}(Icl(A)))$  implies  $A \subseteq Icl(\text{Iint}(Icl(A)))$ . Hence  $A$  is intuitionistic  $\beta$ -open set in  $ITS(X)$ .

The converse of the above theorem need not be true as seen from the following example.

**Example 3.9:** Let  $X = \{a, b, c\}$  with  $\tau = \{X, \phi, A, B, C, D, E, F\}$  where  $A = \langle X, \phi, \{a, b\} \rangle$ ,  $B = \langle X, \{c\}, \{a, b\} \rangle$  and  $C = \langle X, \phi, \{b, c\} \rangle$ ,  $D = \langle X, \{c\}, \{b\} \rangle$ ,  $E = \langle X, \{a, c\}, \{b\} \rangle$  and  $F = \langle X, \phi, \{b\} \rangle$ .

Let  $U = \langle X, \phi, \{a\} \rangle$  is a intuitionistic  $\beta$ -open set in  $ITS(X)$ , but not in intuitionistic pre open sets in intuitionistic topological space  $(X, \tau)$ .

**Corollary 3.10:** Every intuitionistic  $\alpha$ -open set is intuitionistic pre-open in  $ITS(X)$ .

**Corollary 3.11:** Every intuitionistic  $\alpha$ -open set is intuitionistic semi-open in  $ITS(X)$ .

**Theorem 3.12:** Every intuitionistic  $\alpha$ -open set is intuitionistic  $\beta$ -open in  $ITS(X)$ .

**Proof:** Let  $A$  be a intuitionistic  $\alpha$ -open set in  $ITS(X)$ , WKT every intuitionistic  $\alpha$ -open set is intuitionistic semi-open set from above theorem, therefore  $A \subseteq (Icl(\text{Iint}(A)))$  implies  $A \subseteq Icl(\text{Iint}(Icl(A)))$ . Hence  $A$  is intuitionistic  $\beta$ -open set in  $ITS(X)$ .

The converse of the above theorem need not be true as seen from the following example.

**Example 3.13:** Let  $X = \{a, b, c\}$  with  $\tau = \{X, \phi, A, B, C, D, E, F\}$  where  $A = \langle X, \phi, \{a, b\} \rangle$ ,  $B = \langle X, \{c\}, \{a, b\} \rangle$  and  $C = \langle X, \phi, \{b, c\} \rangle$ ,  $D = \langle X, \{c\}, \{b\} \rangle$ ,  $E = \langle X, \{a, c\}, \{b\} \rangle$  and  $F = \langle X, \phi, \{b\} \rangle$ .

Let  $V = \langle X, \phi, \{a\} \rangle$  is a intuitionistic  $\beta$ -open set in  $ITS(X)$ , but not in intuitionistic  $\alpha$ -open sets in intuitionistic topological space  $(X, \tau)$ .

**Theorem 3.14:** Every intuitionistic regular-open set is a intuitionistic  $\beta$ -open in  $ITS(X)$ .

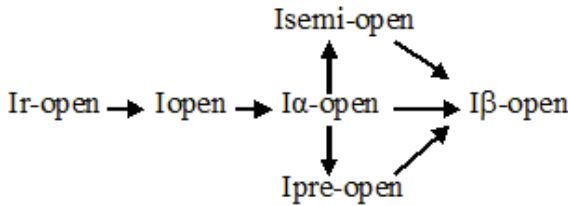
**Proof:** Let  $A$  be a intuitionistic regular-open set in  $ITS(X)$ , then  $A = \text{Iint}(Icl(A))$  implies  $A \subseteq Icl(\text{Iint}(Icl(A)))$ . Hence  $A$  is intuitionistic  $\beta$ -open set in  $ITS(X)$ .

The converse of the above theorem need not be true from the following example.

**Example 3.15:** Let  $X=\{a, b, c\}$  with  $\tau=\{X, \phi, A, B, C, D, E, F\}$  where  $A=\langle X, \phi, \{a, b\} \rangle$ ,  $B=\langle X, \{c\}, \{a, b\} \rangle$  and  $C=\langle X, \phi, \{b, c\} \rangle$ ,  $D=\langle X, \{c\}, \{b\} \rangle$ ,  $E=\langle X, \{a, c\}, \{b\} \rangle$  and  $F=\langle X, \phi, \{b\} \rangle$ .

Let  $V=\langle X, \{a\}, \{c\} \rangle$  is an intuitionistic  $\beta$ -open set in  $ITS(X)$ , but not an intuitionistic regular- open sets in  $ITS(X)$ .

The following diagram shows that relationship between  $I\beta$ -open sets and some other intuitionistic sets.



$A \rightarrow B$  represents  $A$  implies  $B$  but not conversely

**Remark 3.16:** The intersection of two  $I\beta$ -open sets need not be  $I\beta$ -open as seen from the following example.

**Example 3.17:** Let  $X=\{a, b\}$  with intuitionistic topology  $\tau=\{X, \phi, \langle X, \{b\}, \phi \rangle, \langle X, \phi, \{a\} \rangle\}$ . The subset  $A = \langle X, \{b\}, \phi \rangle$  and  $B=\langle X, \{a\}, \phi \rangle$  both are  $I\beta$ -open sets in  $ITS(X)$ . But the  $A \cap B=\langle X, \phi, \phi \rangle$  is not in  $I\beta$ -open set.

**Theorem 3.18:** Let  $A$  be an intuitionistic  $\beta$ -open set in  $ITS(X)$  and suppose  $A \subseteq B \subseteq Icl(A)$ , then  $B$  is an intuitionistic  $\beta$ -open set in  $ITS(X)$ .

**Proof:** Since  $A$  is an intuitionistic  $\beta$ -open set in  $ITS(X)$ , then there exists an intuitionistic pre open set  $U$  in  $X$  such that  $U \subseteq A \subseteq Icl(U)$ , as  $A \subseteq B$ ,  $U \subseteq A \subseteq B$  implies that  $U \subseteq B$ , also  $Icl(A) \subseteq Icl(Icl(U))=Icl(U)$  and thus  $B \subseteq Icl(U)$ , hence  $U \subseteq B \subseteq Icl(U)$  which implies that  $B$  is an intuitionistic  $\beta$ -open set in  $ITS(X)$ .

**Theorem 3.19:** If  $A$  is an intuitionistic  $\beta$ -open set which is an intuitionistic semi-closed set in  $ITS(X)$ , is an intuitionistic semi open set in  $ITS(X)$ .

**Proof:** Let  $A = \langle X, A_1, A_2 \rangle$  is an intuitionistic semi-closed, then  $\text{Iint}(Icl(A)) \subseteq A$  implies  $\text{Iint}(Icl(A)) \subseteq \text{Iint}(A)$  implies  $Icl(\text{Iint}(Icl(A))) \subseteq Icl(\text{Iint}(A))$ . Since  $A$  is an intuitionistic  $\beta$ -open  $A \subseteq Icl(\text{Iint}(Icl(A)))$ , therefore  $A \subseteq Icl(\text{Iint}(Icl(A))) \subseteq Icl(\text{Iint}(A))$  which implies  $A \subseteq Icl(\text{Iint}(A))$ . Therefore  $A$  is an intuitionistic semi open set in  $ITS(X)$ .

**Theorem 3.20:** If  $B$  is an intuitionistic  $\beta$ -closed and an intuitionistic semi-open in  $ITS(X)$ , then  $B$  is an intuitionistic semi closed set in  $ITS(X)$ .

**Proof:** Let  $B = \langle X, B_1, B_2 \rangle$  is an intuitionistic semi-open, then  $B \subseteq Icl(\text{Iint}(A))$  implies  $Icl(A) \subseteq Icl(\text{Iint}(B))$  implies  $\text{Iint}(Icl(B)) \subseteq \text{Iint}(Icl(\text{Iint}(B)))$ . Since  $B$  is an intuitionistic  $\beta$ -closed,  $\text{Iint}(Icl(\text{Iint}(B))) \subseteq B$  therefore  $\text{Iint}(Icl(B)) \subseteq \text{Iint}(Icl(\text{Iint}(B))) \subseteq B$  which implies

$\text{Iint}(Icl(A)) \subseteq B$ . Therefore  $B$  is an intuitionistic semi-closed set in  $ITS(X)$ .

**Intuitionistic  $\beta$ -Closure and Intuitionistic  $\beta$ -Interior:** Analogous to the definitions of  $\beta$ -closure and  $\beta$ -interior in general topology, here is defined  $I\beta$ -closure and  $I\beta$ -interior in intuitionistic topological space.

**Definition 4.1:** Let  $(X, \tau)$  be an intuitionistic topological space and let  $A = \langle X, A_1, A_2 \rangle$  be an intuitionistic set of  $X$ , then  $I\beta\text{-cl}(A) = \bigcap \{F : F \text{ is intuitionistic } \beta\text{-closed in } X \text{ and } A \subseteq F\}$ .

**Definition 4.2:** Let  $(X, \tau)$  be an intuitionistic topological space and let  $A = \langle X, A_1, A_2 \rangle$  be an intuitionistic set of  $X$ , then  $I\beta\text{-int}(A) = \bigcup \{F : F \text{ is intuitionistic } \beta\text{-open in } X \text{ and } F \subseteq A\}$ .

**Lemma 4.3:** Let  $A$  and  $B$  be an intuitionistic sets of an  $ITS(X)$ , then the following results are obvious.

1.  $I\beta\text{-cl}(X) = X$  and  $I\beta\text{-cl}(\phi) = \phi$
2. If  $A \subseteq B$ , then  $I\beta\text{-cl}(A) \subseteq I\beta\text{-cl}(B)$
3.  $I\beta\text{-cl}(I\beta\text{-cl}(A)) = I\beta\text{-cl}(A)$
4.  $I\beta\text{-int}(I\beta\text{-int}(A)) = I\beta\text{-int}(A)$

**Theorem 4.4:** Let  $A = \langle X, A_1, A_2 \rangle$  be an intuitionistic sets of an  $ITS(X)$ , then  $I\beta\text{-cl}(A) = AU \text{Iint}(Icl(\text{Iint}(A)))$ .

**Proof:** We observe that

$$\begin{aligned} \text{Iint}(Icl(\text{Iint}(AU \text{Iint}(Icl(\text{Iint}(A)))))) &\subseteq \text{Iint}(Icl(\text{Iint}(AU(Icl(\text{Iint}(A)))))) \\ &\subseteq \text{Iint}(Icl(\text{Iint}(A) \cup Icl(\text{Iint}(A)))) \\ &= \text{Iint}(Icl(\text{Iint}(A))) \end{aligned}$$

$$I\beta\text{-cl}(A) \subseteq AU \text{Iint}(Icl(\text{Iint}(A))) \rightarrow (i)$$

On the other hand  $I\beta\text{-cl}(A)$  is an intuitionistic  $\beta$ -closed, so

$$\begin{aligned} \text{Iint}(Icl(\text{Iint}(A))) &\subseteq \text{Iint}(Icl(\text{Iint}(I\beta\text{-cl}(A)))) \\ &\subseteq I\beta\text{-cl}(A) \text{ and hence} \end{aligned}$$

$$AU \text{Iint}(Icl(\text{Iint}(A))) \subseteq I\beta\text{-cl}(A) \rightarrow (ii)$$

From (i) and (ii) we have

$$I\beta\text{-cl}(A) = AU \text{Iint}(Icl(\text{Iint}(A))).$$

**Theorem 4.5:** Let  $A = \langle X, A_1, A_2 \rangle$  be an intuitionistic set of an intuitionistic topological space  $(X, \tau)$ , then  $I\beta\text{-int}(A) = A \cap Icl(\text{Iint}(Icl(A)))$ .

**Proof:** We observe that

$$\begin{aligned} A \cap Icl(\text{Iint}(Icl(A))) &= Icl(\text{Iint}(Icl(A))) \\ &\subseteq Icl(\text{Iint}(Icl(A) \cap \text{Iint}(Icl(A)))) \\ &\subseteq Icl(\text{Iint}(Icl(A) \cap \text{Iint}(Icl(A)))) \end{aligned}$$

Here  $A \cap Icl(\text{Iint}(Icl(A)))$  is  $I\beta$ -open and hence

$$A \cap Icl(\text{Iint}(Icl(A))) \subseteq I\beta\text{-int}(A) \rightarrow (i).$$

On the other hand since  $I\beta\text{-int}(A)$  is an intuitionistic  $\beta$ -open, we have

$$\begin{aligned} I\beta\text{-int}(A) &\subseteq Icl(\text{Iint}(Icl(I\beta\text{-int}(A)))) \\ &\subseteq Icl(\text{Iint}(Icl(A))) \\ &\subseteq A \cap Icl(\text{Iint}(Icl(A))) \rightarrow (ii) \end{aligned}$$

From (i) and (ii) we have

$$I\beta\text{-int}(A) = A \cap Icl(\text{Iint}(Icl(A))).$$

**Theorem 4.6:** Let  $A = \langle X, A_1, A_2 \rangle$  be an intuitionistic set of  $ITS(X)$ , then  $I\beta\text{-int}(I\beta\text{-cl}(A)) = I\beta\text{-cl}(I\beta\text{-int}(A))$ .

**Proof:** Let  $A = \langle X, A_1, A_2 \rangle$  be subset of intuitionistic topological space  $(X, \tau)$ . Then we have to prove that  $\beta\text{-int}(\beta\text{-cl}(A)) = \beta\text{-cl}(\beta\text{-int}(A))$ .

By applying theorem 4.3 and 4.4 we have

$$\begin{aligned} &\beta\text{-int}(\beta\text{-cl}(A)) \\ &= \beta\text{-cl}(A) \cap \text{Icl}(\text{Iint}(\text{Icl}(\beta\text{-cl}(A)))) \\ &= (\text{AUint}(\text{Icl}(\text{Iint}(A)))) \cap (\text{Icl}(\text{Iint}(\text{Icl}(\beta\text{-cl}(A)))) \\ &= (\text{AUint}(\text{Icl}(\text{Iint}(A)))) \cap \text{Icl}(\text{Iint}(\text{Icl}(A))) \\ &= (\text{A} \cap \text{Icl}(\text{Iint}(\text{Icl}(A)))) \cup (\text{Iint}(\text{Icl}(\text{Iint}(A)))) \\ &= \beta\text{-int}(A) \cup \text{Iint}(\text{Icl}(\text{Iint}(\beta\text{-int}(A)))) \\ &= \beta\text{-cl}(\beta\text{-int}(A)). \end{aligned}$$

**Theorem 4.7:** Let  $A = \langle X, A_1, A_2 \rangle$  be subset of ITS(X). Then  $\text{Icl}(\beta\text{-int}(A)) = \beta\text{-int}(\text{Icl}(A)) = \text{Icl}(\text{Iint}(\text{Icl}(A)))$

**Proof:** Since  $A = \langle X, A_1, A_2 \rangle$  be subset of ITS(X), Then we take  $\text{Icl}(\beta\text{-int}(A)) = \text{Icl}(\text{Icl}(\text{Iint}(\text{Icl}(A))))$   
 $= \text{Icl}(\text{Iint}(\text{Icl}(A))) \rightarrow (1)$ .

Take  $\beta\text{-int}(\text{Icl}(A)) = \text{Icl}(\text{Iint}(\text{Icl}(\text{Icl}(A))))$   
 $= \text{Icl}(\text{Iint}(\text{Icl}(A))) \rightarrow (2)$

From (1) and (2) we have  $\text{Icl}(\beta\text{-int}(A)) = \beta\text{-int}(\text{Icl}(A)) = \text{Icl}(\text{Iint}(\text{Icl}(A)))$ .

**Theorem 4.8:** Let  $A = \langle X, A_1, A_2 \rangle$  be subset of ITS(X). Then  $\text{Iint}(\beta\text{-cl}(A)) = \beta\text{-cl}(\text{Iint}(A)) = (\text{Iint}(\text{Icl}(\text{Icl}(A))))$ .

**Proof:** obvious.

**Corollary 4.9:** Let  $A = \langle X, A_1, A_2 \rangle$  be subset of ITS(X), then the following results are hold.

1.  $\text{Isc}(A) = A \cup \text{Iint}(\text{Icl}(A))$
2.  $\text{Isint}(A) = A \cap \text{Icl}(\text{Iint}(A))$
3.  $\text{Ipcl}(A) = A \cup \text{Icl}(\text{Iint}(A))$
4.  $\text{Ipsint}(A) = A \cap \text{Iint}(\text{Icl}(A))$ .

**Theorem 4.10:** Let  $A = \langle X, A_1, A_2 \rangle$  be subset of ITS(X). Then  $\text{Isint}(\beta\text{-cl}(A)) = \beta\text{-cl}(\text{Isint}(A)) = \text{Isc}(\text{Isint}(A))$ .

**Proof:** Let  $A = \langle X, A_1, A_2 \rangle$  be subset of ITS(X), then applying 4.8 we have  $\text{Isint}(\beta\text{-cl}(A)) = \beta\text{-cl}(A) \cap \text{Icl}(\text{Iint}(\beta\text{-cl}(A)))$

$$\begin{aligned} &= \beta\text{-cl}(A \cap \text{Icl}(\text{Iint}(A))) \\ &= \beta\text{-cl}(\text{Isint}(A)) \rightarrow (1). \end{aligned}$$

we take

$$\begin{aligned} \text{Isc}(\text{Isint}(A)) &= \text{Isint}(A) \cup \text{Iint}(\text{Icl}(\text{Isint}(A))) \\ &= \text{Isint}(A \cup \text{Iint}(\text{Icl}(A))) \\ &= \text{Isint}(A \cup \text{Icl}(\text{Iint}(\text{Icl}(A)))) \\ &= \text{Isint}(\beta\text{-cl}(A)) \rightarrow (2). \end{aligned}$$

From (1) and (2) we have

$$\begin{aligned} \text{Isint}(\beta\text{-cl}(A)) &= \beta\text{-cl}(\text{Isint}(A)) \\ &= \text{Isc}(\text{Isint}(A)). \end{aligned}$$

**Theorem 4.11:** Let  $A = \langle X, A_1, A_2 \rangle$  be subset of ITS(X). Then  $\text{Isc}(\beta\text{-int}(A)) = \beta\text{-int}(\text{Isc}(A)) = \text{Isint}(\text{Isc}(A))$ .

**Proof:** obvious.

**Theorem 4.12:** Let  $A = \langle X, A_1, A_2 \rangle$  be subset of ITS(X). Then  $\text{Ipcl}(\beta\text{-int}(A)) = \beta\text{-int}(\text{Ipcl}(A)) = (A \cup \text{Icl}(\text{Iint}(A))) \cap \text{Icl}(\text{Iint}(\text{Icl}(A)))$ .

**Proof:** Let  $A = \langle X, A_1, A_2 \rangle$  be subset of ITS(X), then we take

$$\begin{aligned} \text{Ipcl}(\beta\text{-int}(A)) &= \beta\text{-int}(A) \cup \text{Icl}(\text{Iint}(\beta\text{-int}(A))) \\ &= \beta\text{-int}(A \cup \text{Icl}(\text{Iint}(A))) \\ &= \beta\text{-int}(\text{Ipcl}(A)) \rightarrow (1). \\ \text{Take } \beta\text{-int}(\text{Ipcl}(A)) &= \text{Ipcl}(A) \cap \text{Iint}(\text{Icl}(\text{Iint}(\text{Ipcl}(A)))) \\ &= (A \cup \text{Icl}(\text{Iint}(A))) \cap \text{Iint}(\text{Icl}(\text{Iint}(\text{Ipcl}(A)))) \\ &= (\text{AUcl}(\text{Iint}(A))) \cap \text{Iint}(\text{Icl}(\text{Iint}(\text{Icl}(\text{Iint}(A)))) \\ &= (A \cup \text{Icl}(\text{Iint}(A))) \cap \text{Iint}(\text{Icl}(\text{Iint}(\text{Icl}(A)))) \\ &= (A \cup \text{Icl}(\text{Iint}(A))) \cap (\text{Icl}(\text{Iint}(\text{Icl}(A)))) \rightarrow (2) \end{aligned}$$

From (1) and (2)

$$\begin{aligned} \text{Ipcl}(\beta\text{-int}(A)) &= \beta\text{-int}(\text{Ipcl}(A)) \\ &= (\text{AUcl}(\text{Iint}(A))) \cap \text{Icl}(\text{Iint}(\text{Icl}(A))). \end{aligned}$$

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A.Singaravelan/ Research Scholar/ Post graduate and Research/ Department of Mathematics/  
Government Arts College/Coimbatore-641018/Tamilnadu/India/