.SOME PROPERTIES OF INTUITIONISTIC α - OPEN SET

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Abstract: In this paper a new class of sets called Iα – open set (Intuitionistic α–open set) is studied. Also Iαcl(Intuitionistic α-closure) and Iαint(Intuitionistic α-interior) are defined and some of their properties are established. Further a systematic study on Iα – open set is developed and various properties induced by them are obtained.

Introduction: Open sets play a vital role in general topology and they are now the research topics of many researchers worldwide. In 1965, O.Njastad[8] introduced the concept of α sets in topology. Andrijevic[1] has given some properties of the topology of α-sets. Later the concepts of intuitionistic sets and intuitionistic points were introduced by Coker[3].Intuitionistic space was introduced by Coker[5] and then studied by Younis J.Yaseen, Asmaa G. Raouf [9]. Gnanambal Ilango, S.Selvanayaki[6] introduced generalized preregular closed sets in Intuitionistic topological spaces. Gnanambal Ilango, and S.Girija[7] has given some more results on Intuitionistic sets. In this paper, Iα-sets are studied and their properties are investigated.

Preliminaries: Throughout this paper X denote a non-empty set and (X,τ) represents the ITS. In this section, we shall present the fundamental definitions and propositions.

Definition 2.1 [7]: An intuitionistic set (IS) A is an object having the form $<X,A_1,A_2>$ where A_1 and A_2 are subsets of X satisfying $A_1 \bigcap A_2 = \Phi$. The set A_1 is called the set of members of *A*, while A_2 is called the set of nonmembers of A. Furthermore, let $\{A_i : i \in I\}$ be an arbitrary family of IS's in *X*, where $A_i = < X , A_i^1, A_i^2>$ then

1. $\Phi_\sim=<X,\Phi,X>$,$X_\sim = < X,X, \Phi >$
2. $A \subseteq B$ if $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$
3. $\overline{A} = < X, A_2, A_1 >$
4. $A-B = A \bigcap \overline{B}$
5. $[\ ]A = < X , A_1, A_1^c >$
6. $< >A= < X , A_2^c, A_2 >$
7. $\bigcap A_i = < X ,\bigcap A_i^1 ,\bigcup A_i^2>$ and $\bigcup A_i = < X ,\bigcup A_i^1 , \bigcap A_i^2>$

Definition 2.2 [7]: An intuitionistic topological space (ITS) on a nonempty set X is a family τ of IS's in X satisfying the following axioms:

1. $\Phi_\sim , X_\sim \in \tau$
2. $G_1 \bigcap G_2 \in \tau$ for $G_1,G_2 \in \tau$
3. $\bigcup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J \} \subseteq \tau$.

In this case the pair (X,τ) is called intuitionistic topological space and any intuitionistic set in τ is known as an intuitionistic open set in X, and the complement of X is known as intuitionistic closed set in X.

Definition 2.3 [7]: Let (X,τ) be an intuitionistic topological space and $<X,A_1,A_2>$ be an intuitionistic set in X. Then the intuitionistic interior and intuitionistic closure of *A* are defined by

1. $Iint(A) = \bigcup \{G/G$ is an intuitionistic open set in X and $G \subseteq A\}$
2. $Icl(A) = \bigcap \{K/K$ is an intuitionistic in X and $A \subseteq K\}$

Definition 2.4 [3]: Let X be a nonempty set and $p \in X$ a fixed element in X. Then the IS p defined by $p =< X,\{p\},\{p^c\}>$ is called an intuitionistic point (IP) in X.

Definition 2.5 [3]: Let (X,τ) be an intuitionistic topological space and $< X, A_1, A_2 >$ be an intuitionistic set in *X*. Then the set A is called intuitionistic dense in X if Icl(A) =X. Also a subset A of an ITS of X is said to be nowhere dense if the intuitionistic closure of A contains no intuitionistic interior points.

Definition 2.6 [7]: A set A in an intuitionistic topological space (X,τ) will be termed intuitionistic semi-open if there exists an intuitionistic open set U such that $U \subseteq A \subseteq Icl(U)$. Also A is said to be an intuitionistic semi closed if its complement is intuitionistic semi-open.

Definition 2.7 [9]: A set A in an intuitionistic topological space(X,τ) is said to be intuitionistic g-closed (generalized closed set) if $Icl(A) \subseteq U$ whenever $A \subseteq U$ and U is Iopen in X.

Definition 2.8[9]: A set A in an intuitionistic topological space (X,τ) is said to be intuitionistic preopen if $A \subseteq Iint(Icl(A))$ and intuitionistic preclosed set if $Icl(Iint(A)) \subseteq A$

Definition 2.9 [9]: A set A in an intuitionistic topological space (X,τ) is said to be intuitionistic β-open (semi-preopen set) if $A \subseteq Icl(Iint(Icl(A)))$ and the intuitionistic β -closed set if $Iint(Icl(Iint(A))) \subseteq A$.

Intuitionistic α-Sets:

Definition 3.1: Let (X,τ) be a non-empty ITS and let $A = <X,A_1,A_2>$ be IS. Then A is said to be

1. Iα-gp closed if $I\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is Ipreopen in X.
2. $I(\alpha p)^*$closed if $I\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is Isemi-preopen in X.
3. Ig^* p- closed if $Ipcl(A) \subseteq U$ whenever $A \subseteq U$ and U is Ig-open in X.

4. I(sp)*-closed if Icl(A) $\subseteq$ U whenever A $\subseteq$ U and U is Isemipreopen in X.

Theorem 3.2: Every Iα–open set is Ipreopen in X

Proof: Let A be an Iα–open set.Then A $\subseteq$ Iint (Icl(Iint(A)) $\subseteq$ Iint(Icl(A)).Therefore A is an Ipreopen set.

The converse of the above statement is not true and is shown in the following example.

Example 3.3: Let X = {a ,b ,c}, τ = { ϕ , X , < X ,{a}, { b ,c }>,< X ,{a ,b},{c}>}.Then the intuitionistic subset < X,{c},{ ϕ } > is Ipreopen but not Iα-open.

Theorem 3.4: Every Iα – open set is Isemi-open set in X.

Proof: Let A be an Iα–open set then A $\subseteq$ Iint(Icl(Iint(A)) $\subseteq$ Icl(Iint(A)).Therefore A is an Isemi-open set.

The converse of the above statement is not true and is shown in the following example.

Example 3.5: Let X ={a ,b}, τ ={ ϕ , X ,< X ,{a},{ ϕ }>, <X,{a},{b}>,<X,{ ϕ },{b}>.Then intuitionistic subset < X ,{ ϕ } ,{b}> is Isemiopen but not Iα-open.

Theorem 3.6: Every Iα-open set is Iβ-open set in X

Proof: Let A be a Iα–open set. Since every Iα – open set is Isemi-open set this implies A $\subseteq$ Icl(Iint(A)) $\subseteq$ Icl(Iint(Icl(A))). Hence A is Iβ-open set.

The converse of the above statement is not true and is shown in the following example.

Example 3.7: Let X = {a,b} and τ={ ϕ ,X,< X,{a}, ϕ }> <X,{a},{b}>,<X, ϕ ,{b}>.Then intuitionistic set <X,{a}, ϕ > is Iβ-open but not Iα-open.

Theorem 3.8: Every Iα-closed set is Iα-gpclosed set in X

Proof: Let A be Iα-closed set and A $\subseteq$ U where U is preopen. Since A is Iα-closed set, Iαcl(A) = A $\subseteq$ U. Therefore A is Iα-gpclosed.

Converse of the above proposition need not be true and is shown in the following example.

Example 3.9: Let X={a,b,c} and τ = { ϕ , X ,< X ,{a}, {b,c} > ,< X ,{a,b},{c}>}.Then the intuitionistic set <X,{a},{c}> is Iα-gpclosed but not Iα-closed

Theorem 3.10: Every Iα-closed set is I(αp)*closed in X.

Proof: Let A be an Iα-closed set and A $\subseteq$ U where U is I semipreopen. Since A is Iα-closed set, Iαcl(A) = A $\subseteq$ U which implies Iαcl(A) $\subseteq$ U and U is Isemipreopen in X.

The converse of the above statement is not true and is shown in the following example.

Example 3.11: Let X={a,b,c} and τ = { ϕ , X ,< X, {a}, {b,c}>, < X ,{a ,b},{c} >}.Then the intuitionistic set < X,{b,c},ϕ> is I(αp)*closed but not Iα-closed set.

Theorem 3.12: Every Iα-closed set is Ig*p-closed set in X.

Proof: Let A be Iα-closed and A $\subseteq$ U where U is Ig-open. Since A is Iα-closed, Iαcl(A) =A $\subseteq$ U. Since Iα-closed set is Ipreclosed, Ipcl(A) $\subseteq$ Iαcl(A) $\subseteq$ U .Hence Ipcl(A) $\subseteq$ U.

Theorem 3.13: Every I(sp)*-closed set is I(αp)*-closed set in X.

Proof: Let A be a I(sp)*-closed set. So Icl(A) $\subseteq$ U whenever A $\subseteq$ U and U is Isemi-preopen. To prove that A is a I(αp)*-closed set, let A $\subseteq$ U and U is Isemi-preopen which implies Icl(A) $\subseteq$ U ,since A is I(sp)*-closed set. Since Iαcl(A) $\subseteq$ Icl(A) $\subseteq$ U $\Rightarrow$ Iαcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is Isemi-preopen. Therefore A is I(αp)*-closed set

Theorem 3.14: Every Iα-gpclosed set is Igp-closed set in X.

Proof: Let A be Iα-gpclosed set and A $\subseteq$ U where U is Ipreopen. Since A is Iα-gpclosed Iαcl(A) $\subseteq$ U. We know that Iα-closed set is Ipreclosed, Ipcl(A) $\subseteq$ Icl(A).Hence A is Igp-closed.

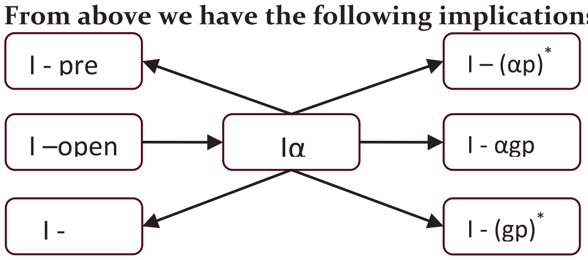
Theorem 3.15: Every Iαg-closed set is Iα-gpclosed set in X.

Proof: Let A be a Iαg-closed set in X and A $\subseteq$ U where U is Iopen. Since every Iopen set is Ipreopen and A is Iαg-closed, Iαcl(A) $\subseteq$ U. Hence A is Iα-gpclosed.

Theorem 3.16: Every Igα-closed set is Iα-gpclosed set in X

Proof: Let A be a Igα-closed set in X and A $\subseteq$ U where U is Iα-open. Since every Iα-open set is Ipreopen and A is Igα-closed, Iαcl(A) $\subseteq$ U. Hence A is Iα-gpclosed

From above we have the following implications:



Theorem 3.17: If A and B are Iα-gpclosed sets, then A $\cup$ B is Iα-gpclosed in X.

Proof: If A $\cup$ B $\subseteq$ U and U is Ipreopen then A $\subseteq$ U and B $\subseteq$ U. Since A and B are Iα-gpclosed, Iαcl(A) $\subseteq$ U , Iαcl(B) $\subseteq$ U and hence Iαcl(A) $\cup$ Iαcl(B) $\subseteq$ U which implies Iαcl(A $\cup$ B) $\subseteq$ U. Thus A $\cup$ B is Iα-gpclosed in X

Remark 3.18: Union of any two I(αp)*closed is I(αp)*closed and the union of any two Ig*p-closed is Ig*p-closed.

Theorem 3.19: If A is both Isemi-preopen and I(αp)*closed then A is Iα-closed.

Proof: Let A be both Isemi-preopen and $I(\alpha p)^*$closed. Let $A \subseteq A$, where A is Isemi-preopen then $I\alpha cl(A) \subseteq A$ as A is $I(\alpha p)^*$closed in (X,τ).But $A \subseteq I\alpha cl(A)$ is always true. Therefore A= Iαcl(A).Hence A is Iα-closed set in (X,τ)

Theorem 3.20: If A is an $I(\alpha p)^*$closed of (X,τ) such that $A \subseteq B \subseteq I\alpha cl(A)$,then B is $I(\alpha p)^*$closed in (X,τ)

Proof: Let U be an Isemi-preopen set such that $A \subseteq A$. We have to prove that $I\alpha cl(A) \subseteq U$.Then $I\alpha cl(A) \subseteq U$ since A is a $I(\alpha p)^*$closed. Also $B \subseteq I\alpha cl(A)$ this implies $I\alpha cl(B) \subseteq I\alpha cl(I\alpha cl(A)) = I\alpha cl(A) \subseteq U \Rightarrow I\alpha cl(B) \subseteq U$. Therefore B is an $I(\alpha p)^*$closed.

Properties of intuitionistic α – set:

Theorem 4.1: Let $\{A_s: s \in S\}$ be a family of Iα – open sets in an ITS (X,τ). Then $\bigcup A_s$ is also an Iα – open set.

Proof: Let $A_s = < X, A_s^1, A_s^2>$ be Iα – open set. Since each A_s is Iα – open for each $s \in S$, $\bigcup A_s \subseteq \bigcup [Iint(Icl(Iint\ A_s))] \subseteq Iint[\bigcup (Icl(Iint\ A_s))] \subseteq Iint[(Icl(\bigcup (Iint\ A_s))] \subseteq Iint[Icl(Iint\ (\bigcup A_s))]$. Hence $\bigcup A_s$ is an Iα – open set.

Proposition 4.2: Let A be a subset of a space X then

1. $I\alpha cl(A) \subseteq A \bigcup Icl[Iint(Icl(A))]$
2. $I\alpha int(A) \subseteq A \bigcap Iint[Icl(Iint(A))]$

Proof is obvious.

Lemma 4.3: For any subsets A,B of an ITS (X,τ) we have $Icl(Iint(Icl(A \bigcup B))) = Icl[Iint(Icl(A))] \bigcup Icl[Iint(Icl(B))]$

Proof: Since A is Iclosed we obtain

$Iint[Icl(A \bigcup B)] = Iint[Icl(A) \bigcup Icl(B)]$

$\subseteq Icl(A) \bigcup Iint(Icl(B))$

$\subseteq Icl(A) \bigcup IclIint(Icl(B))$. Hence we have

$Iint[Icl(A \bigcup B)] \subseteq Iint[Icl(A) \bigcup IclIint(B)]$ Since $Icl[Iint(B)]$ is Iclosed we get $Iint[Icl(A \bigcup B)] \subseteq Icl(Iint(Icl(A))) \bigcup IclIint(Icl(B))]$.

This implies $Icl\ [Iint\ [Icl(A \bigcup B)\ ]\ ]$

$\subseteq Icl[Iint[Icl(A)]] \bigcup Icl[Iint[Icl(B)]]$.

On the otherhand,

$Icl[Iint[Icl(A \bigcup B)] = Icl[Iint(Icl(A))] \bigcup Icl[Iint(Icl(B))]$.

Theorem 4.4: Let (X, τ) be an arbitrary ITS. Define $ICL(A)= A \bigcup Icl\ (Iint(Icl((A)))$ for every subset A of X. Then ICL is a closure operator on X.

Proof: From properties we know that $ICL(\phi) = I\phi$ and $A \subseteq ICL(A)$.

From theorem 4.3, $Icl[Iint[Icl(A \bigcup B)]$

$=Icl[Iint[Icl(A)]] \bigcup Icl[Iint[Icl(B)]]$

$ICL(A \bigcup B)$

$= (A \bigcup B) \bigcup [Icl[Iint[Icl(A \bigcup B)]]$

$= (A \bigcup B) \bigcup \{ [Icl[Iint[Icl(A)]] \bigcup [Icl[Iint[Icl(B)]]\}$

$= ICL(A) \bigcup ICL(B)$.

Finally we have, ICL ICL(A)

$= ICL[A \bigcup Icl(Iint(Icl((A)))]$

$=A \bigcup Icl[(Iint(Icl((A)))] \bigcup Icl[Iint(A \bigcup [Icl(Iint(Icl(A)))]$

$=A \bigcup Icl[(Iint(Icl((A)))] \bigcup Icl[Iint[Icl(A)]$

$= A \bigcup Icl[Iint[Icl(A)] = ICL(A)$.

Theorem 4.5: For every Iopen set U in an ITS (X,τ) and every $A \subseteq X$ we have $Iint[Icl(U \bigcap A)] = IintIcl(U) \bigcap IintIcl(A)$.

Proof: Let U be an Iopen set in (X,τ) ,

$U \bigcap IintIcl(A) \subset U \bigcap Icl(A) \subset Icl(U \bigcap A)$ Hence $Icl[U \bigcap IintIcl(A)] \subset Icl(U \bigcap A)$.Since $Iint(Icl(A))$ is Iopen we have $Iint(Icl(U)) \bigcap IintIcl(A) \subset Icl(U) \bigcap IintIcl(A) \subset Icl[U \bigcap Iint(A)]$.So $Iint(Icl(U)) \bigcap IintIcl(A) \subset Icl(U \bigcap A)$ which implies $Iint(Icl(U)) \bigcap IintIcl(A) \subset Iint(Icl(U \bigcap A))$.On the otherhand, $Iint(Icl(U \bigcap A)) \subset Iint[Icl(U) \bigcap Icl(A)] = Iint(Icl(U)) \bigcap Iint(Icl(A))$.

Relations between I-Closure and I-Interior operators in the (X, T) and (X, T_α): In this section, ICL and IINT refers to the intuitionistic closure and intuitionistic interior in (X, τ_α) and Icl, Iint refers to the intuitionistic closure and intuitionistic interior in (X, τ)

Lemma 5.1: For any intuitionistic set A of a ITS (X,τ) the following holds:

1. ICL(Iint(A)) = Icl(Iint(A))
2. Icl(IINT(A)) = Icl(Iint(A))
3. Iint(ICL(A)) = Iint(Icl(A))
4. IINT(Icl(A)) = Iint(Icl(A))

Proof: (i)$ICLIint(A) = Iint(A) \bigcup IclIint[IclIint(A)]$

$= Iint(A) \bigcup IclIint(A)$

$= IclIint(A)$

Similar to this we prove (ii) ,(iii) and (iv).

Theorem 5.2: For any subset A of a ITS (X,τ) we have

1. ICL(IINT(A)) = Icl(Iint(A))
2. IINT(ICL(A)) = Iint(Icl(A))

Proof: (i)$ICLIINT(A) \subset Icl(I\ INT(A))$

$= Icl(Iint(A))$

$= ICLIint(A)$

$\subset ICLIINT(A)$

Hence ICL(IINT(A)) = Icl(Iint(A)) similarly we prove (ii)

Acknowledgement: I thank my supervisor Dr.Gnanambal Ilango and Professsor S.Selvanayaki for guiding me to prepare this article.

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