FINDING CRITICAL PATH IN A PROJECT NETWORK UNDER FUZZY ENVIRONMENT

ELIZABETH. S, ABIRAMI. M, SUJATHA. L

Abstract: Critical path problem is one of the simplest and most widely used concepts in non-fuzzy networks. In this paper, we have proposed a new method to identify the critical path for project network under fuzzy environment where each edge weights of a network takes the estimated values. This leads to the use of triangular fuzzy numbers for representing imprecise data values. A dynamic programming recursion formulation is developed to identify the critical path in fuzzy sense, where α-cut ranking technique is utilized to defuzzify the triangular fuzzy numbers. The main result from our study is that, the fuzzy critical path obtained from the developed model corresponds to actual path in the network and it is an extension of the crisp problem. Various discussions are made based on the parameters chosen by the decision makers. Finally, examples are illustrated for the proposed model and simulation results are shown using C language.

Keywords: Decision Making, Fuzzy critical path network problem, Triangular fuzzy number, α-cut ranking technique.

Introduction: Critical path method is a network-based method designed for planning and scheduling complex project. The main purpose of critical path method is to evaluate project performance and to identify the critical activities on the critical path so that the available resources could be utilized on these activities in the project network in order to reduce project completion time. The successful implementation of critical path method requires the availability of clear determined time duration for each activity. However in real life situations, project activities are subject to considerable uncertainty that may lead to numerous schedule disruptions. This uncertainty may arise from a number of possible sources like: activities may take more or less time than originally estimated, resources may become unavailable, material may arrive behind schedule, due dates may have to be changed, new activities may have to be incorporated or activities may have to be dropped due to changes in the project scope, weather conditions may cause severe delays, etc. As a result, the conventional approaches tend to be less effective in conveying the imprecision or vagueness nature of the linguistic assessment. Consequently, the fuzzy set theory proposed by Zadeh [13] can play a significant role in this kind of decision making environment to tackle the unknown or the vagueness about the time duration of activities in a project network. This gives rise to fuzzy environment. There have been several attempts in the literature to apply fuzzy numbers to the critical path method since the late 1970's and it has led to the development of fuzzy critical path method. The first method called FPERT, was proposed by Chanas and Kamburowski [1]. Yao and Lin [12] used signed distance ranking of fuzzy numbers to find critical path in a fuzzy project network. Chen and Chang [3] used defuzzification method to find possible critical paths in a fuzzy project network. Chanas and Zielinski [2] assumed that the operation time of each activity can be represented as a crisp value, interval or a fuzzy number. Dubois, Fargier and Galvagnon [5] assigned a different level of importance to each activity on a critical path for a randomly chosen set of activities. Chen and Huang [4] proposed a new model that combines fuzzy set theory with the PERT technique to determine the critical degrees of activities and paths, latest and earlier starting time and floats. Thus numerous papers [8]-[10] have been published in Fuzzy Critical Path Problem (FCPP). Yao and Lin [11] proposed a theorem based on signed distance ranking method for triangular fuzzy number to identify the fuzzy shortest path from vertex i to vertex n. Elizabeth and Sujatha [6] proposed a theorem using signed distance ranking method for trapezoidal fuzzy number to identify the fuzzy critical path from vertex 1 to vertex j. Based on the concepts of two research articles [6],[12] we have proposed a theorem for FCPP from vertex i to vertex n.

The paper is organized as follows: In section 2, basic definitions are reviewed and binary operation is introduced for α-cut ranking technique. Section 3, focus on formulation of critical path network problem under fuzzy environment. In section 4, suitable examples are illustrated to demonstrate the proposed approach. Finally, simulation results using C language are included for the same.

Preliminaries:

Definition 2.1. (Triangular fuzzy number): The triangular fuzzy number $\tilde{A}$=(a, b, c), a<b<c, is a fuzzy set defined on $\Re$ with membership function as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, & if\, a \le x < b \\ 1, & if \quad x = b \\ \frac{c-x}{c-b}, & if\ b < x \le c \\ 0, & otherwise \end{cases}$$

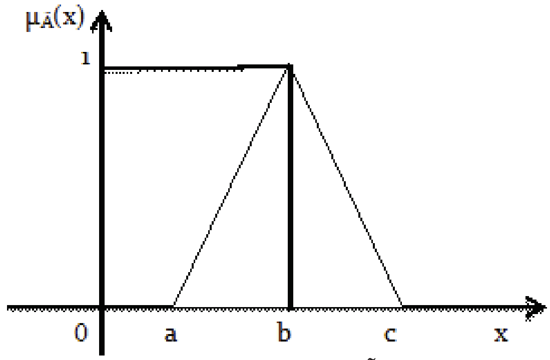


Fig.2.1. Triangular fuzzy number Ã

Definition 2.2. (Arithmetic operation on triangular fuzzy number): Let $\tilde{A}=(a_1,b_1,c_1)$ and $\tilde{B}=(a_2,b_2,c_2)$ be two triangular fuzzy numbers then

$\tilde{A} \oplus \tilde{B} = (a_1,b_1,c_1) \oplus (a_2,b_2,c_2)$
$= (a_1+a_2,b_1+b_2,c_1+c_2)$

$\tilde{A} \ominus \tilde{B} = (a_1,b_1,c_1) \ominus (a_2,b_2,c_2)$
$= (a_1-c_2,b_1-b_2,c_1-a_2)$

Definition 2.3. (α-cut Ranking technique): [7] Let $\tilde{A} = (a, b, c)$ be a triangular fuzzy number. Then the α-cut ranking technique of Ã is,

$$R(\tilde{A})= \int_0^1 [a+\alpha(b-a)]\alpha\, d\alpha + \int_0^1 [c-\alpha(c-b)]\alpha\, d\alpha = \frac{a+4b+c}{6}$$

If Ã and $\tilde{B}$ are two triangular fuzzy numbers then $\tilde{A} \succcurlyeq \tilde{B}$ iff $R(\tilde{A}) \geq R(\tilde{B})$. (2.1)

Here we define the binary operation for α-cut ranking technique.

Definition 2.4. (Binary operation on α-cut ranking technique) Let $\tilde{A}=(a_1,b_1,c_1)$ and $\tilde{B}=(a_2,b_2,c_2)$ be two triangular fuzzy numbers. Then $\tilde{A}\oplus\tilde{B}=(a_1+a_2,b_1+b_2,c_1+c_2)$ and

$$R(\tilde{A}\oplus\tilde{B})=\frac{(a_1+a_2)+4(b_1+b_2)+(c_1+c_2)}{6}$$
$$=\frac{a_1+4b_1+c_1}{6}+\frac{a_2+4b_2+c_2}{6} = R(\tilde{A})+R(\tilde{B}). \quad (2.2)$$

Formulation of critical path network problem under fuzzy environment:

Crisp Critical Path Problem (CCPP): A dynamic programming (DP) formulation for the critical path problem can be given as follows: In a network with an acyclic directed graph G = (V, E) with n vertices numbered from 1 to n such that 1 is the source and n is the destination. Then,

$f(n)=0$

$f(i)=\max\limits_{i<j}\{c_{ij}+f(j)/\langle i,j\rangle \in E\}$ (3.1)

Here c_{ij} is the weight of the directed edge <i, j>, and f(i) is the length of the critical (longest) path from vertex i to vertex n.

Fuzzy Critical Path Problem (FCPP): In the FCPP, we consider the edge weights in the network, i.e., c_{ij} is imprecise in nature, due to the vagueness in the data. A tolerable range for each c_{ij} as an interval $[c_{ij}-\Delta_{ij1}, c_{ij}+\Delta_{ij2}]$, where $0\leq \Delta_{ij1}<c_{ij}$ and $0< \Delta_{ij2}$. Using this interval, the decision maker (DM) can choose an appropriate value for each c_{ij}. The range $[c_{ij}-\Delta_{ij1},c_{ij}+\Delta_{ij2}]$ corresponds to the following triangular fuzzy number,

$\tilde{c}_{ij}=(c_{ij}-\Delta_{ij1},c_{ij},c_{ij}+\Delta_{ij2})$, where $0<\Delta_{ij1}<c_{ij}$, $0<\Delta_{ij2}$ (3.2)

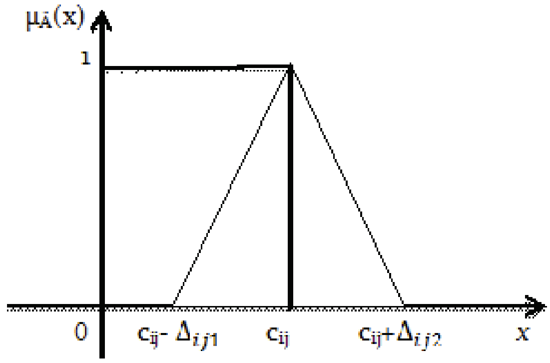


Fig.3.1. The fuzzy number $\tilde{c}_{ij}$

The length of the tolerable range $[c_{ij}-\Delta_{ij1}, c_{ij}+\Delta_{ij2}]$ is $\Delta_{ij}=\Delta_{ij2}-\Delta_{ij1}$. From definition 2.3 we obtain

$$c^*_{ij} = R(\tilde{c}_{ij}) = \frac{c_{ij}-\Delta_{ij1}+4c_{ij}+c_{ij}+\Delta_{ij2}}{6} \quad (3.3)$$
$$=\frac{6c_{ij}+(\Delta_{ij2}-\Delta_{ij1})}{6}$$
$$c^*_{ij} = c_{ij}+\frac{\Delta_{ij2}-\Delta_{ij1}}{6} \quad (3.4)$$
$$c^*_{ij} = c_{ij} + \frac{\Delta_{ij}}{6} >0 \quad (3.5)$$

In (3.5), when $\Delta_{ij1}=\Delta_{ij2}$, we obtain $c^*_{ij} = c_{ij}$. Thus the fuzzy problem becomes the crisp one. We call $c^*_{ij} = c_{ij} + \frac{\Delta_{ij}}{6}$ an estimate of the edge weight < i, j > in the fuzzy sense.

Because there are finite paths from node 1 to node n in a network, we conclude that there are also finite paths from node i to node n in the network. Thus, there must exists a path $p = \langle i,i_1,i_2,\ldots,i_{m(i)},n\rangle$ (i.e., $\langle i,i_1\rangle, \langle i_1,i_2\rangle, \ldots, \langle i_{m(i)},n\rangle \in E$) for $f(i) = c_{ii_1}+c_{i_1i_2}+\ldots+c_{i_{m(i)}n}$. Note that f(i) is the length of the critical path from vertex i to vertex n. Therefore we have

$f(i)= c_{ii_1}+c_{i_1i_2}+\ldots+c_{i_{m(i)}n} \geq c_{ik_1}+c_{k_1k_2}+\ldots+c_{k_{p(k)}n}$ (3.6)

where at least one equal sign holds for all possible paths, $p= \langle i,k_1,k_2,\ldots,k_{p(k)},n \rangle$, from vertex i to vertex n. Thus,

$f(i)= \max\{c_{ik_1}+c_{k_1k_2}+\ldots+c_{k_{p(k)}n}$ / for all paths $p= \langle i,k_1,k_2,\ldots,k_{p(k)},n \rangle\}$.

Fuzzifying both sides of (3.6) yields

$\tilde{c}_{ii_1}\oplus\tilde{c}_{i_1i_2}\oplus\ldots\oplus\tilde{c}_{i_{m(i)}n} \succcurlyeq \tilde{c}_{ik_1}\oplus \tilde{c}_{k_1k_2}\oplus\ldots\oplus\tilde{c}_{k_{p(k)}n}$ (3.7)

From (2.1), we obtain,

$$R(\tilde{c}_{ii_1} \oplus \tilde{c}_{i_1i_2}\oplus \ldots \oplus \tilde{c}_{i_{m(i)}n}) \geq R(\tilde{c}_{ik_1}\oplus \tilde{c}_{k_1k_2}\oplus\ldots \oplus \tilde{c}_{k_{p(k)}n})$$

$$R(\tilde{c}_{ii_1}) + R(\tilde{c}_{i_1i_2}) + \ldots + R(\tilde{c}_{i_{m(i)}n}) \geq R(\tilde{c}_{ik_1}) + R(\tilde{c}_{k_1k_2})+\ldots+R(\tilde{c}_{k_{p(k)}n}) \text{ (by (2.2))}$$

$c^*_{ii_1}+c^*_{i_1i_2}+\ldots+c^*_{i_{m(i)}n} \geq c^*_{ik_1}+c^*_{k_1k_2}+\ldots+c^*_{k_{p(k)}n}$ (3.8)

where at least one equal sign holds for all possible paths from vertex i to vertex n. Let

$f^*(i)$ be the length of the critical path from vertex i to vertex n in network G=(V,E) with $\{c^*_{ij}\ /<i, j> \in E\}$.

We know that, f(i)= $c_{ii_1}+c_{i_1i_2}+\ldots+c_{i_{m(i)}n}$.

From (3.8), we obtain

$$f^*(i) = c^*_{ii_1}+c^*_{i_1i_2}+\ldots+c^*_{i_{m(i)}n} .$$

Similarly, we obtain

$$f^*(j)=c^*_{jj_1}+c^*_{j_1j_2}+\ldots+c^*_{j_{m(j)}n} \quad (3.9)$$

We rewrite (3.1) as follows: for any fixed i,

$f(i) \geq c_{ij} + f(j)$, $\forall$ i<j, (i, j) ∈ E ,where at least one equal sign holds. Then,

$$c_{ii_1}+c_{i_1i_2}+\ldots+c_{i_{m(i)}n} \geq c_{ij}+c_{jj_1}+c_{j_1j_2}+\ldots+c_{j_{m(j)}n}, \forall i<j,(i,j)\in E \quad (3.10)$$

where at least one equal sign holds. Fuzzifying both sides of (3.10) yields

$$\tilde{c}_{ii_1}\oplus\tilde{c}_{i_1i_2}\oplus\ldots\oplus\tilde{c}_{i_{m(i)}n} \succcurlyeq \tilde{c}_{ij}\oplus\tilde{c}_{jj_1}\oplus\tilde{c}_{j_1j_2}\ldots\oplus\tilde{c}_{j_{m(j)}n}, \forall i<j, (i,j)\in E \quad (3.11)$$

where at least one ≈ holds. From (2.1),(2.2) and (3.5), we obtain

$$c^*_{ii_1}+c^*_{i_1i_2}+\ldots+c^*_{i_{m(i)}n} \geq c^*_{ij}+c^*_{jj_1}+\ldots+c^*_{j_{m(j)}n}, \forall i<j,(i,j)\in E \quad (3.12)$$

where at least one equal sign holds. Then, according to (3.5), (3.10) and (3.12), the DM should choose appropriate values for parameters

$\Delta_{ii_1},\Delta_{i_1i_2},\ldots,\Delta_{i_{m(i)}n}$, $\Delta_{ij},\Delta_{jj_1},\ldots$, and $\Delta_{j_{m(j)}n}$ to satisfy

$$\Delta_{ii_1}+\Delta_{i_1i_2}+\ldots+\Delta_{i_{m(i)}n} \geq \Delta_{ij}+\Delta_{jj_1}+\ldots+\Delta_{j_{m(j)}n}, \forall i<j,(i, j) \in E \quad (3.13)$$

From (3.9) and (3.12), the DP recursion of the first type of critical path problem in the fuzzy sense can be given by

$f^*(i) = \max_{i<j}\{c^*_{ij} + f^*(j)\ /<i, j> \in E\}$ and $f^*(n) = 0$.

Finally, we summarize the above description in the following Theorem.

Theorem 3.1: Consider a network G= (V, E) with n vertices numbered from 1 to n with the edge weights $\{c_{ij}\ /<i, j>\in E\}$. An estimate of the edge weight c^*_{ij}, is based on a triangular fuzzy number given in (3.2), defined by $c^*_{ij} = c_{ij} + \frac{1}{6}(\Delta_{ij2} - \Delta_{ij1}) = c_{ij} + \frac{1}{6}\Delta_{ij}$, where Δ_{ij1} and Δ_{ij2} are parameters whose values are determined by the DM to satisfy (3.13), thus creating a set of edge weights in the fuzzy sense, $\{c^*_{ij}\ /<i, j> \in E\}$. The DP recursion of the critical path problem in the fuzzy sense is given by

$$f^*(i) = \max_{i<j}\{c^*_{ij} + f^*(j)\ /<i, j> \in E\}, \text{ and}$$
$$f^*(n) = 0, \quad (3.14)$$

where $f^*(i)$ is the length of the critical path in the fuzzy sense from vertex i to vertex n. When $\Delta_{ij2}=\Delta_{ij1}$ for each edge <i, j>∈ E in Theorem 3.1, we obtain $c^*_{ij} = c_{ij}$. As a result, the fuzzy critical path problem becomes a crisp problem.

Illustrative Example: Consider the network as shown in Fig.4.1.

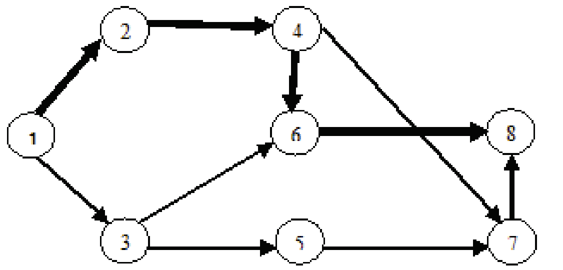


Fig. 4.1. Project network

Crisp Case: Assume the edge weights of the network as follows: $c_{12} = 3$, $c_{13} = 3$, $c_{24} = 3$, $c_{35} = 2$, $c_{36} = 5$, $c_{46} = 4$, $c_{47} = 4$, $c_{57} = 4$, $c_{68} = 5$ and $c_{78} = 4$. From (3.1), we have

$f(8)= 0$, $f(7)= c_{78}+f(8)= 4$, $f(6)= c_{68}+f(8)= 5$,

$f(5)=c_{57}+f(7)=8$, $f(4)=\max\{c_{46}+f(6),c_{47}+f(7)\}= 9$,

$f(3)= \max\{c_{36}+f(6),c_{35}+f(5)\} = 10$,

$f(2)=c_{24}+f(4)=12$, $f(1)=\max\{c_{12}+f(2),c_{13}+f(3)\}=15$.

Here, $f(1)=c_{12}+c_{24}+c_{46}+c_{68}=15$.This is the length of the critical path from vertex 1 to vertex 8 where the critical path is 1→2→4→6→8 in the crisp case.

For i = 1, 3 and 4 we get the following inequalities.

When i=1, $c_{12}+c_{24}+c_{46}+c_{68} > c_{13}+c_{36}+c_{68}$;

$c_{12}+c_{24}+c_{46}+c_{68} > c_{13}+c_{35}+c_{57}+c_{78}$;

$c_{13} + c_{36} + c_{68} = c_{13} + c_{35} + c_{57} + c_{78}$.

When i=3, $c_{36} + c_{68} = c_{35} + c_{57} + c_{78}$

When i=4, $c_{46} + c_{68} > c_{47} + c_{78}$ (4.1)

Fuzzy case: Then, based on the parameters (3.13) and the above inequalities (4.1) we have the following:

$\Delta_{12}+\Delta_{24}+\Delta_{46}+\Delta_{68} > \Delta_{13}+\Delta_{36}+\Delta_{68}$;

$\Delta_{12}+\Delta_{24}+\Delta_{46}+\Delta_{68} > \Delta_{13}+\Delta_{35}+\Delta_{57}+\Delta_{78}$;

$\Delta_{13} + \Delta_{36} + \Delta_{68} = \Delta_{13} + \Delta_{35} + \Delta_{57} + \Delta_{78}$;

$\Delta_{36} + \Delta_{68} = \Delta_{35} + \Delta_{57} + \Delta_{78}$;

$\Delta_{46} + \Delta_{68} > \Delta_{47} + \Delta_{78}$. (4.2)

4.2 a) If the DM chooses the values of parameters as $\Delta_{ij1}<\Delta_{ij2}$ i.e.,$\Delta_{121}= 2$, $\Delta_{122}= 4$,$\Delta_{131}= 1$, $\Delta_{132}= 3$, $\Delta_{241}= 1$, $\Delta_{242}= 2$, $\Delta_{351}= 1$, $\Delta_{352}= 2$, $\Delta_{361}= 1$, $\Delta_{362}= 3$, $\Delta_{461}= 1$, $\Delta_{462}= 3$, $\Delta_{471}= 2$, $\Delta_{472}= 3$, $\Delta_{571}= 3$, $\Delta_{572}= 4$, $\Delta_{681}= 1$,$\Delta_{682}= 2$, $\Delta_{781}= 1$, $\Delta_{782}= 2$.

i.e.,$\Delta_{12}= \Delta_{122} - \Delta_{121}= 2$, $\Delta_{13}= 2$, $\Delta_{24}= 1$, $\Delta_{35}=1$, $\Delta_{36}= 2$, $\Delta_{46}= 2$, $\Delta_{47}= 1$, $\Delta_{57}= 1$, $\Delta_{68}= 1$, $\Delta_{78}=1$ to satisfy the conditions in (4.2), then the fuzzy numbers in (3.2) are determined as follows:

$\tilde{c}_{12}=(1,3,7)$, $\tilde{c}_{13}=(2,3,6)$, $\tilde{c}_{24}=(2,3,5)$, $\tilde{c}_{35}=(1,2,4)$, $\tilde{c}_{36}=(4,5,8)$, $\tilde{c}_{46}=(3,4,7)$, $\tilde{c}_{47}=(2,4,7)$, $\tilde{c}_{57}=(1,4,8)$, $\tilde{c}_{68}=(4,5,7)$, $\tilde{c}_{78}=(3,4,6)$.

From Theorem 3.1 we obtain the estimate of the edge weights in the fuzzy sense as follows: $\tilde{c}^*_{12}$=3.33, $\tilde{c}^*_{13}$=3.33, $\tilde{c}^*_{24}$=3.16, $\tilde{c}^*_{35}$=2.16, $\tilde{c}^*_{36}$=5.33, $\tilde{c}^*_{46}$=4.33, $\tilde{c}^*_{47}$=4.16, $\tilde{c}^*_{57}$=4.16, $\tilde{c}^*_{68}$=5.16, $\tilde{c}^*_{78}$=4.16.

From (3.14), we have $f^*(8)$= 0, $f^*(7)$= 4.16, $f^*(6)$= 5.16, $f^*(5)$= 8.32, $f^*(4)$= 9.49, $f^*(3)$= 10.49, $f^*(2)$= 12.65, $f^*(1)=\tilde{c}^*_{12}+\tilde{c}^*_{24}+\tilde{c}^*_{46}+\tilde{c}^*_{68}$ = 15.98.

$f^*(1)$ is the length of the fuzzy critical path from vertex 1 to vertex 8 where the critical path in fuzzy sense is 1→2→4→6→8.

4.2 b) If the DM chooses the values of parameters as $\Delta_{ij1}>\Delta_{ij2}$ i.e.,Δ_{121}= 2, Δ_{122}= 1, Δ_{131}= 3, Δ_{132}= 1, Δ_{241}= 2, Δ_{242}= 1, Δ_{351}= 2 Δ_{352}= 1, Δ_{361}= 4, Δ_{362}= 2, Δ_{461}= 2, Δ_{462}= 1, Δ_{471}= 4, Δ_{472}= 1, Δ_{571}= 3, Δ_{572}= 1, Δ_{681}= 3, Δ_{682}= 1, Δ_{781}= 2, Δ_{782}= 1.

i.e.,$\Delta_{12}=\Delta_{122}-\Delta_{121}$=-1, Δ_{13}=-2, Δ_{24}=-1, Δ_{35}=-1, Δ_{36}=-2, Δ_{46}=-1, Δ_{47}=-3, Δ_{57}=-2, Δ_{68}=-2, Δ_{78}=-1 to satisfy the conditions in (4.2), then the fuzzy numbers in (3.2) are determined as follows:

$\tilde{c}_{12}$=(1,3,4), $\tilde{c}_{13}$=(0,3,4), $\tilde{c}_{24}$=(1,3,4), $\tilde{c}_{35}$=(0,2,3), $\tilde{c}_{36}$=(1,5,7), $\tilde{c}_{46}$=(2,4,5), $\tilde{c}_{47}$=(0,4,5), $\tilde{c}_{57}$=(1,4,5), $\tilde{c}_{68}$=(2,5,6), $\tilde{c}_{78}$=(2,4,5).

From Theorem 3.1 we obtain the estimate of the edge weights in the fuzzy sense as follows: $\tilde{c}^*_{12}$=2.83, $\tilde{c}^*_{13}$=2.66, $\tilde{c}^*_{24}$=2.83, $\tilde{c}^*_{35}$=1.83, $\tilde{c}^*_{36}$=4.66, $\tilde{c}^*_{46}$=3.83, $\tilde{c}^*_{47}$=3.5, $\tilde{c}^*_{57}$=3.66, $\tilde{c}^*_{68}$=4.66, $\tilde{c}^*_{78}$=3.83.

From (3.14), we have $f^*(8)$ = 0, $f^*(7)$ = 3.83, $f^*(6)$= 4.66, $f^*(5)$= 7.49, $f^*(4)$= 8.49, $f^*(3)$= 9.32, $f^*(2)$= 11.32, $f^*(1)=\tilde{c}^*_{12}+\tilde{c}^*_{24}+\tilde{c}^*_{46}+\tilde{c}^*_{68}$ = 14.15. $f^*(1)$ is the length of the fuzzy critical path from vertex 1 to vertex 8 where the critical path in fuzzy sense is 1→2→4→6→8.

4.2 c) If the DM chooses the values of parameters as $\Delta_{ij1}=\Delta_{ij2}$, $\Delta_{ij1}<\Delta_{ij2}$ and $\Delta_{ij1}>\Delta_{ij2}$ i.e.,Δ_{121}=1, Δ_{122}=1, Δ_{131}=2,Δ_{132}= 3,Δ_{241}= 1 , Δ_{242}= 2, Δ_{351}= 1, Δ_{352}= 2, Δ_{361}= 3, Δ_{362}= 2, Δ_{461}= 1, Δ_{462}= 1, Δ_{471}= 3, Δ_{472}= 2, Δ_{571}= 3, Δ_{572}= 1, Δ_{681}= 1, Δ_{682}= 2, Δ_{781}= 1, Δ_{782}=2.

i.e.,$\Delta_{12}=\Delta_{122}-\Delta_{121}$= 0, Δ_{13}= 1, Δ_{24}= 1, Δ_{35}= 1, Δ_{36}=-1, Δ_{46}=0, Δ_{47}=-1, Δ_{57}= -2, Δ_{68}= 1, Δ_{78}= 1 to satisfy the conditions in (4.2), then the fuzzy numbers in (3.2) are determined as follows:

$\tilde{c}_{12}$=(2,3,4), $\tilde{c}_{13}$=(1,3,6), $\tilde{c}_{24}$=(2,3,5), $\tilde{c}_{35}$=(1,2,4), $\tilde{c}_{36}$=(2,5,7), $\tilde{c}_{46}$=(3,4,5), $\tilde{c}_{47}$=(1,4,6), $\tilde{c}_{57}$=(1,4,5), $\tilde{c}_{68}$=(4,5,7), $\tilde{c}_{78}$=(3,4,6).

From Theorem 3.1 we obtain the estimate of the edge weights in the fuzzy sense as follows: $\tilde{c}^*_{12}$=3, $\tilde{c}^*_{13}$=3.16, $\tilde{c}^*_{24}$=3.16, $\tilde{c}^*_{35}$=2.16, $\tilde{c}^*_{36}$=4.83, $\tilde{c}^*_{46}$=4, $\tilde{c}^*_{47}$=3.83, $\tilde{c}^*_{57}$=3.66, $\tilde{c}^*_{68}$=5.16, $\tilde{c}^*_{78}$=4.16.

From (3.14), we have $f^*(8)$= 0, $f^*(7)$= 4.16, $f^*(6)$= 5.16, $f^*(5)$= 7.82, $f^*(4)$= 9.16, $f^*(3)$= 9.99, $f^*(2)$= 12.32, $f^*(1)=\tilde{c}^*_{12}+\tilde{c}^*_{24}+\tilde{c}^*_{46}+\tilde{c}^*_{68}$= 15.32.

$f^*(1)$ is the length of the fuzzy critical path from vertex 1 to vertex 8 where the critical path in fuzzy sense is 1→2→4→6→8.

Table 4.1: Results and discussions

Parameters chosen by DM	Edge weights	CCPP	FCPP	Results
$\Delta_{ij_1}=\Delta_{ij_2}$	$c^*_{ij}=c_{ij}$	15	-	FCPP = CCPP
$\Delta_{ij_1}<\Delta_{ij_2}$	$c^*_{ij}>c_{ij}$	-	15.98	FCPP is an extension of CCPP.
$\Delta_{ij_1}>\Delta_{ij_2}$	$c^*_{ij}<c_{ij}$	-	14.15	CCPP is an extension of FCPP.
$\Delta_{ij_1}=\Delta_{ij_2}$ $\Delta_{ij_1}<\Delta_{ij_2}$ $\Delta_{ij_1}>\Delta_{ij_2}$	$c^*_{ij}=c_{ij}$ $c^*_{ij}>c_{ij}$ $c^*_{ij}<c_{ij}$	-	15.32	FCPP is an extension of CCPP.

The comparison of critical path in the fuzzy sense with the crisp critical path is as follows:

(1) If $\Delta_{ij_1}<\Delta_{ij_2}$, then $\frac{f^*(1)-f(1)}{f(1)}\times 100$ = 6.53 %

(2) If $\Delta_{ij_1}>\Delta_{ij_2}$, then $\frac{f(1)-f^*(1)}{f^*(1)}\times 100$ = 6.00 %

(3) If $\Delta_{ij_1}=\Delta_{ij_2}$, $\Delta_{ij_1}<\Delta_{ij_2}$ and $\Delta_{ij_1}>\Delta_{ij_2}$, then $\frac{f^*(1)-f(1)}{f(1)}\times 100$ = 2.13 %.

Table 4.2: Results of the network

Paths	Crisp Path Length	Fuzzy Path Length		
	$\Delta_{ij_1}=\Delta_{ij_2}$	$\Delta_{ij_1}<\Delta_{ij_2}$	$\Delta_{ij_1}>\Delta_{ij_2}$	$\Delta_{ij_1}=\Delta_{ij_2}$, $\Delta_{ij_1}<\Delta_{ij_2}$ and $\Delta_{ij_1}>\Delta_{ij_2}$
1→2→4→6→8	**15**	**15.98**	**14.15**	**15.32**
1→2→4→7→8	14	14.81	12.99	14.15
1→3→5→7→8	13	13.81	11.98	13.14
1→3→6→8	13	13.82	11.98	13.15

Here path 1→2→4→6→8 is identified as the critical path in crisp and fuzzy case.

Simulation result using C language:

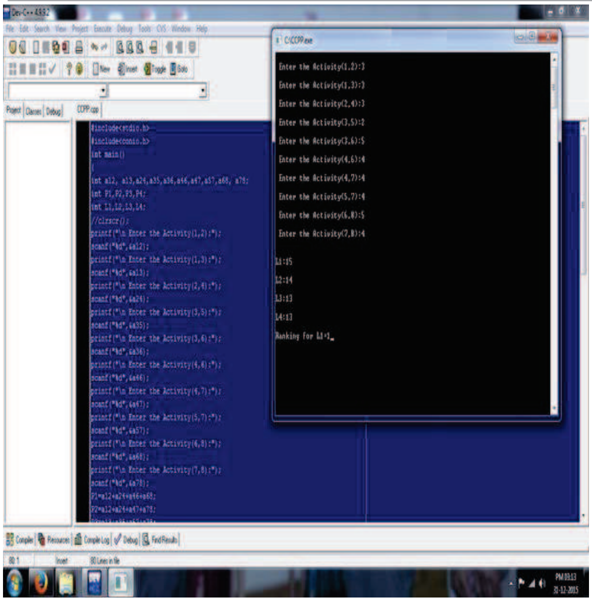
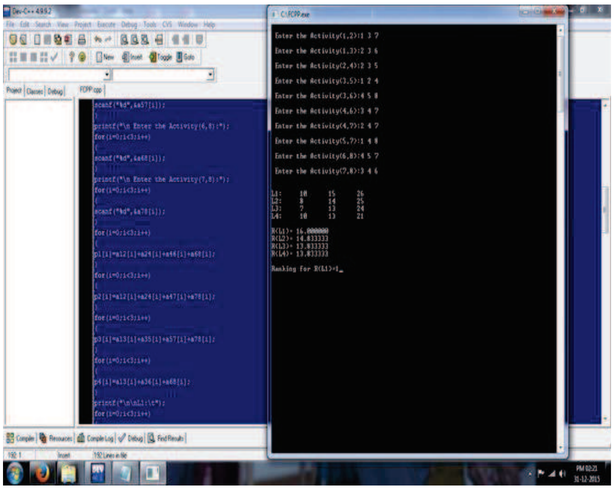
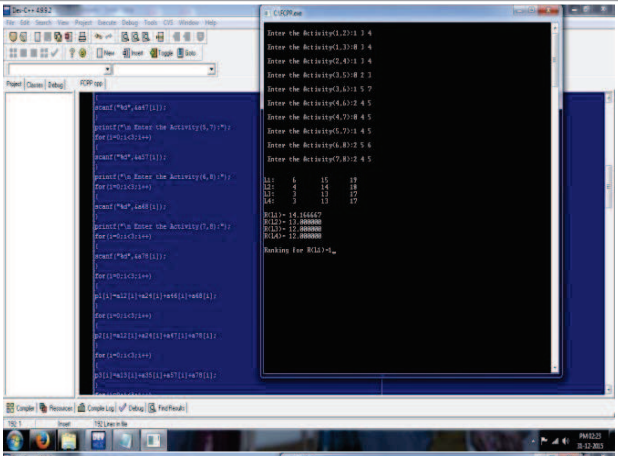


Fig.4.2. crisp critical path





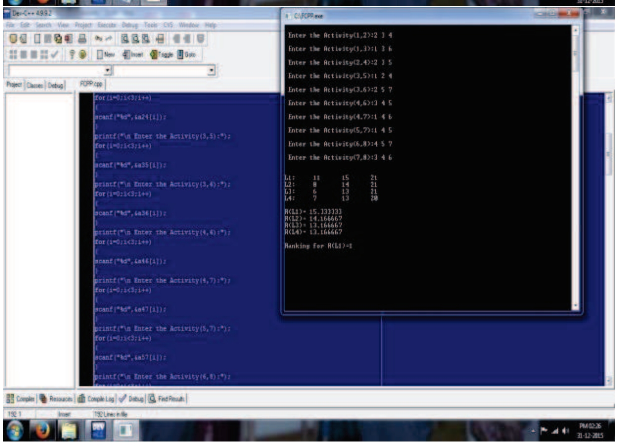


Fig.4.3. Fuzzy critical path

Conclusion: In the real word, the project is executed in an environment where the uncertainty being the principle features of it. One of these uncertainties in project planning process is estimating duration of activities. The duration of activities is usually estimated by the experts. The experts use terms like “almost”, “a little more”, “about’, “more or less”, etc. These terms clearly show some kind of uncertainty in scheduling the project. In this work, as a main task we have investigated a more realistic problem namely FCPP. It aims at providing a decision maker with the optimal solution using imprecise data which is very often used for solving problems of engineering and management sciences, where the decision maker can control the time and cost of the project and improve the efficiency of resource allocation to ensure the project quality.

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Elizabeth. S/ Auxilium College (Autonomous)/ Gandhi Nagar/ Vellore-6/ Head PG and Research/
Dept. of Mathematics / Thiruvalluvar University/
Abirami. M/ Auxilium College (Autonomous)/ Gandhi Nagar/ Vellore-6/ M.Phil./, Scholar/
Dept.of Mathematics/ Thiruvalluvar University/
Sujatha. L/ Auxilium College (Autonomous)/ Gandhi Nagar/ Vellore-6/ Asst. Prof./
Dept.of Mathematics / Thiruvalluvar University/