RELIABILITY ANALYSIS OF TIME – DEPENDENT SYSTEM WHEN THE NUMBER OF CYCLES FOLLOW BINOMIAL DISTRIBUTION AND STRESS- STRENGTH FOLLOW PARETO DISTRIBUTION

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Abstract: This paper deals with the stress strength problem incorporating time dependent system. Assuming that all the components in the system for both stress and strength are independent and stress, strength have three uncertainties i.e. deterministic, random-fixed, random-independent .Different models were developed for reliability whenever the number of cycles follows binomial distribution and stress and strength both follows pareto distribution and also computed numerically with different values of their parameters..

Keywords: Stress- strength model, Time- dependent system, Binomial distribution, Pareto distribution, Reliability.

Introduction: Reliability of a system is the probability that a system will adequately perform its intended purpose for a given period of time under stated environmental conditions [14]. In some cases system failures occur due to certain type of stresses acting on them. Thus system composed of random strengths will have its strength as random variable and the stress applied on it will also be a random variable. A system fails whenever an applied stress exceeds strength of the system. The systems will be failed due to different causes and the times to failure due to different reasons are likely to follow different distributions. Knowledge of these distributions is essential to eliminate cause of failures and thereby to improve the reliability. A multi variate exponential distribution was proposed by Marshal et al[1]. Bhattacharyya et al[2,3,4] studied estimation of reliability in a multi component stress strength model and also reliability estimation from survivor count data. Reliability analysis of time dependent cascade system is given by M.N.Gopalan[5,7]. T.S.Umamaheswari[10] is explained the reliability of single stress under strengths of life distribution. A multivariate pareto distribution and estimation of system reliability from stress – strength relationship is proposed by Hanagal, D.D[11,12]

In the present paper, the uncertainty about the stress and strength variables is classified into three categories:

1) Deterministic: the variable assumes values that are exactly known a priory.

2) Random fixed: the variable is random at any particular instant of time; the word fixed in this classification refers to the behaviour of the random variable with respect to time and/or cycles ; it means that the random variable changes or varies with time in a known manner.

3) Random independent: the variable is not only random but unlike the random fixed case, the successive values assumed by the variables are statistically independent, in accordance with Kapur and Lamberson and Schatz, et al. Here, in this paper, the components are assumed to be identical and the number of cycles for any time period t is assumed to be random. Expression for system reliability have been attained when number of cycles can be follow binomial distribution and stress and strength both follow Pareto distribution. In this paper, we considered nine models for the three uncertainties i.e. deterministic stress & deterministic strength, deterministic stress & random- fixed strength, deterministic stress & random-independent strength, random- fixed stress and deterministic strength, random- fixed stress and random- fixed strength, random- fixed stress and random independent strength, random- independent stress and deterministic strength, random independent stress& random –fixed strength, random-independent stress and random independent strength.

Notations:

R_n: Reliability after n cycles

$R(t)$: Reliability at time t

R:Reliability, independent of the cycle number (fixed)

X:Stress variable

Y:Strength variable

x_0:Deterministic stress

y_0:Deterministic Strength
X_i:Stress on the i^{th} cycle
Y_i:Strength on the i^{th} cycle
$f(x)$:Probability density function of a random variable X
$g(y)$:Probability density function of a random variable Y
E_i:event no failure occurs on the i^{th} cycle
$f_0(x_0)$:Probability density function of a random variable x_0
$g_0(y_0)$:Probability density function of a random variable y_0
$\pi_i(t)$:Probability of i cycles occurring in the time interval [o, t]
p:Probability of success in any trial
$q = 1 - p$:Probability of failure

Reliability Evaluation: If the cycles occur at random times, then

$$R(t) = \sum_{i=0}^{\infty} \pi_i(t) R_i \tag{1}$$

Where $\pi_i(t)$ is the probability of i cycles occurring in the time interval [o, t] and R_i is, as before, the probability of all i success. Clearly the case of deterministic cycle times becomes a special case above equation. In some cases it is appropriate to assume that the number of cycles occurring in a given time interval are Binomial distributed. Hence

$$\pi_i(t) = P(X = x) = \binom{n}{x} p^x q^{n-x} ;$$
$$x = 0,1,2, \ldots . . n. \tag{2}$$

Case 1: Deterministic Stress and Deterministic Strength: Let $x_i \; and \; y_i, \quad i = 1,2, \ldots . n$ be the stress and strength, respectively, on the i^{th} cycle. Then

$$R_n = P[E_1, E_2, \ldots . E_n]$$

Where$R_n = reliability \; after \; n \; cycle$,
$E_i = event \; no \; failure \; occurs \; on \; the \; i^{th}$ cycle

$$\text{Hence, } R_n = \begin{Bmatrix} 0 & if \;\; x_i > y_i \;\; for \; some \;\; i, \;\; 1 \le i \le n \\ 1 & if \; x_i \le y_i \qquad for \; all \; i, \;\; 1 \le i \le n \end{Bmatrix} \tag{3}$$

Let the stress x_i be known and non decreasing and strength y_i be known and non increasing. Let n^* be such that $R_{n^*} = 1 \; and \; R_{n^*+1} = 0$. Then we have $R_i = 1 \; for \; i = 0,1,2, \ldots . n^*$ and $R_i = 0 \; for \; i = n^* + 1, n^* + 2, \ldots .$

$$R(t) = \sum_{i=0}^{n} \pi_i(t) R_i = \sum_{i=0}^{n} \binom{n}{i} p^i q^{n-i} R_i$$

$$R(t) = \pi_0(t) R_0 = \binom{n}{0} p^0 q^n = q^n$$

$$R(t) = \sum_{i=0}^{n} \pi_i(t) R_i = 1. \sum_{i=0}^{n} \pi_i(t) = \sum_{i=0}^{n} \binom{n}{i} p^i q^{n-i}$$

$$R(t) = q^n \left[1 + \frac{p}{q}\right]^n = 1 \tag{4}$$

Case 2: Deterministic Stress and random- fixed Strength: Let the stress be x_0 , a constant, and the strength on the i^{th} cycle be Y_i given by

$$Y_i = Y_0 - a_i \,, \;\; i = 1,2, \ldots \tag{5}$$

Where $a_i \ge 0$ are known constants. Further, the a_i's are assumed nondecreasing in time . The probability density function of Y_0, $g_0(y_0)$ is assumed known. Then $P[E_n] = P(x_n \le Y_n)$

$$= P(x_0 \le y_0 - a_n)$$
$$= P(x_0 + a_n \le y_0)$$
$$= \int_{x_0 + a_n}^{\infty} g_0(y_0) dy_0$$

Hence

$$R_n = P[E_n] = \int_{x_0 + a_n}^{\infty} g_0(y_0) dy_0 \tag{6}$$

Let $Y_i = Y_0$, $i = 1,2, \ldots .$be the strength random variable with a known probability density function $f_0(y_0)$. Then

$$R_i = P[E_i] = \int_{x_0}^{\infty} f_0(y_0) dy_0 \triangleq R \tag{7}$$

The expression for R_i is independent of the cycle number i. Hence

$$R(t) = \sum_{i=0}^{\infty} \pi_i(t) R_i = \pi_0(t) R_0 + \sum_{i=1}^{\infty} \pi_i(t) R_i$$
$$= q^n(1) + R \sum_{i=1}^{n} \pi_i(t) = q^n + R\big(1 - \pi_0(t)\big)$$
$$R(t) = q^n + R(1 - q^n) \; or \; R + (1 - R) q^n \tag{8}$$

Case 3: Deterministic Stress and Random independent Strength: Let the stress be constant at x_0. Let $g_i(y)$ be the probability density function of the random variable strength Y_i during the cycle i. Since successive values of Y_i are independent, we get

$$R_n = P[E_1, E_2, \ldots . E_n] = P[E_1] * P[E_2] * \ldots \ldots * P[E_n]$$

Where

$$P[E_i] = P(x_0 \le y_i) = \int_{x_0}^{\infty} g_i(y) dy$$

In particular, if the probability density function remains unchanged over time, that is, if

$$g_1(y) = g_2(y) = \cdots = g_n(y) = g(y)$$

Then

$$R_n = (P[E_i])^n = \left\{\int_{x_0}^{\infty} g(y) dy\right\}^n \tag{9}$$

$$Let \quad R_i = R^i \,, \quad i = 0,1,2, \ldots \ldots n$$

$$where \quad R = \int_{x_0}^{\infty} g(y)dy$$

$$R(t) = \sum_{i=0}^{\infty} \pi_i(t)R_i = \sum_{i=0}^{n} \binom{n}{i} p^i q^{n-i} R^i$$

$$= q^n \sum_{i=0}^{n} \left(\frac{pR}{q}\right)^i \binom{n}{i} = q^n \left(1 + \frac{pR}{q}\right)^n \; or \; (q + pR)^n \quad (10)$$

Case 4: Random- fixed Stress and deterministic Strength: Let the strength be y_0 , a constant, and the stress on the i^{th} cycle be x_i given by

$$X_i = X_0 + b_i\,, \quad i = 1,2, \ldots.$$

Where $b_i \geq 0$ are known constants. Further, theb_i's are assumed nondecreasing in time. The probability density function of X_0, $f_0(x_0)$ is assumed known. Then $R_n = P[E_n] = P(X_n \leq y_n)$

$$= P(x_0 + b_n \leq y_0)$$
$$= P(x_0 \leq y_0 - b_n)$$

$$R_n = \int_0^{y_0 - b_n} f_0(x_0)dx_0$$

Then

$$R_i = P[E_i] = \int_0^{y_0} f_0(x_0)dx_0 \triangleq R \quad (11)$$

The expression for R_i is independent of the cycle number i. Hence

$$R(t) = \sum_{i=0}^{\infty} \pi_i(t)R_i = \pi_0(t)R_0 + \sum_{i=1}^{\infty} \pi_i(t)R_i$$

$$= q^n(1) + R\sum_{i=1}^{n} \pi_i(t) = q^n + R\big(1 - \pi_0(t)\big)$$

$$R(t) = q^n + R(1 - q^n) \; or \; R + (1 - R)q^n \quad (12)$$

$$where \; R = \int_0^{y_0} f_0(x_0)dx_0$$

Case 5: Random- fixed Stress and random- fixed Strength: Let the stress be given by

$X_i = X_0 + b_i\,, \quad i = 1,2, \ldots.$

Where $b_i \geq 0$ are known constants. Further, the b_i's are assumed nondecreasing in time .

Let the strength be given by

$Y_i = Y_0 - a_i\,, \quad i = 1,2, \ldots.$

Where $a_i \geq 0$ are known constants. Further, thea_i's are assumed nondecreasing in time. The probability density functions $f_0(x_0)$ *and* $g_0(y_0)$ are assumed known. We have to require the stress to be nondecreasing and strength to be nonincreasing. Hence

$R_n = P[E_n]$

$= P(X_n \leq Y_n)$

$= P(x_0 + b_n \leq y_0 - a_n)$

$= P(x_0 \leq y_0 - a_n - b_n)$

$$= \int_0^{\infty} g_0(y_0)\left(\int_0^{y_0 - a_n - b_n} f_0(x_0)dx_0\right)dy_0$$

Let X_0 and Y_0 be the random fixed stress and strength with known probability density functions $f_0(x_0)$ and $g_0(y_0)$ respectively. X_0 and Y_0 will be assumed not vary with time that is $a_i = b_i = 0, i = 1,2, \ldots..$ Hence

$$R_i = \int_0^{\infty} g_0(y_0) \int_0^{y_0} f_0(x_0)dx_0dy_0 = R, \quad i = 1,2, \ldots.$$

$$R(t) = \sum_{i=0}^{\infty} \pi_i(t)R_i = \pi_0(t)R_0 + \sum_{i=1}^{\infty} \pi_i(t)R_i$$

$$= q^n(1) + R\sum_{i=1}^{n} \pi_i(t) = q^n + R\big(1 - \pi_0(t)\big) \quad (13)$$

$$R(t) = q^n + R(1 - q^n) \; or \; R + (1 - R)q^n$$

where R is given by above

Case 6: Random- independent Stress and deterministic Strength: Let the strength be constant at y_0. Let $f_i(x)$ be the probability density function of the random variable stress X_i during the cycle i. Since successive values of X_i are independent, we get

$R_n = P[E_1, E_2, \ldots . E_n] = P[E_1] * P[E_2] * \ldots \ldots * P[E_n]$

Where

$$P[E_i] = P(X_i \leq y_0) = \int_0^{y_0} f_i(x)dx$$

In particular, if the probability density function remains unchanged over time, that is, if

$f_1(x) = f_2(x) = \cdots = f_n(x) = f(x)$

Then

$$R_n = (P[E_i])^n = \left\{\int_0^{y_0} f(x)dx\right\}^n$$

Let $R_i = R^i$

We get

$$R(t) = \sum_{i=0}^{\infty} \pi_i(t)R_i = \sum_{i=0}^{n} \binom{n}{i} p^i q^{n-i} R^i$$

$$= q^n \sum_{i=0}^{n} \left(\frac{pR}{q}\right)^i \binom{n}{i} = q^n \left(1 + \frac{pR}{q}\right)^n \; or \quad (q + pR)^n$$

$$R(t) = (q + pR)^n \; where \;\; R = \int_0^{y_0} f(x)dx \quad (14)$$

Case 7: Random- independent Stress and random- independent Strength: Let $f_i(x)$*and* $g_i(y)$ be the probability density functions of stress X_i and strength Y_i respectively in cycle i=1,2,.... . Then, since $X_i's$ and $Y_i's$ are independent,

$$R_n = P[E_1, E_2, \dots . E_n]$$
$$= P[E_1].P[E_2]. \dots . P[E_n]$$
$$= \prod_{i=1}^{n} P(E_i)$$

Where

$$P = P(X_i < Y_i)$$
$$= \int_0^\infty f_i(x) \int_x^\infty g_i(y)dydx$$

Let $f(x)$ and $g(y)$ represent the probability density functions for stress X and strength Y respectively. Further the random variables be independent on each cycle. Then

$$R_i = R^i, \quad i = 0,1,2, \dots \dots n$$
$$R(t) = \sum_{i=0}^{\infty} \pi_i(t) R_i = \sum_{i=0}^{n} \binom{n}{i} p^i q^{n-i} R^i$$
$$= q^n \sum_{i=0}^{n} \left(\frac{pR}{q}\right)^i \binom{n}{i} = q^n \left(1 + \frac{pR}{q}\right)^n$$
$$R(t) = (q + pR)^n$$

$where$ R

$$= \int_0^\infty f(x) \int_x^\infty g(y)dydx \tag{15}$$

Reliability computations:

In Binomial Distribution, the Stress and Strength Follow Pareto Distribution

$$f(x) = \frac{\lambda k^\lambda}{x^{\lambda+1}}, \quad k < x < \infty$$

Case 2: Deterministic Stress and Random - fixed Strength

$$R(t) = q^n + R(1 - q^n)$$

$$where \quad R = \int_{x_0}^{\infty} f_0(y_0)dy_0$$

$$R = \int_{x_0}^{\infty} \frac{\mu k^\mu}{{y_0}^{\mu+1}} dy_0 = \left(\frac{k}{x_0}\right)^\mu$$

$$R(t) = q^n + \left(\frac{k}{x_0}\right)^\mu (1 - q^n) \tag{16}$$

Case 3: Deterministic Stress and Random- independent Strength

$$R(t) = (q + pR)^n$$

$$where \quad R = \int_{x_0}^{\infty} g(y)dy$$

$$R = \int_{x_0}^{\infty} \frac{\mu k^\mu}{y^{\mu+1}} dy = \left(\frac{k}{x_0}\right)^\mu$$

$$R(t) = \left(q + p\left(\frac{k}{x_0}\right)^\mu\right)^n \tag{17}$$

Case 4: Random- fixed stress and deterministic Strength

$$R(t) = q^n + R(1 - q^n), \quad where \ R = \int_k^{y_0} f_0(x_0)dx_0$$

$$R = \int_k^{y_0} \frac{\lambda k^\lambda}{{x_0}^{\lambda+1}} dx_0 = \left(1 - \left(\frac{k}{y_0}\right)^\lambda\right)$$

$$R(t) = q^n + \left(1 - \left(\frac{k}{y_0}\right)^\lambda\right)(1 - q^n)$$

$$= 1 - \left(\frac{k}{y_0}\right)^\lambda (1 - q^n) \tag{18}$$

Case 5: Random- fixed Stress and random- fixed Strength

$$R(t) = q^n + R(1 - q^n)$$

$$where \quad R = \int_k^\infty g_0(y_0) \int_k^{y_0} f_0(x_0)dx_0dy_0$$

$$= \int_k^\infty \frac{\mu k^\mu}{{y_0}^{\mu+1}} \int_k^{y_0} \frac{\lambda k^\lambda}{{x_0}^{\lambda+1}} dx_0 \, dy_0$$

$$R = \frac{\lambda}{(\lambda + \mu)}$$

$$R(t) = q^n + \frac{\lambda}{(\lambda + \mu)}(1 - q^n)$$

$$R(t) = \frac{1}{(\lambda + \mu)}[\lambda + q^n \mu] \tag{19}$$

Case 6: Random- independent Stress and deterministic Strength

$$R(t) = (q + pR)^n \quad where \quad R = \int_k^{y_0} f(x)dx$$

$$R = \int_k^{y_0} \frac{\lambda k^\lambda}{x^{\lambda+1}} dx = \left(1 - \left(\frac{k}{y_0}\right)^\lambda\right)$$

$$R(t) = \left(q + p\left(1 - \left(\frac{k}{y_0}\right)^\lambda\right)\right)^n$$

$$= \left(1 - p\left(\frac{k}{y_0}\right)^\lambda\right)^n \tag{20}$$

Case 7: Random- independent Stress and random- independent Strength: $R(t) = (q + pR)^n$

$$where \quad R = \int_k^\infty f(x) \int_x^\infty g(y)dydx$$

$$R = \int_k^{\infty} \lambda e^{-\lambda x} \int_x^{\infty} \mu e^{-\mu y} dx\, dy = \frac{\lambda}{(\lambda + \mu)} \qquad R(t) = \left(q + p\left(\frac{\lambda}{(\lambda + \mu)}\right)\right)^n \qquad (21)$$

Numerical Results

Case 2: Deterministic stress and random fixed strength

Table 1(q=0.5, n=1, xo=0.2, μ=0.5.)

k	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
R	0.3012	0.3846	0.4455	0.5024	0.55	0.5929	0.6324	0.6692	0.7037	0.7363

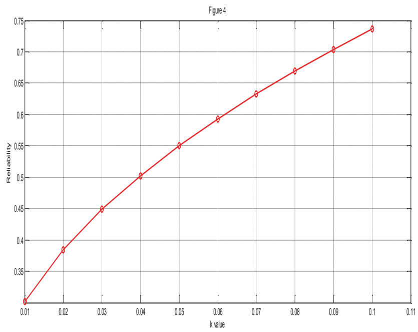
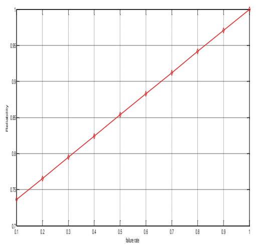
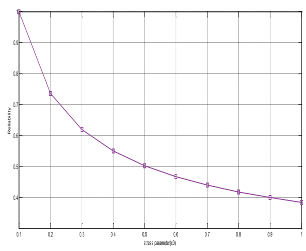


Table 2: (n=1, k=0.1, μ=0.5)

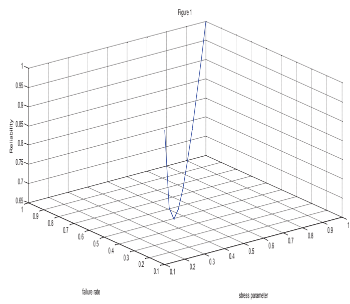
x_0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
q	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	1	0.7656	0.7041	0.7	0.7236	0.7633	0.8134	0.8707	0.9333	1



Graph 1: failure rate- Reliability



Graph 2: stress parameter- Reliability

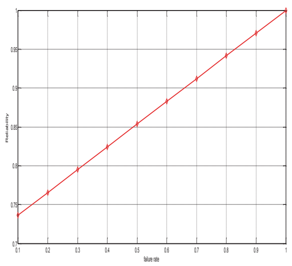


Graph 3: failure rate, stress parameter- Reliability

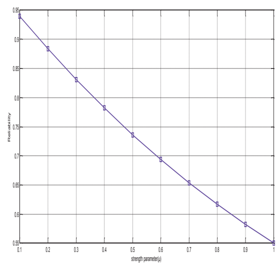
As failure rate increases reliability increases and as stress parameter increases reliability decreases. If the change of both parameters failure rate and stress parameters, reliability decreases when the parameters from 0.1 to 0.4 and reliability increases when the parameters values from 0.5 to 1.

Table 3:(n=1, xo=0.2, k=0.1)

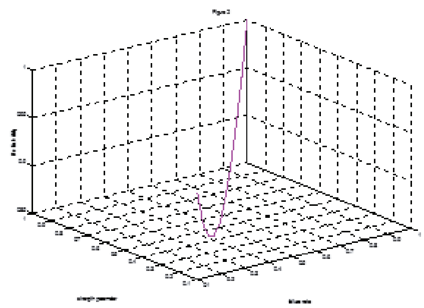
q	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
μ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.9397	0.8964	0.8686	0.8547	0.8535	0.8639	0.8847	0.9149	0.9536	1



Graph 1: failure rate-Reliability



Graph 2: strength parameter-Reliability

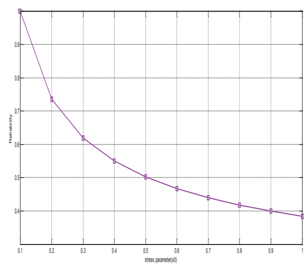


Graph 3: failure rate, strength parameter-Reliability

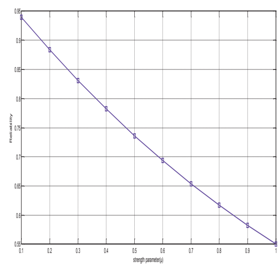
As failure rate increases reliability increases and as strength parameter increases reliability decreases. If the change of both parameters failure rate and strength parameters, reliability decreases when the parameters from 0.1 to 0.5 and reliability increases when the parameters values from 0.6 to 1

Table 4(n=1, q=0.5, k=0.1)

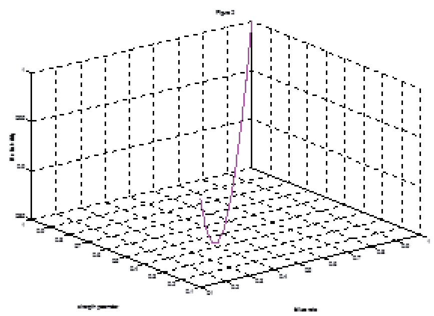
x_0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
μ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	1	0.9353	0.8596	0.7872	0.7236	0.6706	0.6281	0.5947	0.5692	0.55



Graph 1: stress parameter-Reliability



Graph 2: strength parameter- Reliability



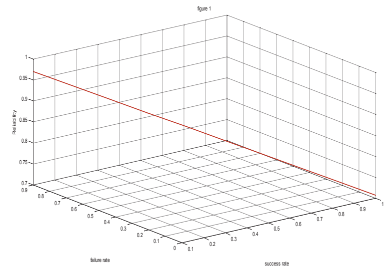
Graph 3: failure rate, strength parameter-Reliability

As failure rate increases reliability increases and as strength parameter increases reliability decreases. If the change of both parameters failure rate and strength parameters, reliability decreases

Case 3: deterministic stress and random independent strength

Table 1:(n=1, k=0.1, μ=0.5, $x_0 = 0.2$)

p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
q	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
R	0.9707	0.9414	0.9121	0.8828	0.8536	0.8243	0.795	0.7657	0.7364	0.7071

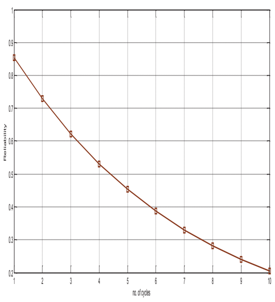


Graph : success rate, failure rate – Reliability

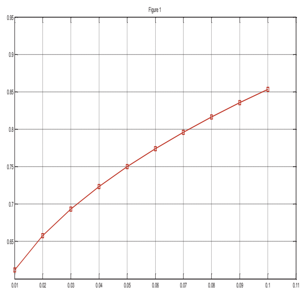
If the change of both parameters success rate and failure rate, reliability decreases.

Table 2:(p=0.5, q=0.5, μ=0.5, $x_0 = 0.2$)

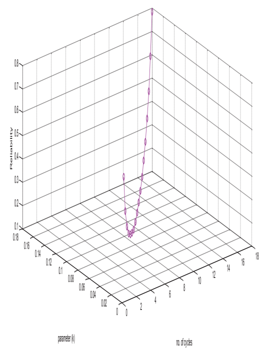
n	1	2	3	4	5	6	7	8	9
k	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
R	0.6118	0.4331	0.3337	0.2742	0.2374	0.2148	0.1970	0.1982	0.2053
n	10	11	12	13	14	15	16	17	18
k	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18
R	0.2184	0.2381	0.2659	0.3034	0.3534	0.4199	0.5083	0.6263	0.7850



Graph 1: number of cycles - Reliability



Graph 2: parameter k - Reliability

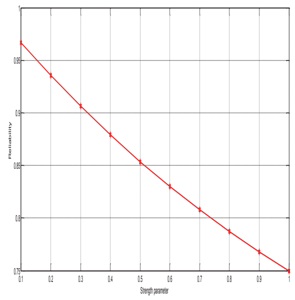


Graph 3: number of cycles, parameter k - Reliability

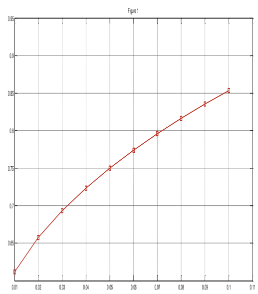
As no. of cycles increases reliability decreases and as parameter k increases reliability increases. If the change of both parameters no. of cycles and parameter k, reliability decreases when the parameters no. of cycles from 1 to 7 , parameter k from 0.01 to 0.07 and reliability increases when the parameters no. of cycles from 8 to 18 , parameter k from 0.08 to 0.18.

Table 3:(p=0.5, q=0.5, μ=0.5, $x_0 = 0.2$)

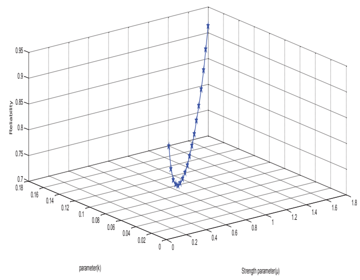
μ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
k	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
R	0.8706	0.8155	0.7830	0.7627	0.75	0.7428	0.7398	0.7402	0.7437
μ	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
k	0.1	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18
R	0.75	0.7590	0.7708	0.7861	0.8036	0.8248	0.8499	0.8793	0.9136



Graph 1: strength parameter-Reliability



Graph 2: parameter k- Reliability

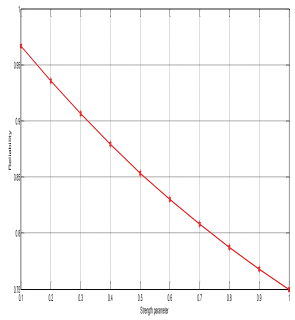


Graph 3: strength parameter, parameter k - Reliability

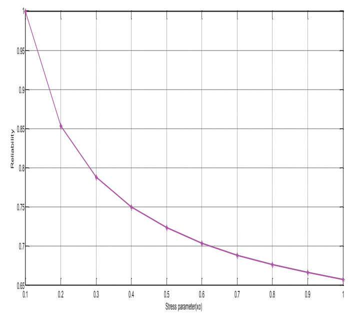
As strength parameter increases, reliability decreases and as parameter k increases, reliability increases. If the change of both parameters strength parameter and parameter k, reliability decreases when the strength parameters from 0.1 to 0.7 , parameter k from 0.01 to 0.07 and reliability increases when the strength parameter from 0.8 to 1.8 , parameter k from 0.08 to 0.18.

Table 4: (p=0.5, q=0.5, k=0.1, n= 1)

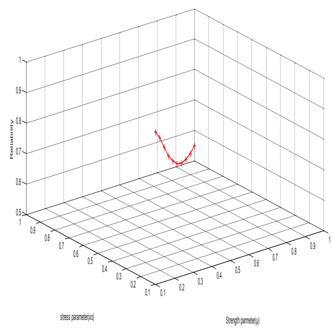
μ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
x_0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	1	0.9353	0.8596	0.7872	0.7236	0.6706	0.6281	0.5947	0.5692	0.55



Graph 1: strength parameter-Reliability



Graph 2: stress parameter - Reliability



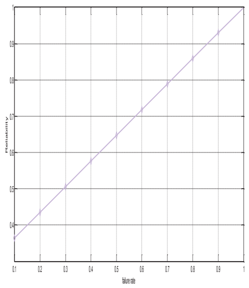
Graph 3: strength parameter, stress parameter - Reliability

As strength and stress parameters increases, reliability decreases. If the change of both parameters strength parameter and stress parameter, reliability decreases.

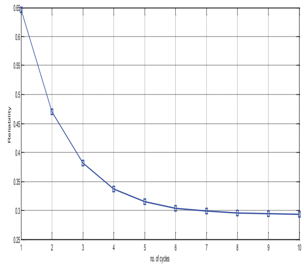
Case 4 : Random- fixed stress and deterministic strength

Table 1(k=0.1,y_0=0.2, λ=0.5)

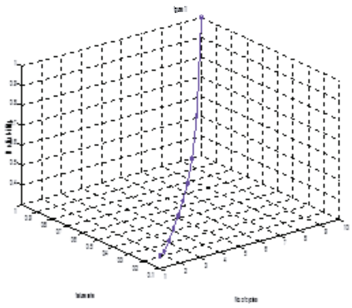
q	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
n	1	2	3	4	5	6	7	8	9	10
R	0.3636	0.3212	0.3119	0.3109	0.3149	0.3259	0.3511	0.4115	0.5668	1



Graph 1: failure rate- Reliability



Graph 2: no. of cycles- Reliability

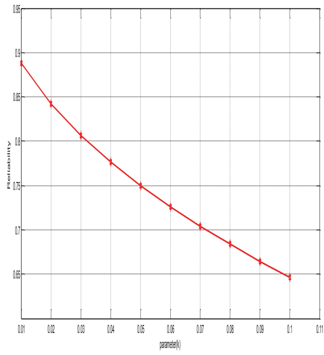


Graph 3: failure rate, no. of cycles-Reliability

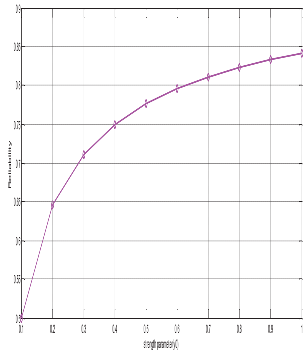
As failure rate increases, reliability increases and as no. of cycles increases, reliability decreases. If the change of both parameters failure rate and no. of cycles, reliability decreases when the failure rate from 0.1 to 0.4 , no. of cycles from 1 to 4 and reliability increases when the failure rate from 0.5 to 1 , no. of cycles from 5 to 10

Table 2: (q=0.5, n=1, λ=0.5)

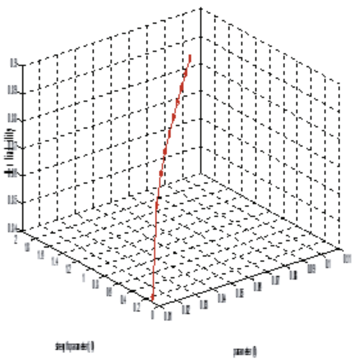
k	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
y_0	0.1	0.3	0.5	0.7	0.9	1.1	1.3	1.5	1.7	1.9
R	0.8419	0.8709	0.8775	0.8805	0.8821	0.8832	0.8839	0.8845	0.8849	0.8853



Graph 1: parameter k - Reliability



Graph 2: strength parameter - Reliability

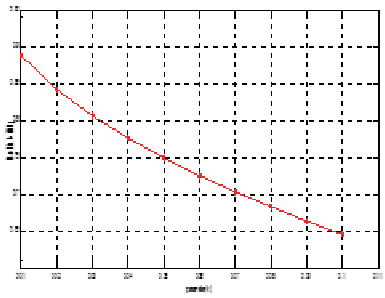


Graph 3: strength parameter, parameter k – Reliability

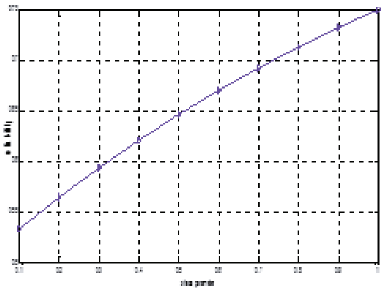
As parameter k increases, reliability decreases and as strength parameter increases, reliability increases. If the change of both parameters strength parameter and parameter k, reliability increases

Table 3: (q=0.5, n=1, y_0=0.2)

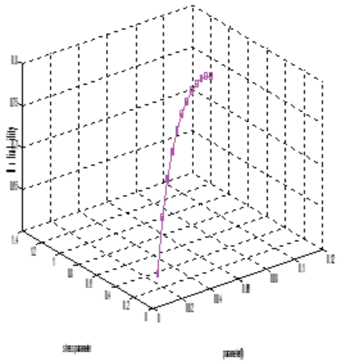
k	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.11	0.12
λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
R	0.6294	0.6845	0.7169	0.7373	0.75	0.7572	0.7602	0.7598	0.7563	0.75	0.7409	0.7281



Graph 1: parameter k - Reliability



Graph 2: stress parameter - Reliability



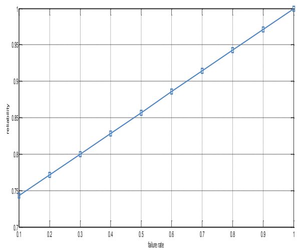
Graph 3: stress parameter, parameter k - Reliability

As parameter k increases, reliability decreases and as stress parameter increases, reliability increases. If the change of both parameters parameter k and stress parameter, reliability decreases when the parameter k from 0.01 to 0.04 , stress parameter from 0.1 to 0.4 and reliability increases when the parameter k from 0.05 to 0.12 , stress parameter from 0.5 to 1

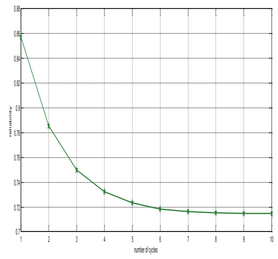
Case 5: Random- fixed Stress and random- fixed Strength

Table 1(λ=0.5, μ=0.2)

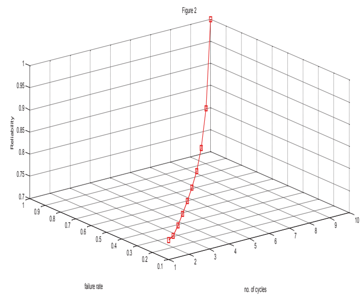
q	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
n	1	2	3	4	5	6	7	8	9	10
R	0.7428	0.7257	0.722	0.7216	0.7232	0.7276	0.7378	0.7622	0.8249	1



Graph 1: failure rate - Reliability



Graph 2: no. of cycles - Reliability

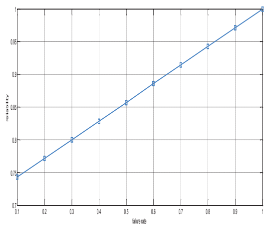


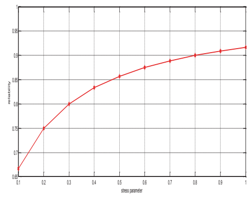
Graph 3: failure rate, no. of cycles - Reliability

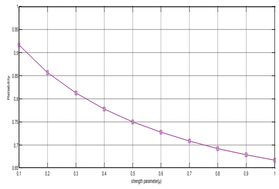
As failure rate increases, reliability increases and as no. of cycles increases, reliability decreases. If the change of both parameters failure rate and no. of cycles, reliability decreases when the failure rate from 0.1 to 0.3 , stress parameter from 0.1 to 0.4 and reliability increases when the parameter k from 0.05 to 0.12 , stress parameter from 0.5 to 1

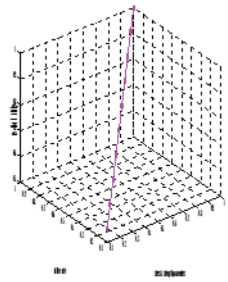
Table 2: (n=1)

λ= μ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
q	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1


Graph 1: failure rate - Reliability


Graph 2: stress parameter – Reliability


Graph 3 : strength parameter – Reliability

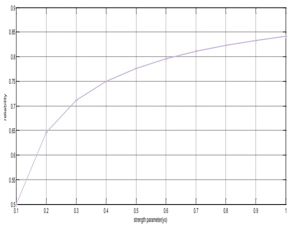

Graph 4: failure rate, stress, strength parameters - Reliability

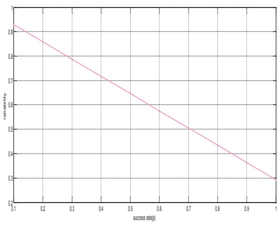
As failure rate increases, reliability increases, as stress parameter increases, reliability increases and as strength parameter increases, reliability decreases. If the change of parameters failure rate, stress and strength parameters, reliability increases

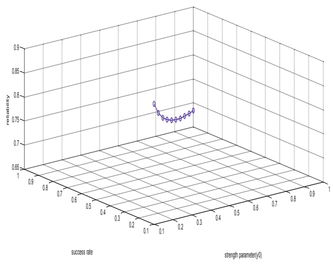
Case 6: Random- independent Stress and deterministic Strength

Table 1:(n=1,k=0.1, λ=0.5)

y_0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.9	0.8586	0.8268	0.8	0.7764	0.7551	0.7354	0.7172	0.7	0.6838


Graph 1: strength parameter - Reliability

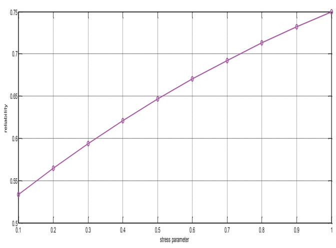

Graph 2: success rate - Reliability

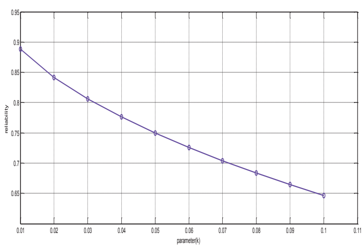

Graph 3: strength parameter, success rate - Reliability

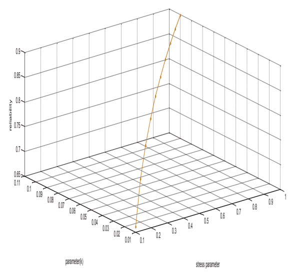
As strength parameter increases, reliability increases and as success rate increases, reliability decreases. If the change of both parameters strength parameter, success rate increases, reliability decreases.

Table 2: (p=0.5, n=1, $y_0 = 0.2$)

λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
k	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
R	0.6619	0.7373	0.7850	0.8179	0.8419	0.8599	0.8737	0.8846	0.8932	0.9


Graph 1: stress parameter - Reliability


Graph 2: parameter k- Reliability

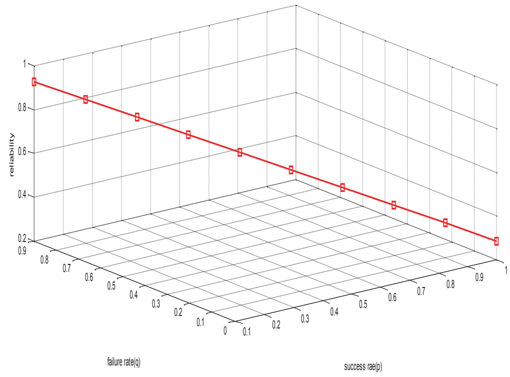

Graph 3: stress parameter, parameter k - Reliability

As stress parameter increases, reliability increases and parameter k increases, reliability decreases. If the change of both parameters stress parameter, parameter k increases, reliability increases.

Case 7: Random- independent Stress and random- independent Strength

Table 1: (λ=0.2, μ=0.5, n=1)

p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
q	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
R	0.9285	0.8571	0.7857	0.7142	0.6428	0.5714	0.5	0.4285	0.3571	0.2857

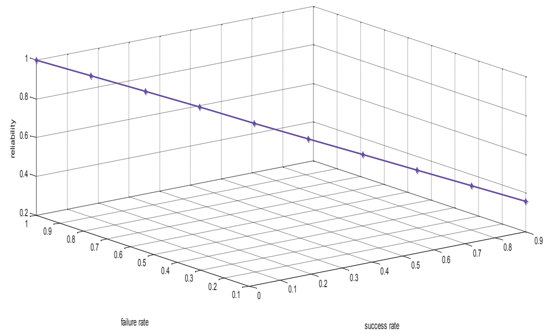


Graph : success rate, failure rate - Reliability

As increase of success rate and decrease of failure rate, reliability decreases.

Table 2: (λ=0.2, μ=0.5, n=1)

p	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
q	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.3571	0.4286	0.5	0.5714	0.6428	0.7142	0.7857	0.8571	0.9285	1

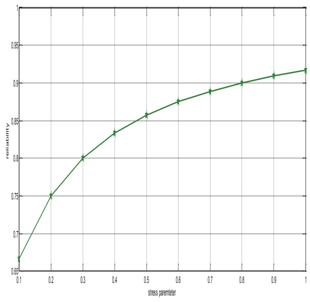


Graph: success rate, failure rate - Reliability

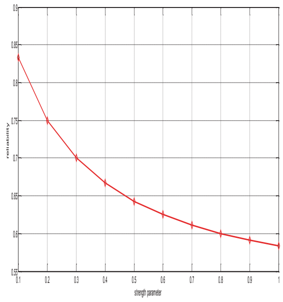
As decrease of success rate and increase of failure rate, reliability increases.

Table 3:(n=1, p=0.5,q=0.5)

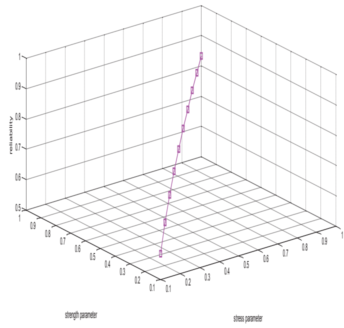
λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
μ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.5833	0.6429	0.6875	0.7222	0.75	0.7727	0.7917	0.8077	0.8214	0.8333



Graph 1: stress parameter - Reliability



Graph 2: strength parameter- Reliability



Graph 3: stress parameter, strength parameter - Reliability

As stress parameter increases, reliability increases and strength parameter increases, reliability decreases. If the change of both parameters stress parameter, strength parameter increases, reliability increases.

Conclusion: In the present work, numerical calculations for reliability have been done for nine models for three uncertainties of stress strength models where number of cycles follows binomial distribution and stress and strength follows Pareto distribution. In this we can find that strength parameter increase, reliability value decrease as well as stress parameter increases reliability value decreases. Since this may arise, here we use no. of cycles follows binomial distribution so results are arise opposite. This type of results use in real time problems.

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