DEPENDENT ELEMENTS OF SEMIDERIVATIONS ON SEMIPRIMESEMIRINGS

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Abstract: Laradji and Thaheem initiated the study of dependent elements of endomorphism of semiprime rings in "On dependent elements in semiprime rings" .In "On rings of operators" F.J.Murray and J.VonNeuman introduced the notions of dependent elements and free action. Choda, Kashahara and Nakamoto generalized the concept of freely acting automorphisms to C ∗algebras by introducing dependent elements associated to automorphisms in "Dependent elements of an automorphism of a C∗-algebra". Vukman and Ireana investigate some properties of dependent elements of derivations, Generalized derivations and automorphisms of prime and semiprime rings in On dependent elements in rings. M.S.Samman and M.Anwarhave studied some properties of dependent elements of left centralizers. Vukman ,KosiUlbl have made further study of dependent elements of various mappings related to automorphisms, derivations, (α, β) derivations and generalized derivations of semi-prime rings in On dependent elements and related problems in rings. Motivated by all these, in this paper we authors study and investigate dependent elements of semiderivations and generalized semiderivations on semprime semiring and also we study the semiderivation and generalized semiderivation of semiprime semiring are free actions.

In this paper we characterize dependent elements of semiderivations and generalized semiderivations on prime and semiprime semirings and also we study the semiderivation f and generalized semiderivation F on a semiprime semiring S are free actions.

Key words: Semiring, Prime semiring, Semiprime semiring, derivation, semiderivation, generalized semiderivation, dependent element, free action.

Introduction: This research has been motivated by the work of Laradji and Thaheem [1]. Throughout, S will represent an associative semiring with center Z(S). In [2] Josovukman and Irena Kosi Ulbl worked on dependent elements of derivations on rings.Dependent elements were implicitly used by Kallman [3] to extend the notion of free action of automorphisms of abelian von Neumann algebras of Murray and von Neumann [4,5]. They were later on introduced by Choda et al. [6]. Several other authors have studied dependent elements in operator algebras. A brief account of dependent elements in W∗-algebras has been also appeared in the book of Strˇatilˇa [7]. The purpose of this paper is to investigate dependent elements of some mappings related to semiderivations and generalized semiderivations on semiprime semirings. In this paper we characterize dependent elements of semiderivations and generalized semiderivations on semiprime semirings and also we study the semiderivation f and generalized semiderivation F on a semiprime semiring S are free actions.

Preliminaries:

Definition 2.1: A semiring S is a nonempty set S equipped with two binary operations $+$ and $\bullet$ such that

1. $(S,+)$ is a commutative monoid with identity element 0
2. $(S,\bullet)$ is a monoid with identity element 1
3. Multiplication left and right distributes over addition.

Definition 2.2: A semiring S is said to be prime if $xsy = 0$ implies $x = 0$ or $y = 0$ $for\, all\, x, y \in S.$

Definition 2.3: A semiring S is said to be semiprime if $xsx = 0$ implies $x = 0$ $for\, all\, x \in S.$

Definition 2.4: An additive mapping $d : S \to S$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds $for\, all\, x, y \in S$.

Definition 2.5: An additive mapping $f : S \to S$ is called a semiderivation associated with a function $g : S \to S$ if for all $x, y \in S$

1. $f(xy) = f(x)g(y) + xf(y) = f(x)y + g(x)f(y)$
2. $f(g(x)) = g(f(x))$.

If $g = I$, i.e., an identity mapping of S then all semiderivations associated with g are merely ordinary derivations. If g is any endomorphism of S, then semiderivations are of the form $f(x) = x - g(x)$.

Definition 2.6: Let S be a semiprime semiring, $F : S \to S$ be an additive map. If there is a semiderivation $f : S \to S$ associated with the function $g : S \to S$ such that

$$F(xy) = F(x)y + g(x)f(y) = f(x)g(y) + xF(y)$$

and $F(g(x)) = g(F(x))\, for\, all\, x, y \in S$

then (F, f) is called a generalized semiderivation associated with the function g and the semiderivation f.

Definition 2.7: Let S be a semiprime semiring. An element $a \in S$ is called a dependent element on a mapping $f : S \to S$ if $f(x)a = ax$ for all $x \in S$.

Definition 2.8: A mapping $F : S \to S$ is called a **free action** in case zero is the only element dependent on F.

Results:

Theorem 3.1: Let S be a semiprimesemiring with a non-zero semiderivation f associated with a function $g : S \to S$. Then f is a free action.

Proof: Let $a \in S$ be a dependent element of f. Hence we have to prove that $a = 0$.

Now, for all $x \in S$ and $a \in S$, we have the relation

$$f(x)a = ax \quad (1)$$

Put $xy\ for\ x$ in (1)

$$f(xy)a = axy\ for\ all\ x, y \in S$$
$$f(x)g(y)a + xf(y)a = axy,\ for\ all\ x, y \in S$$
$$f(x)g(y)a + xay = axy$$
$$f(x)g(y)a + (xa - ax)y = 0,\ for\ all\ x, y \in S \quad (2)$$

Put $yz\ for\ y$ in (2)

$$f(x)g(yz)a + (xa - ax)yz = 0,\ for\ all\ x, y, z \in S \quad (3)$$

Right multiply equation (2) by z and subtracting from (3) we get

$$f(x)g(y)(g(z)a - az) = 0,\ for\ all\ x, y, z \in S \quad (4)$$

Replace $y\ by\ ay$ in(4)

$$f(x)g(ay)(g(z)a - az) = 0$$
$$f(x)g(a)g(y)(g(z)a - az) = 0,\ for\ all\ x, y, z \in S$$
$$f(x)ay(za - az) = 0,\ \ since\ g\ is\ surjective$$
$$axy(za - az) = 0, by\ (1)\ for\ all\ x, y, z \in S \quad (5)$$

Replace $x\ by\ zx$ in (5)

$$azxy(za - az) = 0,\ for\ all\ x, y, z \in S \quad (6)$$

Left multiply equation (5) by z and subtracting from (6) we get

$$(az - za)xy(az - za) = 0,\ for\ all\ x, y, z \in S$$

Hence $(az - za)s(az - za) = 0,\ for\ all\ z \in S$

Since S is semiprime we get

$$(az - za) = 0,\ for\ all\ z \in S \quad (7)$$

Substituting (7) in (2) for all $z \in S$ we get

$f(z)g(y)a = 0$

Ie, $f(z)ya = 0\ since\ g\ is\ surjective.$

Putting $y = ay, f(z)aya = 0$

Ie, $azya = 0\ by\ (1)\ for\ all\ y, z \in S$

Hence $asa = 0$

By the semiprimeness of S we have $a = 0$.

Thus f is a free action.

Theorem 3.2: Let S be a semiprimesemiring, and F be a non-zero generalized semiderivation associated with a semiderivation f. Then F is a free action.

Proof: Let $a \in S$ be a dependent element of F. Hence we have to prove that $a = 0$.

Now, for all $x \in S$ and $a \in S$, we have the relation

$$F(x)a = ax \quad (1)$$

Put $xy\ for\ x$ in (1)

$$F(xy)a = axy, for\ all\ x, y \in S$$
$$F(x)ya + g(x)f(y)a = axy, for\ all\ x, y \in S$$
$$F(x)ya + g(x)ay = axy\ by\ (1)$$
$$F(x)ya + (g(x)a - ax)y = 0, for\ all\ x, y \in S \quad (2)$$

Put $yz\ for\ y$ in (2)

$$F(x)yza + (g(x)a - ax)yz = 0, for\ all\ x, y, z \in S \quad (3)$$

Right multiply equation (2) by z and subtracting from (3) we get

$$F(x)y(za - az) = 0, for\ all\ x, y, z \in S \quad (4)$$

Replace $y\ by\ ay$ in (4)

$$F(x)ay(za - az) = 0$$
$$axy(za - az) = 0 = 0, for\ all\ x, y, z \in S \quad (5)$$

Replace $x\ by\ zx$ in (5)

$$azxy(za - az) = 0, for\ all\ x, y, z \in S \quad (6)$$

Left multiply equation (5) by z and subtracting from (6) we get

$$(az - za)xy(az - za) = 0, for\ all\ x, y, z \in S$$

Hence $(az - za)s(az - za) = 0$

Since S is semiprime we get $(az - za) = 0$ (7)

Substituting (7) in (2) for all $z \in S$ we get

$F(z)ya = 0$

Putting $y = ay, F(z)aya = 0$

ie, $azya = 0\ by\ (1)\ for\ all\ y, z \in S$

Hence $asa = 0$

By the semiprimeness of S we have $a = 0$.

Thus F is a free action.

Theorem 3.3: Let S be a semiprimesemiring and F be a generalized semiderivation on S associated with a non zerosemiderivation f. If $a \in S$ and a is dependent element of F, then $a \in Z(S)$.

Proof: Since $a \in S$ and a is dependent element of F, we have $F(x)a = ax, for\ all\ x \in S$ (1)

Replacing $x\, by\, xy$ and using (1) we get

$$(F(x)a - xa)y = f(x)ya, for\, all\, x, y \in S \quad (2)$$

Right multiply (2) by z

$$(F(x)a - xa)yz = f(x)yaz, for\, all\, x, y, z \in S$$

Replacing $y\, by\, yz$ in (2) and subtracting we get

$$f(x)y(az - za) = 0, for\, all\, x, y, z \in S$$

Since S is prime and f is non zero we get $(az - za) = 0$ which means $a \in Z(S)\, for\, all\, z \in S$.

Corollary 3.4: Let S be a semiprimesemiring and let $a, b \in S$ be fixed elements. Suppose that c ∈ S is dependent element of $F(x) = ax + xb.$ Then $c \in Z(S)$

Proof: For all $x \in S$ we have $F(x)a=ax$, for all $x \in S$

Replacing $x\, by\, xy$

$$F(xy)a = axy, for\, all\, x, y \in S$$

Also we have $F(x) = ax + xb.$

$$F(xy) = axy + xyb$$
$$= axy + xyb - xby + xyb$$
$$= F(x)y + x(yb - by)$$
$$= F(x)y + g(x)f(y)$$
$$where\, x = g(x)\, and\, yb - by = f(y)$$

Hence F is a generalized derivation. Thus it follows from theorem 3 that c ∈ Z(S).

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