

ORTHOGONAL GENERALIZED SEMIDERIVATIONS OF SEMIPRIME SEMIRINGS

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Abstract: Motivated by some results on Orthogonal Generalized Derivations of Semiprime Rings, in [2], the authors defined the notion of derivations and generalized derivations on semirings and investigated some results on the derivations in semirings. In this paper, we also introduce the notion of orthogonal generalized semiderivations of semiprime semirings and derived some interesting results.

Keywords: Semirings, Derivations, Semiderivation, Orthogonal derivations, Generalized orthogonal derivations, Centralizer.

Introduction: This paper has been inspired by the work of Argac, Nakajima and Albas and also Mehsin Jabel Atteya[8] and [7], Throughout this paper S will represent a Semiring with the center $Z(S)$. Bresar and vukman[1] introduced the notation of Orthogonality for a pair d, g of derivations on a Semiprime Ring, and they contributed several necessary and sufficient conditions for d and g to be Orthogonal. Argac, Nakajima and Albas[5] introduced the notation of Orthogonality generalized for a pair D, G of derivations on 2-torsionfree Semiprime Ring, and extended the results of Orthogonal derivations to Orthogonal Generalized derivations. Majeed and Mehsin[6] proved the following result in his paper, if R is a 2-torsionfree Semiprime ring, (D, d) and (G, g) are generalized derivations of R such that R admits to satisfy $[d(x), g(x)] = 0$, for all $x \in R$ and d acts as a left centralizer (resp g acts as a left centralizer), then (D, d) and (G, g) are Orthogonal Generalized derivations of R . In this paper we study and investigate some interesting results concerning a non-zero generalized semiderivations with left cancellation property on semiprime semiring S , when the non-zero additive mapping acts as a left centralizer of S .

Preliminaries: Definition: 2.1:

A Semiring $(S, +, \cdot)$ is a non-empty set S together with two binary operations, $+$ and \cdot such that

1. $(S, +)$ and (S, \cdot) are a Semigroup.
2. For all $a, b, c \in S$, $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(b + c) \cdot a = b \cdot a + c \cdot a$

Definition: 2.2: A semiring S is said to be 2-torsionfree if $2x = 0 \Rightarrow x = 0, \forall x \in S$.

Definition: 2.3: A semiring S is Prime if $xSy = 0 \Rightarrow x = 0$ or $y = 0, \forall x, y \in S$ and S is Semi Prime if $xSx = 0 \Rightarrow x = 0, \forall x \in S$.

Definition: 2.4: An additive map $d: S \rightarrow S$ is called a derivation if $d(xy) = d(x)y + x d(y), \forall x, y \in S$

Definition: 2.5: An additive map $d: S \rightarrow S$ is called left centralizer if $d(xy) = d(x)y, \forall x, y \in S$.

Definition: 2.6: Let d, g be two additive maps from S to S . They are said to be Orthogonal if $d(x)Sg(y) = 0 = g(y)Sd(x), \forall x, y \in S$.

Definition: 2.7: An additive mapping $D: S \rightarrow S$ is called a generalized derivation if there exists a

derivation $d: S \rightarrow S$ such that $D(xy) = D(x)y + x d(y), \forall x, y \in S$.

Definition: 2.8: Two generalized derivations (D, d) and (G, g) of S are called Orthogonal if $D(x)Sg(y) = 0 = G(y)Sd(x), \forall x, y \in S$

Definition: 2.9: An additive mapping $d: S \rightarrow S$ is called semiderivation associated with a function $d_1: S \rightarrow S, \forall x, y \in S$

1. $d(xy) = d(x)d_1(y) + x d(y) = d(x)y + d_1(x)d(y)$
2. $d(d_1(x)) = d_1(d(x))$

If $d_1 = I$, that is, an identity mapping of S then all semiderivations associated with d_1 are merely ordinary derivations. If d_1 is any endomorphism of S , then semiderivations are of the form $d(x) = x - d_1(x)$

Definition: 2.10: Let S be a semiprime semiring $D: S \rightarrow S$ be an additive. If there is a semiderivation $d: S \rightarrow S$ associated with the function $f: S \rightarrow S$ such that $D(xy) = D(x)y + f(x)d(y) = d(x)f(y) + x D(y)$ and $D(f(x)) = f(D(x)), \forall x, y \in S$, then (D, d) is called a generalized semiderivation associated with the function f and the semiderivation d .

We write $[x, y] = xy - yx$ and note that important identity $[xy, z] = x[y, z] + [x, z]y$ and $[x, yz] = y[x, z] + [x, y]z$

Orthogonal Generalized Derivations in Semirings:

Lemma: 3.1: Let S be a 2-torsion free semiprime semiring. Let D and G be two generalized semiderivations of S . If D and G are Orthogonal to g and d respectively, then

1. $dg = 0$ and DG is a left centralizer of S .
2. $gd = 0$ and GD is a left centralizer of S .

Proof:

(i) Since D and g are Orthogonal, we get $D(x)Sg(y) = 0, \forall x, y, s \in S$.

Replacing x by xr we get $D(xr)Sg(y) = 0, \forall x, y, r, s \in S$.

$\Rightarrow [D(x)r + d_1(x)d(r)]Sg(y) = 0 \Rightarrow D(x)rSg(y) + d_1(x)d(r)Sg(y) = 0$

Since d_1 is surjective, $D(x)rSg(y) + x d(r)Sg(y) = 0 \Rightarrow x d(r)Sg(y) = 0$ ($\because D$ and g are Orthogonal)

$\Rightarrow d(r)Sg(y) = 0$ ($\because S$ is Semiprime)

$\Rightarrow g(y)Sd(x) = 0$

$\therefore d$ and g are Orthogonal.

Therefore $dg = o$. Now we prove that DG is a left centralizer of S .

Since D is Orthogonal to g and G is Orthogonal to d we get, $D(x) s g(y) = o$ and $G(x) s d(y) = o$ so $Dg = o$ and $Gd = o, \forall x, y, s \in S$ (1)

Now $DG(xy) = D[G(xy)] = D[G(x)y + g_1(x)g(y)] = D[G(x)y] + D[g_1(x)g(y)]$

$= DG(x)y + d_1(G(x))d(y) + D(g_1(x))g(y) + d_1(x)dg(y)$

Since d_1 and g_1 are surjective, $DG(xy) = DG(x)y + G(x)d(y) + D(x)g(y) + xdg(y)$

$= DG(x)y$ (\because by(1))

$\therefore DG$ is a left centralizer of the Semiring. Similarly we shall prove (ii)

Lemma: 3.2: Let S be a Semiprime Semiring, d a non-zero semiderivation of S , and U a non-zero left ideal of S . If for some positive integers t_0, t_1, \dots, t_n , and all $x \in U$, the identity $[[\dots [[d(x^{t_0}), x^{t_1}], x^{t_2}], \dots] x^{t_n} = o$ holds, then either $d(U) = o$ or else $d(U)$ and $d(S)U$ are contained in a non-zero central ideal of S . In particular, if S is a prime semiring, then S is commutative.

Theorem: 3.3: Let S be a Semiprime Semiring with left Cancellation property, (D, d) and (G, g) be a non-zero generalized semiderivations of S , and U a non-zero ideal of S . If S admits to satisfy $[d(x), g(x)] = o, \forall x \in U$ and a non-zero d acts as a left centralizer (resp a non-zero g acts as a left centralizer), then S contains a non-zero central ideal.

Proof: We have $[d(x), g(x)] = o, \forall x \in S$

Replacing x by xy ,

$[d(x) d_1(y), g(xy)] + [x d(y), g(xy)] = o$
 $\Rightarrow d(x)[d_1(y), g(xy)] + [d(x), g(xy)]d_1(y) + x[d(y), g(xy)] + [x, g(xy)]d(y) = o$

$\Rightarrow d(x)[d_1(y), g(x)g_1(y)] + d(x)[d_1(y), xg(y)] + [d(x), g(x)g_1(y)]d_1(y) + [d(x), xg(y)]d_1(y) + x[d(y), g(x)g_1(y)] + x[d(y), xg(y)] + [x, g(x)g_1(y)]d(y) + [x, xg(y)]d(y) = o$

Since d_1 and g_1 are surjective, we have

$\Rightarrow d(x)[y, g(x)y] + d(x)[y, xg(y)] + [d(x), g(x)y]y + [d(x), xg(y)]y + x[d(y), g(x)y] + x[d(y), xg(y)] + [x, g(x)y]d(y) + [x, xg(y)]d(y) = o, \forall x, y \in S$

$\Rightarrow d(x)g(x)[y, y] + d(x)[y, g(x)y] + d(x)x[y, g(y)] + d(x)[y, x]g(y) + g(x)[d(x), y]y + [d(x), g(x)]y^2 + x[d(x), g(y)]y + [d(x), x]g(y)y + xg(x)[d(y), y] + x[d(y), g(x)]y + x^2[d(y), g(y)] + x[d(y), x]g(y) + g(x)[x, y]d(y) + [x, g(x)]yd(y) + x[x, g(y)]d(y) + [x, x]g(y)d(y) = o, \forall x, y \in U$

Replacing y by x and according to the relation $[d(x), g(x)] = o$

$d(x)[x, g(x)]x + d(x)x[x, g(x)] + g(x)[d(x), x]x + [d(x), x]g(x)x + xg(x)[d(x), x] + x[d(x), x]g(x) + [x, g(x)]xd(x) + x[x, g(x)]d(x) = o, \forall x \in U$

$\Rightarrow d(x)xg(x)x - d(x)g(x)x^2 + d(x)x^2g(x) - d(x)xg(x)x + g(x)d(x)x^2 - g(x)xd(x)x + d(x)xg(x)x - xd(x)g(x)x + xg(x)d(x)x - xg(x)xd(x) + xd(x)xg(x) - x^2d(x)g(x) + xg(x)xd(x) - g(x)x^2d(x) + x^2g(x)d(x) - xg(x)xd(x) = o, \forall x \in U$

$\Rightarrow d(x)x^2g(x) - d(x)g(x)x^2 + g(x)d(x)x^2 - g(x)xd(x)x + d(x)xg(x)x - xd(x)g(x)x + xg(x)d(x)x + xd(x)xg(x) - x^2d(x)g(x) - g(x)x^2d(x) + x^2g(x)d(x) - xg(x)xd(x) = o$
 Since $d(x)g(x) = g(x)d(x)$, then above equation become

$d(x)x^2g(x) - g(x)xd(x)x + d(x)xg(x)x + xd(x)xg(x) - g(x)x^2d(x) - xg(x)xd(x) = o, \forall x \in U$ (1)

Since d acts as a left centralizer, $d(x, x^2)g(x) - g(x)xd(x)x + d(x)xg(x)x + xd(x)xg(x) - g(x)x^2d(x) - xg(x)xd(x) = o, \forall x \in U$

$\Rightarrow d(x)x^2g(x) + xd(x^2)g(x) - g(x)xd(x)x - g(x)x^2d(x) + d(x)xg(x)x + xd(x)g(x)x + xd(x)xg(x) + x^2d(x)g(x) - g(x)x^2d(x) - xg(x)xd(x) = o$

$\Rightarrow xd(x^2)g(x) + xd(x)g(x)x + x^2d(x)g(x) - g(x)x^2d(x) = o, \forall x \in U$ (\because by(1)) (2)

$\Rightarrow xd(x)xg(x) + x^2d(x)g(x) + xd(x)g(x)x + x^2d(x)g(x) - g(x)x^2d(x) = o$

$\Rightarrow xd(x)xg(x) + 2x^2d(x)g(x) + xd(x)g(x)x - g(x)x^2d(x) = o, \forall x \in U$ (3)

Since $d(x)g(x) = g(x)d(x)$,

$xd(x)xg(x) + 2x^2g(x)d(x) + xg(x)d(x)x - g(x)x^2d(x) = o$ (4)

d acts as a left centralizer, $xd(x)xg(x) + 2x^2g(x)d(x) + xg(x)d(x)x - g(x)x^2d(x) = o$

$xd(x)xg(x) + x^2d(x)g(x) + 2x^2g(x)d(x) + xg(x)d(x)x + xg(x)xd(x) - g(x)x^2d(x) = o$

Using (4) in the above equation, $x^2d(x)g(x) + xg(x)xd(x) = o, \forall x \in U$ (5)

Since d acts as a left centralizer, $x^2d(xg(x)) + xg(x)xd(x) = o, \forall x \in U$

$\Rightarrow x^2d(x)g(x) + x^3d(g(x)) + xg(x)xd(x) = o, \forall x \in U$ (6)

Using (5) in (6) we get, $x^3d(g(x)) = o, \forall x \in U$ (7)

Right multiplying by r and d acts as a left centralizer, $x^3d(g(x)r) = o$

$\Rightarrow x^3d(g(x))r + x^3g(x)d(x) = o, \forall x, r \in U$

Using (7) we get, $x^3g(x)d(x) = o, \forall x \in U, r \in S$

Using left cancellation property of $x^3g(x)$ this relation reduces to $d(r) = o, \forall r \in S$

Left multiplying by $x, xd(r) = o, \forall x \in U, r \in S$

Again right multiplying the same relation by $x, d(r)x = o, \forall x \in U, r \in S$

Subtracting these relations with replacing r by x and using lemma 3.2, we obtain S contains a non-zero central ideal.

Theorem 3.4: Let S be a Semiprime Semiring with left Cancellation property, (D, d) and (G, g) be two non-zero generalized semiderivations of S , and U a non-zero ideal of S . If S admits to satisfy $[D(x), G(x)] = o, \forall x \in U$ and a non-zero d acts as a left centralizer (resp a non-zero g acts as a left centralizer), then S contains a non-zero central ideal.

Proof: The theorem is nothing to prove if we replace d by D and g by G and use the generalization property in the above theorem.

Theorem 3.5: Let S be a Semiprime Semiring with left Cancellation property, (D, d) and (G, g) be two non-zero generalized semiderivations of S , and U a

non-zero ideal of S . If S admits to satisfy $[D(x), G(x)] = [d(x), g(x)]$, $\forall x \in U$ and a non-zero d acts as a left centralizer (resp a non-zero g acts as a left centralizer), then S contains a non-zero central ideal.

Proof: Given $[D(x), G(x)] = [d(x), g(x)]$, $\forall x \in U$ (1)

Then $[D(x), G(x)] = d(x)g(x) - g(x)d(x)$, $\forall x \in U$
 d acts as a left centralizer,

$$[D(x), G(x)] = d(xg(x)) - g(x)d(x), \forall x \in U$$

$$= d(x) d_1(g(x)) + xd(g(x)) - g(x)d(x), \forall x \in S$$

Since d_1 is surjective, then $[D(x), G(x)] = d(x) g(x) + xd(g(x)) - g(x)d(x)$, $\forall x \in S$

$$[D(x), G(x)] = [d(x), g(x)] + xd(g(x)), \text{ Using (1),}$$

$$xd(g(x)) = 0, \forall x \in U \quad (2)$$

Right multiplying by y and since d acts as a left centralizer, $xd(g(x), y) = 0$, $\forall x, y \in U$

$$xd(g(x)) d_1(y) + xg(x)d(y) = 0, \forall x, y \in U$$

Since d_1 is surjective, $xd(g(x))y + xg(x)d(y) = 0$, $\forall x, y \in U$, By (2), $xg(x)d(y) = 0$, $\forall x, y \in U$

Using left cancellation property of $xg(x)$ this relation reduces to $d(y) = 0$, $\forall y \in U$

By the same method in Theorem 3.3, we complete our proof.

Theorem 3.6: Let S be a Semiprime Semiring with left Cancellation property, (D, d) and (G, g) be two non-zero generalized semiderivations of S , and U a non-zero ideal of S . If S admits to satisfy $[D(x), G(x)] = [d(x), g(x)]$, $\forall x \in U$ and a non-zero D acts as a left centralizer (resp a non-zero G acts as a left centralizer), then S contains a non-zero central ideal.

Proof: Given $[D(x), G(x)] = [d(x), g(x)]$, $\forall x \in U$

Replacing x by xy ,

$$D(xy)G(xy) - G(xy)D(xy) = [d(xy), g(xy)], \forall x, y \in U$$

$$D(xy)G(x)y + D(xy) f_1(x) g(y) - G(x)yD(xy) - f_1(x) g(y)D(xy) = [d(xy), g(xy)], \forall x, y \in S$$

Since f is surjective,

$$D(xy)G(x)y + D(xy) xg(y) - G(x)yD(xy) - xg(y)D(xy) = [d(xy), g(xy)], \forall x, y \in S$$

$$D(x)yG(x)y + D(x)y xg(y) - G(x)yD(x)y - xg(y)D(x)y = [d(xy), g(xy)], \forall x, y \in U$$

$$[D(x)y, G(x)y] + D(x)y xg(y) - xg(y)D(x)y = [d(xy), g(xy)], \forall x, y \in U$$

$$G(x)[D(x)y, y] + [D(x)y, G(x)]y + D(x)y xg(y) - xg(y)D(x)y = [d(xy), g(xy)], \forall x, y \in U$$

$$G(x)D(x)y^2 - G(x)yD(x)y + D(x)yG(x)y - G(x)D(x)y^2 + D(x)y xg(y) - xg(y)D(x)y = [d(xy), g(xy)], \forall x, y \in U$$

$$D(x)yG(x)y - G(x)yD(x)y + D(x)y xg(y) - xg(y)D(x)y = [d(xy), g(xy)], \forall x, y \in U \quad (1)$$

Since D acts as a left centralizer

$$D(xy)G(x)y - G(x)yD(xy) + D(xy)xg(y) - xg(y)D(xy) = [d(xy), g(xy)], \forall x, y \in U$$

$$\Rightarrow D(x)yG(x)y - G(x)yD(x)y + D(xy)xg(y) - xg(y)D(xy) = [d(xy), g(xy)], \forall x, y \in U$$

$$\Rightarrow D(x)yG(x)y - G(x)yD(x)y + D(x)y xg(y) + d_1(x)d(y)xg(y) - xg(y)D(xy) = [d(xy), g(xy)]$$

Using (1) and d_1 is surjective, $xd(y)xg(y) = 0$, $\forall x, y \in U$

By using left Cancellation property of $xd(y)x$, this relation reduces to $g(y) = 0$, $\forall x \in U$.

By the same argument used in theorem 3.3, we complete our proof.

Theorem 3.7: Let S be a Semiprime Semiring with Cancellation property, (D, d) and (G, g) be two generalized semiderivations of S , and U a non-zero ideal of S . If S admits to satisfy $[D(x), G(x)] = [d(x), g(x)]$, $\forall x \in U$ and D and a non-zero g act as a left centralizers (resp a G and a non-zero d acts as a left centralizers), then S contains a non-zero central ideal.

Proof: Given $[D(x), G(x)] = [d(x), g(x)]$, $\forall x \in U$

Replacing x by xy and Since D and g acts as a left centralizers, we obtain

$$D(x)[y, G(xy)] + [D(x), G(xy)]y = g(x)[d(xy), y] + [d(xy), g(x)]y, \forall x, y \in U$$

$$\Rightarrow D(x)[y, G(x)y] + D(x)[y, f_1(x)g(y)] + [D(x), G(x)y]y + [D(x), f_1(x)g(y)]y = g(x)[d(x) d_1(y), y] + g(x)[xd(y), y] + [d(x) d_1(y), g(x)]y + [xd(y), g(x)]y, \forall x, y \in U$$

Since d_1 and g_1 are surjective, we have

$$\Rightarrow D(x)[y, G(x)]y + D(x)x[y, g(y)] + D(x)[y, x]g(y) + G(x)[D(x), y]y + [D(x), G(x)]y^2 + x[D(x), g(y)]y + [D(x), x]g(y)y = g(x)[d(x), y]y + g(x)x[d(y), y] + g(x)[x, y]d(y) + d(x)[y, g(x)]y + [d(x), g(x)]y^2 + x[d(y), g(x)]y + [x, g(x)]d(y)y, \forall x, y \in U$$

Replacing y by x and according to $[D(x), G(x)] = [d(x), g(x)]$ we obtain

$$D(x)[x, G(x)] + D(x)x[x, g(x)] + G(x)[D(x), x]x + x[D(x), g(x)]x + [D(x), x]g(x)x = g(x)[d(x), x]x + g(x)x[d(x), x] + d(x)[x, g(x)]x + x[d(x), g(x)]x + [x, g(x)]d(x)x, \forall x \in U$$

$$\Rightarrow D(x)xG(x)x - D(x)G(x)x^2 + D(x)x^2g(x) - D(x)xg(x)x + G(x)D(x)x^2 - G(x)xD(x)x + xD(x)g(x)x - xg(x)D(x)x + D(x)xg(x)x - xD(x)g(x)x = g(x)d(x)x^2 - g(x)xd(x)x + g(x)xd(x)x - g(x)x^2d(x) + d(x)xg(x)x - d(x)g(x)x^2 + xd(x)g(x)x - xg(x)d(x)x + xg(x)d(x)x - g(x)xd(x)x$$

According to $[D(x), G(x)] = [d(x), g(x)]$,

$$D(x)xG(x) + D(x)x^2g(x) - G(x)xD(x)x - xg(x)D(x)x + [d(x), g(x)]x^2 = [g(x), d(x)]x^2 - g(x)x^2d(x) + d(x)xg(x)x + xd(x)g(x)x - g(x)xd(x)x$$

$$\Rightarrow D(x)xG(x^2) + (x)x^2g(x) - G(x^2)D(x)x - xg(x)D(x)x = -g(x)x^2d(x) + d(x)xg(x)x + xd(x)g(x)x - g(x)xd(x)x \quad (1)$$

$$D(x)xG(x^2) - G(x^2)D(x)x = -g(x^3)d(x) + d(x)xg(x) + xd(x)g(x^2) - g(x^2)d(x)x, \forall x \in U$$

Since D acts as a left centralizer,

$$D(x)xG(x^2) - G(x^2)D(x)x = -g(x^3)d(x) + d(x)xg(x) + xd(x)g(x^2) - g(x^2)d(x)x, \forall x \in U \quad (2)$$

Since g acts as a left centralizer, (2) becomes

$$D(x)xG(x^2) - G(x^2)D(x)x = -g(x^2)xd(x) + d(x)xg(x)x + xd(x)g(x)x - g(x)xd(x)x, \forall x \in U \quad (3)$$

Substituting (1) in (3),

$$-g(x)x^2d(x) = -g(x^2)xd(x) \quad \forall x \in U$$

Then $xg(x)xd(x) = 0$, $\forall x \in U$

By using left cancellation property of $xg(x)x$ this relation reduces to $d(x) = 0$, $\forall x \in U$

By the same argument used in theorem 3.3, we complete our proof.

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