FUZZY CO-FINITE FILTER

M. MUTHUKUMARI, A. NAGARAJAN, M. MURUGALINGAM

Abstract: We introduce the new concepts fuzzy co-finite filter and α fuzzy co-finite filter in a topological space.

Keywords: Fuzzy co finite filter, α Fuzzy co finite filter, Convergence.

Introduction: In a metric space sequence is a tool to characterize many concepts. In a topological space filter is an important tool to study many properties. The closure of a set A can be characterized using convergent filters containing A. The continuity of a function from one topological space to another can be characterized using convergent filters. In the year 1965 Lotfi A.Zadeh [2] introduced the concept of fuzzy sets and fuzzy logic.In the year 1968 C.L.Chang [1] introduced Fuzzy topological spaces. In the year 2014 We [3] introduced fuzzy sequences in a metric space.In the same year we [6] introduced fuzzy subsequences and limit points. Also we [7] defined disconvergent fuzzy sequences.In the same year we [4], [5] defined fuzzy net and fuzzy filter

In this paper we introduce fuzzy cofinite filter in a topological space and study the properties.

Fuzzy Co-finite Filter: First we recall the definition of fuzzy filter

Let X be a non empty set. A fuzzy set F: P(X) → [0,1] is called a fuzzy filter if

1. F(ϕ) = 0
2. F(A∩B) ≥ min { F(A),F(B) } for all A,B ⊂ X
3. A ⊂ B implies F(A) ≤ F(B) for all A,B ⊂ X

Definition 1: Fuzzy co-finite filter

Let X be an infinite set. A function F : P(X) → [0,1] is called a fuzzy co-finite filter, if F(A) = 1 if A^C is finite and F(A) = 0 otherwise.

Example 1: Let X=N.

Define F: P(N)→[0,1] as for any A⊂N, F(A) = 1 if A^c is finite and 0 otherwise.

Clearly F is a fuzzy co-finite filter.

Theorem 1: Fuzzy co-finite filter is a fuzzy filter.

Proof: Let X be any infinite set. F: P(X) → [0,1] is defined as F(A) = 1 if A^C is finite and 0 otherwise.

1. Consider ϕ ϵ P(X). $ϕ^C$ = X which is not finite. Hence F(ϕ) = 0.
2. Take A, B ϵ P(X). If F(A) = 0 and F(B) = 0 then whatever be the value of F(A∩B), we have F(A∩B) ≥ min {F(A), F(B)}. Now suppose F(A) = 1 and F(B) = 1, then A^C is finite and B^C is finite. Hence $A^C \cup B^C$ is finite which implies $(A\cap B)^C$ is finite. Hence F(A∩B) = 1. Hence F(A∩B) ≥ min {F(A), F(B)}. Now suppose F(A) = 0 and F(B) = 1, then min {F(A), F(B)} = 0. Hence F(A∩B) ≥ min {F(A), F(B)}.
3. Let A⊂B. Then $A^C \supset B^C$. If F(A) = 0 then whatever be the value of F(B), we have F(B) ≥ F(A). If F(A) = 1 then A^C is finite. This implies that B^C is finite. Hence F(B) = 1. Therefore F(A) ≤ F(B). Hence A ⊂ B implies F(A) ≤ F(B). Hence F is a fuzzy filter.

Result 1: Converse is not true.

A fuzzy filter need not be a fuzzy co-finite filter.

Example 2:

Let X=N. Define F: P(X) → [0.1] as F (A) = 1 if A^c is finite and 0.5 otherwise.

Clearly F is a fuzzy filter which is not co-finite.

Definition 2: α- fuzzy co-finite filter:

Let X be an infinite set. Let α ϵ (0,1).A fuzzy filter F on X is called a α-fuzzy co-finite filter if F(A) = 1 if A^C is finite and F(A) < α otherwise.

Result 2: It is clear that Fuzzy co-finite filter is an α-fuzzy co-finite filter for every α ϵ (0,1).

Example 3: Let X = N. Define F: P(N) → [0,1] as F(A) = 1 of A^C is finite, F(ϕ) = 0 and F(A) = 0.1 otherwise. It is clear that for α = 0.2. F is a α- fuzzy co-finite filter.

In the above example F is a α-fuzzy co-finite filter but it is not a fuzzy co-finite filter. Hence we get the result.

Result 3: A α- fuzzy co-finite filter need not be fuzzy co-finite filter.

Result 4: If F is a α fuzzy co finite filter and α ≤ β then F is a β fuzzy co finite filter.

Now we recall the definition of crisp filter.

Let X be a non empty set.F ⊂P(X) is called a crisp filter if

1. Φ ∉ F
2. F is closed under finite intersection.
3. A ⊂B and AϵF implies BϵF.

Now we recall the definition of crisp co-finite filter.

Let X be an infinite set. { A⊂X/ A^C is finite} is called the crisp co-finite filter.

Theorem 2: Let X be an infinite set. Let f be a fuzzy co-finite filter. Let αϵ (0,1).Then α cut of f is a crisp co-finite filter on X.

Proof: X is an infinite set. f : P(X) → [0,1] is a fuzzy co-finite filter. Let F= { A / f(A) ≥ α}. Then F= { A / f(A) =1}. Then F= { A / A^C is finite }. F is crisp co-finite filter on X.

Hence α cut of a fuzzy co-finite filter is a crisp co-finite filter.

Theorem 3: Let X be an infinite set. Let f be a α-fuzzy co-finite filter. Then α cut of f is a crisp co-finite filter on X.

Proof: X is an infinite set. f : P(X) → [0,1] is a α-fuzzy co-finite filter.
Let F= { A / f(A) ≥ α}. Then F= { A / f(A) =1}. Then F= { A / A^C is finite }. Fis crisp co-finite filter on X.
Hence α cut of a α-fuzzy co-finite filter is a crisp co-finite filter.
Now we recall convergence of crisp filter. A crisp filter F is said to converge to x if F contains all open sets containing x.
Now we recall convergence of fuzzy filter. A fuzzy filter F is said to converge to x if F(U)=1 for every open set containing x.
Theorem 4: Let X be an infinite set. Let f be a fuzzy co-finite filter. Let f converge to x.Then α cut of f converges to x.
Proof: X is an infinite set. f : P(X) → [0,1] is a fuzzy co-finite filter. Let F= { A / f(A) ≥ α}. F is the α cut of f. Also it is given that f converges to x. Let U be any open set containing x. Since f converges to x, we have f(U)=1.This implies f(U)≥α. Hence U∈F. Therefore F contains all open sets containing x. Hence F converges to x.
Theorem 5: Let X be an infinite set. Let f be a fuzzy co-finite filter. Let α cut of f converge to x.Then f converges to x.
Proof: X is an infinite set. f : P(X) → [0,1] is a fuzzy co-finite filter. Let F= { A / f(A) ≥ α}. F is the α cut of f. Also it is given that α cut of f converges to x. Let U be any open set containing x. Since α cut of f converges to x, we have U ϵ F. Hence f(U)≥α. Therefore f(U) =1.This implies that f converges to x.
Theorem 6: Let X be an infinite set. Let f be a fuzzy co-finite filter. Then f converges to x.iff α cut of f converges to x.
Proof: Follows from previous theorems.
Theorem 7: Let X be an infinite set. Let f be a α-fuzzy co-finite filter. Let f converge to x.Then α cut of f converges to x.
Proof: X is an infinite set. f : P(X) → [0,1] is a α-fuzzy co-finite filter.
Let F= { A / f(A) ≥ α}. F is the α cut of f. Also it is given that f converges to x. Let U be any open set containing x. Since f converges to x, we have f(U)=1.This implies f(U)≥α. Hence U∈F. Therefore F contains all open sets containing x. Hence F converges to x.
Theorem 8: Let X be an infinite set. Let f be a α-fuzzy co-finite filter. Let α cut of f converge to x.Then f converges to x.
Proof: X is an infinite set. f : P(X) → [0,1] is a α-fuzzy co-finite filter.
Let F= { A / f(A) ≥ α}. F is the α cut of f. Also it is given that F converges to x. Let U be any open set containing x. Since F converges to x, we have U∈F.This implies f(U)≥α. Hence f(U)=1. Therefore f converges to x.
Theorem 9: Let X be an infinite set. Let f be a α-fuzzy co-finite filter. Then f converges to x.iff α cut of f converges to x.
Proof: Follows from previous theorems.
Now we recall convergence of fuzzy filter at level α. A fuzzy filter F is said to converge to x at level α if F(U) ≥ α for every open set containing x.
Theorem 10: Let X be an infinite set. Let T be the co-finite topology in X. Then fuzzy co-finite filter on X converges to every point of X at level α.
Proof: (X,T) is a topological space X is infinite. T is co-finite topology. Take α ϵ (0,1]. Let F be the fuzzy co-finite filter. Then F(A) = 1 if A^C is finite and 0 otherwise. Take any x in X.
Let U be an open set containing x. U is open implies U^C is finite. Hence F(U) = 1. Hence F(U) ≥ α. Therefore for every open set U which contains x, we have F(U) ≥ α. Hence F converges to x at level α. This is true for every x ϵ X. Hence F converges to every point of X at level α.
Theorem 11: Let X be an infinite set. Let T be the co finite topology. Let F be α-fuzzy co-finite filter. Then F converges to every point of X at level α.
Proof: X is infinite set. T is co finite topology. α ϵ (0,1]. F(ϕ) = 0. For A≠ ϕ, F(A) = 1 if A^C is finite. If A is non empty and A^C is not finite then F(A) < α. Take any x ϵ X. Let U be an open set containing x. U is open implies U^C is finite. Since F is α fuzzy co finite filter and U^C is finite, F(U) = 1. Therefore F(U) ≥ α. Hence F converges to x at level α. This is true for every x ϵ X. Hence F converges to every point of X at level α.

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M. Muthukumari/ Research scholar/ V.O.C. College/ Tuticorin/ Tamilnadu/ India/
A. Nagarajan/Department of Mathematics/ V.O.C. College/ Tuticorin/ Tamilnadu/ India/
M. Murugalingam/ Department of Mathematics/ Thiruvalluvar College /Papanasam /Tamilnadu/ India/