DOMINATION AND GLOBAL DOMINATION IN GRAPH STRUCTURE

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Abstract: In this paper, a new domination parameter namely "Optimum domination number of a graph structure" has been introduced. Comparison of this with the already existing domination numbers has been done.

Keywords: graph structure, domination number, global domination, optimum domination

Introduction: Many domination parameters such as domination number, global domination number, etc. have been defined for generalized graph structures. Domination number for graph structures defined in [1] depends only on its underlying graph but not the structure representation given to the graph. This paper introduces a new domination parameter depending upon the underlying graph as well as the structure representation given to it.

Definition: A graph structure $G=(V;R_1,R_2,....,R_k)$ consists of a non-empty set V together with relations $R_1,R_2,...,R_k$ on V which are mutually disjoint such that each R_i, $1\leq i \leq k$, is symmetric and irreflexive.

Definition: In a graph structure $G=(V;R_1,R_2,....,R_k)$, if ab is an R_i-edge in G, we say that a R_i-dominates b, and conversely. A set D \ V is a dominating set if every vertex v in G is R_i dominated by some vertex u in D, for some i, $1\leq i \leq k$. Any R_i-edge going from D to V-D is called dominating edge with respect to D. The domination number of G, $\gamma(G)$ is a minimum cardinality of a domination set in G.

Definition: A dominating set D is R-full if each edge R_i, $1\leq i \leq k$, is a dominating edge with respect to D. The full domination number of G, $\gamma_f(G)$ is a minimum cardinality of a R-full dominating set in G. Clearly, $\gamma(G) \leq \gamma_f(G)$.

Definition: A set $D\subseteq V$ is a $R_1R_2....R_t$-dominating set in G if the set of dominating edges with respect to D is $\{R_1R_2....R_t\}$. Thus a R-full dominating set is a $R_1R_2....R_k$-dominaing set. The $R_1R_2....R_t$-domination number $\gamma(R_1R_2....R_t)(G)$ of G is a minimum cardinality of a $R_1R_2....R_t$-dominating set. Clearly, $\gamma(G) \leq \gamma(R_1R_2....R_t)(G)$.

Definition: Let D be a dominating set in G. Then D is called strong capacity dominating set if for every vertex v in V-D there is a vertex u in D adjacent to v such that $c(u) \geq c(v)$. The minimum cardinality of a strong capacity dominating set is called strong capacity domination number $\gamma_{sc}(G)$.

Definition: Let D be a dominating set in G. Then D is called weak capacity dominating set if for every vertex v in V-D there is a vertex u in D adjacent to v such that $c(u) \leq c(v)$. The minimum cardinality of a weak capacity dominating set is called weak capacity domination number $\gamma_{wc}(G)$.

Clearly, $\gamma(G) \leq \gamma_{sc}(G)$ and $\gamma(G) \leq \gamma_{wc}(G)$.

Definition: In a graph structure $G=(V;R_1,R_2,....,R_k)$, a dominating set D is a strong dominating set if for every vertex u in V-D there exists a vertex v in D adjacent to u such that deg v ≥ deg u.

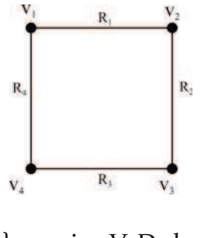
Definition: In a graph structure $G=(V;R_1,R_2,....,R_k)$, a dominating set D is a weak dominating set if for every vertex u in V-D there exists a vertex v in D adjacent to u such that deg v ≤ deg u.

Definition: A dominating set $D \subseteq V$ is a $R_1R_2....R_t$-global dominating set if for each i, $1\leq i \leq k$, every vertex in V-D is R_i dominated by some vertex v in D. Further if D is $R_1R_2....R_k$ -global dominating set then D is called full global dominating set. The $R_1R_2....R_t$ -global domination number $\gamma_g(R_1R_2....R_t)(G)$ of G is a minimum cardinality of a $R_1R_2....R_t$ -global dominating set. The full global domination number $\gamma_{fg}(G)$ of G is a minimum cardinality of a full global dominating set. A γ_{fg}\$-set is a minimum $R_1R_2....R_k$ -global dominating set.

Example: Every graph G can be considered as a R_1R_2-graph structure where R_1 is the set of edges in G and R_2 is the set of non adjacent pairs of vertices in G. A R_1R_2-global dominating set is nothing but the usual global dominating set.

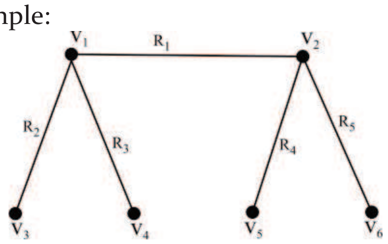
Note: If G is $R_1R_2....R_t$ -global then G is $R_{i1}R_{i2}....R_{ir}$-global, $2 \leq r \leq t$. If D is $R_{i1}R_{i2}....R_{ir}$ -global dominating set then V-D is $R_{i1}R_{i2}....R_{ir}$ -saturated.

Definition: Let $G=(V;R_1,R_2,....,R_k)$, be a graph structure. Let $G_i=G[R_i]$ be the subgraph induced by the R_i-edges in G. A set $D \subseteq V$ is an optimal dominating set of G if for every v in V-D, there exists v_i in D such that vv_i in R_i, whenever v in $V(G_i)$. The optimal domination number $\gamma_{op}(G)$ of G is the minimum cardinality of a optimal dominating set D of G.



$D=\{v_1,v_3\}$. v_2 in V-D has no match in R_3 and R_4. Therefore D is not a global dominating set. But D is an optimal dominating set.

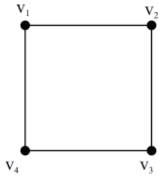
Example:



$D=\{v_1,v_2\}$ is a global dominating set. But D is a minimal optimal dominating set.

Therefore $\gamma(G)=\gamma_{op}(G)=2$.

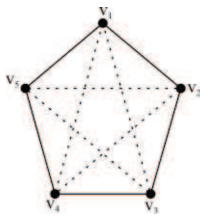
Example:



$R_1=\{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$; $R_2=\{v_1v_3, v_2v_4\}$.

$D=\{v_1, v_4\}$ is an optimal dominating set.

Example:



Consider the graph structure $(G; R_1, R_2)$ where each R_i is a Hamiltonian cycle in G.

$R_1=\{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_1\}$; $R_2=\{v_1v_3, v_3v_5, v_5v_2, v_2v_4, v_4v_1\}$.

$D=\{v_1, v_3, v_5\}$ is an optimal and global dominating set.

Therefore $\gamma_{fg}=\gamma_{op}=3$. But $\gamma=1$ and $\gamma(G_i)=2$, for $i=1,2$.

Theorem: Let $G=(V;R_1,R_2,....,R_k)$ be a graph structure. Let $G_i=G[R_i]$ be a subgraph induced by R_i-edges in G and D_i be a dominating set of G_i, $1\le i \le k$. Then $D=D_1 \cup D_2 \cup .... \cup D_k$ is an optimal dominating set of G.

Proof: Let $G=(V;R_1,R_2,....,R_k)$ be a graph structure. Let $G_i=G[R_i]$ be a subgraph induced by R_i-edges in G and D_i be the dominating set of G_i, $1\le i \le k$.

Let $D=D_1 \cup D_2 \cup .... \cup D_k$.

Let $v\in V(G)$-D and $v\in V(G_i)$.

$\Rightarrow v \notin D_i$. Then there exists $v_i \in D_i$ such that $vv_i \in R_i$.

Therefore D is an optimal dominating set of G.

Note: Let $\gamma_{op}(G)$ be the optimal domination number of G. Then $\gamma_{op}(G) \le |D| = |D_1 \cup D_2 \cup .... \cup D_k| \le \gamma_1 + \gamma_2 + ... + \gamma_k$.

Also, $\gamma(G) \le \gamma_{op}(G) \le \gamma_1+\gamma_2+....+\gamma_k$.

Theorem: Let G be a graph and G_s be the graph structure $(V;R_1=\{e_1\},R_2=\{e_2\},.....,R_k=\{e_k\})$. Then max\{d(v)/v \in V(G)-D\} $\le \gamma_{op}(G_s) \le k$.

Proof: Consider the graph structure $G_s=(V;R_1=\{e_1\},R_2=\{e_2\},.....,R_k=\{e_k\})$.

$v \in V(G)$-D is incident with d(v) edges. So it will appear in d(v) partitions and v is to be dominated by d(v) vertices. Therefore $\gamma_{op}(G_s) \ge \max\{d(v)/v \in V(G)-D\}$. i.e.) $k \ge \gamma_{op}(G_s) \ge$ max\{d(v)/v $\in$ V(G)-D}.

Theorem: Let D be an optimal dominating set for the graph structure $G=(V;R_1,R_2,....,R_k)$ and $D_i=D \cap V(G_i)$. Then D_i is a dominating set of G_i.

Proof: Let D be an optimal dominating set for the graph structure $G=(V;R_1,R_2,....,R_k)$.

Let $D_i=D \cap V(G_i)$.

$v \in V(G_i)-D_i \Rightarrow v \in V(G_i)$ and $v \notin D_i$.

$\Rightarrow v \in V(G)$ and $v \notin D$.

$\Rightarrow$ there exists v_i such that $vv_i \in R_i$.

Hence D_i is a dominating set of G_i.

Any graph can be naturally associated with three graph structures:

1. $G=(V; R)$ where k=1.
2. $G=(V; R_1,R_2,....,R_k)$ where k = number of edges in G.
3. $G=(V; R_1,R_2)$ where R_1 is the set of edges in G and R_2 is the set of non adjacent pairs of vertices in G.

Theorem: Let $G=(V;R_1,R_2,....,R_k)$ be any structure representation of a star graph. Then $\gamma = \gamma_{op} = 1$.

Proof: Let $G=(V;R_1,R_2,....,R_k)$ be any structure representation of a star graph. Let $G_i=G[R_i]$ be the subgraph induced by R_i-edges in G.

Let $V(G)=\{v_o,v_1,...,v_n\}$ and $E(G)=\{v_ov_i \,/\, 1 \le i \le n\}$.

$G_i = G[R_i] \Rightarrow$ Each G_i is a star with v_o as a common vertex

$\Rightarrow D_i=\{v_o\}$ is a minimum dominating set of G_i, $1 \le i \le k \Rightarrow D_{op}=\bigcup_{i=1}^{k} D_i=\{v_o\}$is an optimal dominating set of G

$\Rightarrow \gamma_{op}=1$

Clearly, $\gamma = 1$. Hence $\gamma = \gamma_{op} = 1$.

Note: $\gamma_{fg} = 1$, if k=1.

Let $G=(V; R_1,R_2)$ be a graph structure with R_1 as the set of edges in G and R_2 as the set of non adjacent pairs of vertices in G. Then $\gamma=1$; $\gamma_{fg} \neq 1$; $\gamma_{op} \ge 2$.

Proof: Let G be a graph which is not complete and G has no isolates. Let $G=(V; R_1,R_2)$ be the graph structure with R_1 as the set of edges in G and R_2 as the set of non adjacent pairs of vertices in G.

Then $R_1 \neq \phi$, $R_2 \neq \phi$.

Consider $D \subseteq V(G)$ and $u \in V$-D.

Choose $u \in D$.

Then \$v $\in R_1$ or $uv\in R_2$.

Therefore D is a dominating set of the structure $G=(V; R_1,R_2)$. Then $\gamma=1$.

Consider $D=\{u\}\subseteq V(G)$.

Let $v \in V$-D. Then $uv\notin R_1\cap R_2$.

Therefore D is not a global dominating set.

Therefore $\gamma_{fg} \neq 1$.

Let $V_1=V(G[R_1])$ and $V_2=V(G[R_2])$.

Note that if a vertex in G is of full degree, it is not in V_2 and V_1=V and any vertex which is not of full degree is in V_2.

Any optimal dominating set should intersect V_2.

If possible, let D = {u} where d(u) < n-1 be an optimal dominating set. There is at least one v($\neq$ u) such that d(u) < n-1. Then v $\in V_2$.

If uv$\in R_1$, then uv$\neq R_2$.

i.e, uv$\in R_1 \Rightarrow$uv$\neq R_2$.

Therefore D is not an optimal dominating set. Hence $\gamma_{op} \geq 2$.

Conclusion: This work can be extended to various graph structures.

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