* - ALUTHGE TRANSFORMATION ON POWERS OF N-CLASS A_k OPERATORS

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Abstract: In this paper, we study * - Aluthge transformation on powers of N- class A_k operators. And we have proved adjoint of * - aluthge transformation on powers of N- class A_k operators .

Keywords: Aluthge Transformations, * - Aluthge transformations, powers of N- class $\boldsymbol{A_k}$ operators .

Introduction: Let $B(H)$ be a bounded linear operator on a Hilbert space$H$. An operator T is said to positive if $\langle Tx, x\rangle \geq 0$ for all $x \in H$.In [2] Furuta introduced class $A(k)$ and absolute – k – paranormal operators. In [4] have defined and studied new classes of A_k operators and N – class $A(k)$ operators. In [5] defined and studied powers of N – class A_k operators. In [1] Aluthge has defined a transformation $\tilde{T}$ of T as$\tilde{T} = |T|^{\frac{1}{2}}\, U\, |T|^{\frac{1}{2}}$, where $\tilde{T}$ is the Aluthge transformation. In [6] Yamazaki has defined * - Aluthge transformation and have discussed some properties of * - Aluthge transformation.

*** - Aluthge Transformation on Powers of N- class $\boldsymbol{A_k}$ operators:**

Definition 2.1[4]: An operator $T \in B(H)$ is powers of N-class A_k if $|T|^{2p} \leq N|T^{k+1}|^{\frac{2p}{k+1}}$ for a fixed $N > 0$ and $0 < p \leq 1$.

Proposition 2.2[4]: If $T \in B(H)$ is powers of N-class $A_k$operator if $|T|^{2p} \leq N|T^{k+1}|^{\frac{2p}{k+1}}$ for a fixed $N > 0$ and $0 < p \leq 1$.

1. If p = 1 then the operator is N – class A_k operator.
2. If p = 1 and k = 1 then the operator is N- class A operator.
3. If N = 1 and p = 1 then the operator is class A_k operator.
4. If N = 1, p = 1 and k = 1 then the operator is class A operator.

Theorem 2.3[4]: Let $T = U|T| \in B(H)$ be the polar decomposition of$T$. Then T is a powers of N-class A_k operator if and only if $\||T|^p x\| \leq N \||T|^{k+1} x\|^{\frac{p}{k+1}}$.

Definition 2.4: Let $T = U|T|$ be the polar decomposition of an operator T then adjoint of * - Aluthge transformation T is $\tilde{T}^{(*)} = |T^*|^{\frac{1}{2}}\, U\, |T^*|^{\frac{1}{2}}$.

Definition 2.5: Let $T = U|T|$ be the polar decomposition of an operator T then adjoint of * - Aluthge transformation T is $\left(\tilde{T}^{(*)}\right)^* = |T^*|^{\frac{1}{2}}\, U^*\, |T^*|^{\frac{1}{2}}$.

Theorem 2.6[3]: Let $T = U|T|$ be the polar decomposition of an operator$T$, and $\tilde{T} = |T|^{\frac{1}{2}}\, U\, |T|^{\frac{1}{2}}$ is the Aluthge transformation$\left\|\tilde{\tilde{T}}\right\| = \left\|\tilde{\tilde{T}}^{(*)}\right\|$.

Proposition 2.7[3]: If T is a bounded linear operator on a Hilbert space then we know that

1. $T = U|T| = |T^*|\, U$ is the polar decomposition of an operator T.
2. $T^* = U^*|T^*| = |T|U^*$ is the polar decomposition of an operator T.
3. If $\tilde{T} = |T|^{\frac{1}{2}}\, U\, |T|^{\frac{1}{2}}$ is the Aluthge transformation then adjoint transformation $\tilde{T}^*$ is given by $\tilde{T}^{(*)} = \left(\tilde{T}^*\right)^* = |T|^{\frac{1}{2}}\, U^*\, |T|^{\frac{1}{2}}$.
4. If $\tilde{T}^{(*)} = |T^*|^{\frac{1}{2}}\, U\, |T^*|^{\frac{1}{2}}$and $\left(\tilde{T}^{(*)}\right)^* = \left(\left(\tilde{T}^*\right)^*\right)^* = |T^*|^{\frac{1}{2}}\, U^*\, |T^*|^{\frac{1}{2}}$

Theorem 2.8: Let $T = U|T| \in B(H)$ is powers of N-classA_k and U is isometry operator then $\tilde{T}$ is powers of N- classA_k operator.

Proof: From the definition of powers of N- classA_k operator $|T|^{2p} \leq N|T^{k+1}|^{\frac{2p}{k+1}}$

$$(T^*\,T)^p \leq N\,((T^*\,T)^{k+1})^{\frac{p}{k+1}}$$

$$(U^*|T^*|\,U|T|)^p \leq N\,((U^*|T^*|\,U|T|)^{k+1})^{\frac{p}{k+1}}$$

$$\left(U^*|T^*|^{\frac{1}{2}}\,|T^*|^{\frac{1}{2}}\,U|T|^{\frac{1}{2}}\,|T|^{\frac{1}{2}}\right)^p$$

$$\leq N\left(\left(U^*|T^*|^{\frac{1}{2}}\,|T^*|^{\frac{1}{2}}\,U|T|^{\frac{1}{2}}\,|T|^{\frac{1}{2}}\right)^{k+1}\right)^{\frac{p}{k+1}}$$

$$U\left(\left(\tilde{T}^*\right)^*\tilde{T}^*\right)^p U^* \leq N\,U\left(\left(\left(\tilde{T}^*\right)^*\tilde{T}^*\right)^{k+1}\right)^{\frac{p}{k+1}} U^*$$

$$U\left|\tilde{T}^*\right|^{2p}U^* \leq N\,U\left|\left(\tilde{T}^*\right)^{2(k+1)}\right|^{\frac{p}{k+1}}U^*$$

$$\left|\tilde{T}\right|^{2p} \leq N\left|\left(\tilde{T}\right)^{k+1}\right|^{\frac{2p}{k+1}}$$

Therefore$\tilde{T}$ is powers of N- classA_k operator

Theorem 2.9: Let $T \in B(H)$ and $\tilde{T}$ is powers of N-classA_k operator then $\tilde{T}^*$ is powers of N- classA_k operator.

Proof: From the definition of powers of N- classA_k operator $|T|^{2p} \leq N|T^{k+1}|^{\frac{2p}{k+1}}$

$$(T^*\,T)^p \leq N\,((T^*\,T)^{k+1})^{\frac{p}{k+1}}$$

$$\left(\tilde{T}^*\,\tilde{T}\right)^p \leq N\left(\left(\tilde{T}^*\,\tilde{T}\right)^{k+1}\right)^{\frac{p}{k+1}}$$

$$\left(|T|^{\frac{1}{2}}\,U^*\,|T|^{\frac{1}{2}}\,|T|^{\frac{1}{2}}\,U\,|T|^{\frac{1}{2}}\right)^p$$

$$\leq N\left(\left(|T|^{\frac{1}{2}}\,U^*\,|T|^{\frac{1}{2}}\,|T|^{\frac{1}{2}}\,U\,|T|^{\frac{1}{2}}\right)^{k+1}\right)^{\frac{p}{k+1}}$$

$$\left(U^*|T^*|^{\frac{1}{2}}\,|T|\,|T^*|^{\frac{1}{2}}\,U\right)^p$$

$$\leq N\left(\left(U^*|T^*|^{\frac{1}{2}}\,|T|\,|T^*|^{\frac{1}{2}}\,U\right)^{k+1}\right)^{\frac{p}{k+1}}$$

$$U^*\left(|T^*|^{\frac{1}{2}}\,U^*|T^*|\,U\,|T^*|^{\frac{1}{2}}\right)^p U$$

$$\leq NU^* \left(\left(|T^*|^{\frac{1}{2}} \; U^*|T^*| \; U \; |T^*|^{\frac{1}{2}}\right)^{k+1}\right)^{\frac{p}{k+1}} U$$

$$U^*\left(|T^*|^{\frac{1}{2}} \; U^*|T^*|^{\frac{1}{2}} \; |T^*|^{\frac{1}{2}} U \; |T^*|^{\frac{1}{2}}\right)^{p} U$$

$$\leq NU^* \left(\left(|T^*|^{\frac{1}{2}} \; U^*|T^*|^{\frac{1}{2}} |T^*|^{\frac{1}{2}} \; U \; |T^*|^{\frac{1}{2}}\right)^{k+1}\right)^{\frac{p}{k+1}} U$$

$$U^*\left(|T^*|^{\frac{1}{2}} \; U^*|T^*|^{\frac{1}{2}} \; |T^* \; |^{\frac{1}{2}}U \; |T^*|^{\frac{1}{2}}\right)^{p} U$$

$$\leq NU^* \left(\left(|T^*|^{\frac{1}{2}} \; U^*|T^*|^{\frac{1}{2}} |T^*|^{\frac{1}{2}} \; U \; |T^*|^{\frac{1}{2}}\right)^{k+1}\right)^{\frac{p}{k+1}} U$$

$$U^*\left(\left(\tilde{T}^{(*)}\right)^* \tilde{T}^{(*)}\right)^p U \; \leq N \; U^*\left(\left(\left(\tilde{T}^{(*)}\right)^* \tilde{T}^{(*)}\right)^{k+1}\right)^{\frac{p}{k+1}} U$$

$$U^*\left|\tilde{T}^{(*)}\right|^{2p} U \leq N \; U^* \; \left|\left(\tilde{T}^{(*)}\right)^{2(k+1)}\right|^{\frac{p}{k+1}} U$$

$$U^*\left|\left(\tilde{T}^*\right)^*\right|^{2p} U \leq N \; U^* \; \left|\left(\left(\tilde{T}^*\right)^*\right)^{2(k+1)}\right|^{\frac{p}{k+1}} U$$

$$\left|\tilde{T}^*\right|^{2p} \leq N\left|\left(\tilde{T}^*\right)^{k+1}\right|^{\frac{2p}{k+1}}$$

Therefore $\tilde{T}^*$ is powers of N- classA_k operator.

Theorem 2.10: Let $T \in B(H)$ and$\tilde{T}^*$ is powers of N-classA_k operator then $\left(\tilde{T}^{(*)}\right)^*$ is powers of N- classA_k operator.

Proof: From the definition of powers of N- classA_k operator $|T|^{2p} \leq N|T^{k+1}|^{\frac{2p}{k+1}}$

$$(T^* \, T)^p \; \leq N \; ((T^* \, T)^{k+1})^{\frac{p}{k+1}}$$

$$\left(\left(\tilde{T}^*\right)^* \tilde{T}^*\right)^p \; \leq N \; \left(\left(\left(\tilde{T}^*\right)^* \tilde{T}^*\right)^{k+1}\right)^{\frac{p}{k+1}}$$

$$\left(\; |T^*|^{\frac{1}{2}} U \; |T^*|^{\frac{1}{2}} \; |T|^{\frac{1}{2}} U^* \; |T|^{\frac{1}{2}}\right)^p$$

$$\leq N \; \left(\left(\; |T^*|^{\frac{1}{2}} U \; |T^*|^{\frac{1}{2}} \; |T|^{\frac{1}{2}} U^* \; |T|^{\frac{1}{2}}\right)^{k+1}\right)^{\frac{p}{k+1}}$$

$$\left(U \; |T|^{\frac{1}{2}} \; |T^*|^{\frac{1}{2}} \; |T|^{\frac{1}{2}} \; |T^*|^{\frac{1}{2}} U^* \;\right)^p$$

$$\leq N \; \left(\left(\; U \; |T|^{\frac{1}{2}} \; |T^*|^{\frac{1}{2}} \; |T|^{\frac{1}{2}} \; |T^*|^{\frac{1}{2}} U^* \;\right)^{k+1}\right)^{\frac{p}{k+1}}$$

$$U\left(\; |T|^{\frac{1}{2}} \; U^*|T|^{\frac{1}{2}} U \; |T|^{\frac{1}{2}} \; |T^*|^{\frac{1}{2}}\right)^p U^*$$

$$\leq N \; U\left(\left(\; |T|^{\frac{1}{2}} U^* \; |T|^{\frac{1}{2}} U \; |T|^{\frac{1}{2}} \; |T^*|^{\frac{1}{2}} \;\right)^{k+1}\right)^{\frac{p}{k+1}} U^*$$

$$U\left(|T|^{\frac{1}{2}} \; U^*|T|^{\frac{1}{2}} \; |T^*|^{\frac{1}{2}} \; \mathrm{U} \; |T^*|^{\frac{1}{2}}\right)^p U^*$$

$$\leq N \; U\left(\left(|T|^{\frac{1}{2}} U^* \; |T|^{\frac{1}{2}} \; |T^*|^{\frac{1}{2}} \; \mathrm{U} \; |T^*|^{\frac{1}{2}} \;\right)^{k+1}\right)^{\frac{p}{k+1}} U^*$$

$$U\left(\tilde{T}^* \, \tilde{T}^{(*)}\right)^p U^* \; \leq N \; U\left(\left(\; \tilde{T}^* \; \tilde{T}^{(*)}\right)^{k+1}\right)^{\frac{p}{k+1}} U^*$$

$$U\left(\tilde{T}^* \, \left(\tilde{T}^*\right)^*\right)^p U^* \; \leq N \; U\left(\left(\; \tilde{T}^* \; \left(\tilde{T}^*\right)^*\right)^{k+1}\right)^{\frac{p}{k+1}} U^*$$

$$U \left|\left(\tilde{T}^*\right)^* \right|^{2p} U^* \leq N \; U\left|\left(\left(\tilde{T}^*\right)^* \right)^{2(k+1)}\right|^{\frac{p}{k+1}} U^*$$

$$\left|\left(\tilde{T}^{(*)}\right)^*\right|^{2p} \leq N\left|\left(\left(\tilde{T}^{(*)}\right)^*\right)^{2(k+1)}\right|^{\frac{p}{k+1}}$$

$$\left|\left(\tilde{T}^{(*)}\right)^*\right|^{2p} \leq N\left|\left(\left(\tilde{T}^{(*)}\right)^*\right)^{k+1}\right|^{\frac{2p}{k+1}}$$

Therefore $\left(\tilde{T}^{(*)}\right)^*$ is powers of N- class A_k operator.

Theorem 2.11: Let $T = U|T| \in B(H)$ is powers of N-classA_k and U is isometry operator if and only if $\left(\tilde{T}^{(*)}\right)^*$ is powers of N- classA_k operator.

Proof: Assume $\left(\tilde{T}^{(*)}\right)^*$ is powers of N- classA_k operator.

From the definition of powers of N- classA_k operator

$$|T|^{2p} \leq N|T^{k+1}|^{\frac{2p}{k+1}}$$

$$(T^* \, T)^p \; \leq N \; ((T^* \, T)^{k+1})^{\frac{p}{k+1}}$$

$$\left(\left(\left(\tilde{T}^{(*)}\right)^*\right)^* \left(\tilde{T}^{(*)}\right)^*\right)^p \leq N \; \left(\left(\left(\left(\tilde{T}^{(*)}\right)^*\right)^* \left(\tilde{T}^{(*)}\right)^*\right)^{k+1}\right)^{\frac{p}{k+1}}$$

$$\left(\; |T|^{\frac{1}{2}} U \; |T|^{\frac{1}{2}} \; |T^*|^{\frac{1}{2}} \; \mathrm{U}^* \; |T^*|^{\frac{1}{2}}\right)^p$$

$$\leq N \; \left(\left(\; |T|^{\frac{1}{2}} U \; |T|^{\frac{1}{2}} \; |T^*|^{\frac{1}{2}} \; \mathrm{U}^* \; |T^*|^{\frac{1}{2}}\right)^{k+1}\right)^{\frac{p}{k+1}}$$

$$\left(U \; |T^*|^{\frac{1}{2}} \; |T|^{\frac{1}{2}} \; |T^*|^{\frac{1}{2}} |T|^{\frac{1}{2}} \; \mathrm{U}^* \right)^p$$

$$\leq N \; \left(\left(U \; |T^*|^{\frac{1}{2}} \; |T|^{\frac{1}{2}} \; |T^*|^{\frac{1}{2}} |T|^{\frac{1}{2}} \; \mathrm{U}^*\right)^{k+1}\right)^{\frac{p}{k+1}}$$

$$U\left(\; |T^*|^{\frac{1}{2}} \, \mathrm{U} \;\; |T^*|^{\frac{1}{2}} |T|^{\frac{1}{2}} \, \mathrm{U}^* \; |T|^{\frac{1}{2}} \;\right)^p \mathrm{U}^*$$

$$\leq N \; U\left(\left(\; |T^*|^{\frac{1}{2}} \, \mathrm{U} \;\; |T^*|^{\frac{1}{2}} |T|^{\frac{1}{2}} \, \mathrm{U}^* \; |T|^{\frac{1}{2}} \;\right)^{k+1}\right)^{\frac{p}{k+1}} \mathrm{U}^*$$

$$U\left(\left(\tilde{T}^*\right)^* \tilde{T}^*\right)^p \mathrm{U}^* \; \leq N \; U\left(\left(\left(\tilde{T}^*\right)^* \tilde{T}^*\right)^{k+1}\right)^{\frac{p}{k+1}} \mathrm{U}^*$$

$$U \left|\tilde{T}^* \right|^{2p} U^* \leq N \; U\left|\left(\tilde{T}^* \right)^{2(k+1)}\right|^{\frac{p}{k+1}} U^*$$

$$\left|\tilde{T} \right|^{2p} \leq N\left|\left(\tilde{T} \right)^{k+1}\right|^{\frac{2p}{k+1}}$$

Assume that$T = U|T| \in B(H)$ is powers of N- classA_k and U is isometry operator.

From the definition of powers of N- classA_k operator

$$|T|^{2p} \leq N|T^{k+1}|^{\frac{2p}{k+1}}$$

$$(T^* \, T)^p \; \leq N \; ((T^* \, T)^{k+1})^{\frac{p}{k+1}}$$

$$(U^*|T^*| \; U|T|)^p \; \leq N \; ((U^*|T^*| \;\; U|T|)^{k+1})^{\frac{p}{k+1}}$$

$$\left(U^*|T^*|^{\frac{1}{2}} \, |T^*|^{\frac{1}{2}} \, U|T|^{\frac{1}{2}} \, |T|^{\frac{1}{2}}\right)^p$$

$$\leq N \; \left(\left(U^*|T^*|^{\frac{1}{2}} \, |T^*|^{\frac{1}{2}} \, U|T|^{\frac{1}{2}} \, |T|^{\frac{1}{2}}\right)^{k+1}\right)^{\frac{p}{k+1}}$$

$$U\left(\; |T^*|^{\frac{1}{2}} \, \mathrm{U} \;\; |T^*|^{\frac{1}{2}} |T|^{\frac{1}{2}} \, \mathrm{U}^* \; |T|^{\frac{1}{2}} \;\right)^p \mathrm{U}^*$$

$$\leq N \; U\left(\left(\; |T^*|^{\frac{1}{2}} \, \mathrm{U} \;\; |T^*|^{\frac{1}{2}} |T|^{\frac{1}{2}} \, \mathrm{U}^* \; |T|^{\frac{1}{2}} \;\right)^{k+1}\right)^{\frac{p}{k+1}} \mathrm{U}^*$$

$$U\left(\left(\tilde{T}^*\right)^* \tilde{T}^*\right)^p \mathrm{U}^* \; \leq N \; U\left(\left(\left(\tilde{T}^*\right)^* \tilde{T}^*\right)^{k+1}\right)^{\frac{p}{k+1}} \mathrm{U}^*$$

$$U \left|\tilde{T}^* \right|^{2p} U^* \leq N \; U\left|\left(\tilde{T}^* \right)^{2(k+1)}\right|^{\frac{p}{k+1}} U^*$$

$$\left|\tilde{T} \right|^{2p} \leq N\left|\left(\tilde{T} \right)^{k+1}\right|^{\frac{2p}{k+1}}$$

Hence proved theorem.

Theorem 2.12: Let $T \in B(H)$ and $\tilde{T}^{(*)}$ is powers of N-class A_k operator then $\tilde{T}$ is powers of N- class A_k operator.

Proof: From the definition of powers of N- class A_k operator $|T|^{2p} \le N|T^{k+1}|^{\frac{2p}{k+1}}$

$$(T^* T)^p \le N\,((T^* T)^{k+1})^{\frac{p}{k+1}}$$

$$\left(\left(\tilde{T}^{(*)}\right)^* \tilde{T}^{(*)}\right)^p \le N\left(\left(\left(\tilde{T}^{(*)}\right)^* \tilde{T}^{(*)}\right)^{k+1}\right)^{\frac{p}{k+1}}$$

$$\left(|T^*|^{\frac{1}{2}}\, U^*|T^*|^{\frac{1}{2}}\, |T^*|^{\frac{1}{2}}\, U|T^*|^{\frac{1}{2}}\right)^p \le N\left(\left(|T^*|^{\frac{1}{2}}\, U^*|T^*|^{\frac{1}{2}}\, |T^*|^{\frac{1}{2}}\, U|T^*|^{\frac{1}{2}}\right)^{k+1}\right)^{\frac{p}{k+1}}$$

$$\left(U^*\, |T|^{\frac{1}{2}}\, |T^*|^{\frac{1}{2}}\, |T^*|^{\frac{1}{2}}\, |T|^{\frac{1}{2}}\, U\right)^p \le N\left(\left(U^*\, |T|^{\frac{1}{2}}\, |T^*|^{\frac{1}{2}}\, |T^*|^{\frac{1}{2}}\, |T|^{\frac{1}{2}}\, U\right)^{k+1}\right)^{\frac{p}{k+1}}$$

$$\left(U^*\, |T|^{\frac{1}{2}}\, |T^*|\, |T|^{\frac{1}{2}}\, U\right)^p \le N\left(\left(U^*\, |T|^{\frac{1}{2}}\, |T^*|\, |T|^{\frac{1}{2}}\, U\right)^{k+1}\right)^{\frac{p}{k+1}}$$

$$U^*\left(|T|^{\frac{1}{2}}\, U\, |T| U^*\, |T|^{\frac{1}{2}}\right)^p U \le N\, U^*\left(\left(|T|^{\frac{1}{2}}\, U\, |T|\, U^*\, |T|^{\frac{1}{2}}\right)^{k+1}\right)^{\frac{p}{k+1}} U$$

$$U^*\left(|T|^{\frac{1}{2}}\, U\, |T|^{\frac{1}{2}}\, |T|^{\frac{1}{2}}\, U^*\, |T|^{\frac{1}{2}}\right)^p U \le N\, U^*\left(\left(|T|^{\frac{1}{2}}\, U\, |T|^{\frac{1}{2}}\, |T|^{\frac{1}{2}}\, U^*\, |T|^{\frac{1}{2}}\right)^{k+1}\right)^{\frac{p}{k+1}} U$$

$$U^*\left(\tilde{T}\,\tilde{T}^*\right)^p U \le N\, U^*\left(\left(\tilde{T}\,\tilde{T}^*\right)^{k+1}\right)^{\frac{p}{k+1}} U$$

$$U^*\left|\tilde{T}^*\right|^{2p} U \le N\, U^*\left|\left(\tilde{T}^*\right)^{k+1}\right|^{\frac{2p}{k+1}} U$$

$$U^*\left|\tilde{T}^*\right|^{2p} U \le N\, U^*\left|\left(\tilde{T}^*\right)^{k+1}\right|^{\frac{2p}{k+1}} U$$

$$\left|\tilde{T}\right|^{2p} \le N\left|\left(\tilde{T}\right)^{k+1}\right|^{\frac{2p}{k+1}}$$

Therefore $\tilde{T}$ is powers of N- class A_k operator.

Theorem 2.13: Let $T \in B(H)$ and $\tilde{T}^{(*)}$ is powers of N-class A_k operator then $\left(\tilde{T}^{(*)}\right)^*$ is powers of N- class A_k operator.

Proof: From the definition of powers of N- class A_k operator $|T|^{2p} \le N|T^{k+1}|^{\frac{2p}{k+1}}$

$$(T^* T)^p \le N\,((T^* T)^{k+1})^{\frac{p}{k+1}}$$

$$\left(\left(\tilde{T}^{(*)}\right)^* \tilde{T}^{(*)}\right)^p \le N\left(\left(\left(\tilde{T}^{(*)}\right)^* \tilde{T}^{(*)}\right)^{k+1}\right)^{\frac{p}{k+1}}$$

$$\left(|T^*|^{\frac{1}{2}}\, U^*|T^*|^{\frac{1}{2}}\, |T^*|^{\frac{1}{2}}\, U|T^*|^{\frac{1}{2}}\right)^p \le N\left(\left(|T^*|^{\frac{1}{2}}\, U^*|T^*|^{\frac{1}{2}}\, |T^*|^{\frac{1}{2}}\, U|T^*|^{\frac{1}{2}}\right)^{k+1}\right)^{\frac{p}{k+1}}$$

$$\left(U^*\, |T|^{\frac{1}{2}}\, |T^*|^{\frac{1}{2}}\, |T^*|^{\frac{1}{2}}\, |T|^{\frac{1}{2}}\, U\right)^p \le N\left(\left(U^*\, |T|^{\frac{1}{2}}\, |T^*|^{\frac{1}{2}}\, |T^*|^{\frac{1}{2}}\, |T|^{\frac{1}{2}}\, U\right)^{k+1}\right)^{\frac{p}{k+1}}$$

$$\left(U^*\, |T|^{\frac{1}{2}}\, |T^*|\, |T|^{\frac{1}{2}}\, U\right)^p \le N\left(\left(U^*\, |T|^{\frac{1}{2}}\, |T^*|\, |T|^{\frac{1}{2}}\, U\right)^{k+1}\right)^{\frac{p}{k+1}}$$

$$U^*\left(|T|^{\frac{1}{2}}\, U\, |T| U^*\, |T|^{\frac{1}{2}}\right)^p U \le N\, U^*\left(\left(|T|^{\frac{1}{2}}\, U\, |T|\, U^*\, |T|^{\frac{1}{2}}\right)^{k+1}\right)^{\frac{p}{k+1}} U$$

$$U^*\left(|T|^{\frac{1}{2}}\, U\, |T|^{\frac{1}{2}}\, |T|^{\frac{1}{2}}\, U^*\, |T|^{\frac{1}{2}}\right)^p U \le N\, U^*\left(\left(|T|^{\frac{1}{2}}\, U\, |T|^{\frac{1}{2}}\, |T|^{\frac{1}{2}}\, U^*\, |T|^{\frac{1}{2}}\right)^{k+1}\right)^{\frac{p}{k+1}} U$$

$$U^*\left(\tilde{T}\,\tilde{T}^*\right)^p U \le N\, U^*\left(\left(\tilde{T}\,\tilde{T}^*\right)^{k+1}\right)^{\frac{p}{k+1}} U$$

$$U^*\left|\tilde{T}^*\right|^{2p} U \le N\, U^*\left|\left(\tilde{T}^*\right)^{k+1}\right|^{\frac{2p}{k+1}} U$$

$$U^*\left|\tilde{T}^{(*)}\right|^{2p} U \le N\, U^*\left|\left(\tilde{T}^{(*)}\right)^{k+1}\right|^{\frac{2p}{k+1}} U$$

$$\left|\left(\tilde{T}^{(*)}\right)^*\right|^{2p} \le N\left|\left(\left(\tilde{T}^{(*)}\right)^*\right)^{k+1}\right|^{\frac{2p}{k+1}}$$

Therefore $\left(\tilde{T}^{(*)}\right)^*$ is powers of N- class A_k operator.

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