N(A)– SEMIGROUPS

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Abstract: In this paper, the terms, 'A-potent', 'left A-divisor', 'right A-divisor', 'A-divisor' elements and ideals, 'N(A)-semigroup' for an ideal A of a semigroup are introduced. If A is an ideal of a semigroup S then it is proved that (1) $A \subseteq N_2(A) \subseteq N_1(A) \subseteq N_0(A)$, (2) $N_0(A) = A_2$, $N_1(A)$ is a semiprime ideal of S containing A, $N_2(A) = A_4$ are equivalent, where $N_0(A)$ = The set of all A-potent elements in S, $N_1(A)$ = The largest ideal contained in $N_0(A)$, $N_2(A)$ = The union of all A-potent ideals. If A is a semipseudo symmetric ideal of a semigroup then it is proved that $N_0(A) = N_1(A) = N_2(A)$. It is also proved that if A is an ideal of a semigroup such that $N_0(A) = A$ then A is a completely semiprime ideal. Further it is proved that if A is an ideal of semigroup S then R(A), the divisor radical of A, is the union of all A-divisor ideals in S. In a N(A)-semigroup it is proved that $R(A) = N_1(A)$. If A is a semipseudo symmetric ideal of a semigroup S then it is proved that S is an N(A)-semigroup iff $R(A) = N_0(A)$. It is also proved that if M is a maximal ideal of a semigroup S containing a pseudo symmetric ideal A then M contains all A-potent elements in S or S\M is singleton which is A-potent.

Introduction: The algebraic theory of semigroups was widely studied by CLIFFORD [2], [3], PETRICH [4] and LJAPIN [5]. The ideal theory in general semigroups was developed by ANJANEYULU [1]. In this paper we introduce the notions of 'A-potent', 'left A-divisor', 'right A-divisor', 'A-divisor' elements and ideals, 'N(A)-semigroup' for an ideal A of a semigroup. We obtained some characterizations of N(A)-semigroups.

Preliminaries:

Definition 2.1: A system S = (S, .), where S is a nonempty set and **.** is an associative binary operation on S, is called a *semigroup.*

Definition2.2: A semigroup S is said to be commutative provided *ab* = *ba* for all *a, b* ∈ S.

Definition 2.3: A semigroup S is said to be *normal* provided *aS* = *Sa* for all *a* ∈ S.

Definition 2.4: An element *a* of a semigroup S is said to be a *two sided identity* or an *identity* provided *as* =*sa*= *s* for all *s* ∈ S.

Definition 2.5: An element *a* of a semigroup S is said to be a *two sided zero* or *zero* of S provided *sa* =*as* = *a* for all *s* ∈ S.

Definition 2.6: A nonempty subset A of a semigroup S is said to be a *left ideal (right ideal)* of S provided SA⊆A (AS⊆A).

Definition 2.7: A nonempty subset A of a semigroup S is said to be a *two sided ideal* or *ideal* of S provided it is both a left ideal and a right ideal of S.

Definition2.8: An ideal A of a semigroup S is said to be a *proper ideal* of S provided A ≠ S.

Definition 2.9: An ideal A of a semigroup S is said to be a *trivial ideal* of S provided S\A is singleton.

Theorem 2.10: The nonempty intersection of any family of ideals of a semigroup S is an ideal of S.

Theorem 2.11: The union of any family of ideals of a semigroup S is an ideal of S.

Definition 2.12: Let S be a semigroup. The intersection of all ideals of S containing a nonempty set A is called the *ideal genetated by A***.** It is denoted by < A >.

Definition 2.13: An ideal A of a semigroup S is said to be a *principal ideal* provided A is an ideal generated by single element set. If an ideal A is generated by *a,* then A is denoted as <*a*> or J[*a*].

Definition 2.14: An ideal A of a semigroup S is said to be a *maximal ideal* provided A is a proper ideal of S and is not properly contained in any proper ideal of S.

Definition 2.15: An ideal A of a semigroup S is said to be a *minimal ideal* provided A does not contain any ideal of S properly.

Definition 2.16: An ideal A of a semigroup S is said to be a *completely prime ideal* provided *x, y* ∈ S, *xy*∈ A, implies either *x*∈ A or *y*∈ A.

Definition 2.17 : An ideal A of a semigroup S is said to be a *prime ideal* provided X,Y are ideals of S, XY ⊆ A implies either X ⊆ A orY ⊆ A.

Theorem 2.18: Every completely prime ideal of a semigroup is prime.

Definition 2.19: If A is an ideal of a semigroup S, then the intersection of all prime ideals containing A is called *prime radical* or simply *radical* of A and it is denoted by $\sqrt{A}$ or *rad* A.

Definition 2.20: An ideal A of a semigroup S is said to be *completely semiprime* provided $x \in S$, $x^n \in A$ for some natural number n implies $x \in A$.

Theorem 2.21 : Every completely semiprime ideal A in a semigroup S is a pseudo symmetric ideal.

Theorem 2.22: If S is a globally idempotent semigroup then every maximal ideal M of S is a prime ideal of S.

Definition 2.23: An ideal A of a semigroup S is said to be *semiprime* provided X is an ideal of S, $X^n \subseteq A$ for some natural number n implies $X \subseteq A$.

Theorem 2.24: An ideal Q of a semigroup S is a semiprime ideal of S iff $\sqrt{Q} = Q$.

Definition2.25: An ideal A of a semigroup S is said to be *pseudo symmetric* provided $x, y \in S$, $xy \in A$ implies $xsy \in A$ for all $s \in S$.

Theorem 2.26: Let A be an ideal of a semigroup S. Then A is completely prime iff A is prime and pseudo symmetric.

Definition 2.27: A semigroup S is said to be *pseudo symmetric* provided every ideal in S is a pseudo symmetric ideal.

Definition 2.28: An ideal A in a semigroup S is said to be *semipseudo symmetric* provided for any natural number n, $x \in S$, $x^n \in A$, $\Rightarrow \langle x^n \rangle \subseteq A$.

Theorem 2.29: Every pseudo symmetric ideal of a semigroup is a semipseudo symmetric ideal.

Theorem 2.30: If A is an ideal of a semigroup S, then the following are equivalent.

1. A is completely prime.
2. A is prime and pseudo symmetric.
3. A is prime and semipseudo symmetric.

Theorem 2.31: Let A be a semipseudo symmetric ideal of a semigroup S. Then the following are equivalent.

1. A_1= The intersection of all completely prime ideals of S containing A.
2. A_1^1 = The intersection of all minimal completely prime ideals of S containing A.
3. A_1^{11} = The minimal completely semiprime ideal of S relative to containing A.
4. $A_2 = \{x \in S : x^n \in A$ for some natural number n$\}$
5. A_3= The intersection of all prime ideals of S containing A.
6. A_3^1 = The intersection of all minimal prime ideals of S containing A.
7. A_3^{11} = The minimal semiprime ideal of S relative to containing A.
8. $A_4 = \{x \in S : \langle x \rangle^n \subseteq A$ for some natural number n$\}$

Theorem 2.32: If M is a maximal ideal of a semigroup S with $M_4 \neq S$, then the following are equivalent.

1. M is completely prime.
2. M is completely semiprime.
3. M is pseudo symmetric.
4. M is semipseudo symmetric.

Corollary 2.32: An ideal Q of a semigroup S is a semiprime ideal iff Q is the intersection of all prime ideals of S contains Q.

Definition 2.33: Let P be prime ideal in a semigroup S. A primary ideal A in S is said to be *P-primary* or P is a prime ideal belonging to A provided $\sqrt{A} = P$.

Theorem 2.34: If $A_1, A_2, \ldots, A_n$ are P- primary ideals in a semigroup S, then $\bigcap_{i=1}^{n} A_i$ is also a P-primary ideal.

Definition 2.35: An ideal A in a semigroup S is said to have *left primary decomposition* if $A = A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n$ where each A_i is a left primary ideal. If no A_i contains $A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_{i-1} \cap A_{i+1} \cap \ldots \cap A_n$ and the radicals P_i of the ideals A_i are all distinct, then the left primary decomposition is said to be *reduced*. If P_i is minimal in the set $\{P_1, P_2, P_3, \ldots, P_n\}$ then P_i is said to be *isolated prime.*

Definition 2.36: An ideal A in a semigroup S is said to have *right primary decomposition* if $A = A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n$ where each A_i is a right primary ideal. If no A_i contains $A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_{i-1} \cap A_{i+1} \cap \ldots \cap A_n$ and the radicals P_i of the ideals A_i are all distinct, then the right primary decomposition is said to be *reduced*. If P_i is minimal in the set $\{P_1, P_2, P_3, \ldots, P_n\}$ then P_i is said to be *isolated prime.*

Definition 2.37: An ideal A of a semigroup S is said to have a primary *decomposition* if $A = A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n$ where each A_i is a right primary ideal. If no A_i contains $A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_{i-1} \cap A_{i+1} \cap \ldots \cap A_n$ and the radicals P_i of the ideals A_i are all distinct, then the primary decomposition is said to be ***reduced***. If P_i is minimal in the set $\{P_1, P_2, P_3, \ldots, P_n\}$ then P_i is said to be *isolated prime.*

Theorem 2.38: If an ideal A in a semigroup S has a primary decomposition, then A has a reduced primary decomposition.

N(A) – Semigroups:

Definition 3.1: Let A be an ideal in a semigroup S. An element $x \in S$ is said to be *A-potent* there exists a natural number n such that $x^n \in A$.

Definition 3.2: Let A be an ideal in a semigroup S. An ideal B of S is said to be *A-potent ideal* provided there exists a natural number n such that $B^n \subseteq A$.

Notation 3.3: $N_o(A)$ = The set of all A-potent elements in S.

$N_1(A)$ = The largest ideal contained in $N_o(A)$.

$N_2(A)$ = The union of all A-potent ideals.

Theorem 3.4: If A is an idealof a semigroup S, then $A \subseteq N_2(A) \subseteq N_1(A) \subseteq N_0(A)$.

Proof: Since A is itself an A-potent ideal, and $N_2(A)$ is the union of all A-potent ideals. Therefore $A \subseteq N_2(A)$. Let $x \in N_2(A) \Rightarrow x$ belongs to atleast one A-potent ideal

$\Rightarrow x$ is an A-potent element. Hence $x \in N_o(A)$. Therefore $N2(A) \subseteq N_o(A)$.

Clearly $N_2(A)$ is an ideal of S. Since $N_1(A)$ is the largest ideal contained in $N_o(A)$,

we have $N_2(A) \subseteq N_1(A) \subseteq N_0(A)$. Hence $A \subseteq N_2(A) \subseteq N_1(A) \subseteq N_0(A)$.

Theorem 3.5 : If A is an ideal in a semigroup S, then the following are true.

1. $N_o(A) = A_2$.
2. $N_1(A)$ is a semiprime ideal of S containing A.
3. $N_2(A) = A_4$.

Proof : (1) $N_o(A)$ = The set of all A-potent elements = $\{ x \in S: x^n \in A$ for some natural number n$\} = A_2$.

(2) Suppose that $<x>^2 \subseteq N_1(A)$. Suppose if possible $x \notin N_1(A)$. $N_1(A)$, $<x>$ are the ideals implies $N_1(A) \cup <x>$ is an ideal. Since $N_1(A)$ is the largest ideal in $N_o(A)$,

We have $N_1(A) \cup <x> \not\subset N_o(A) \Rightarrow <x> \not\subset N_o(A)$. Hence there exists an element y such that

$y \in <x> \backslash N_o(A)$. Now $y^2 \in <x>^2 \subseteq N_1(A) \subseteq N_o(A) \Rightarrow y^2 \in N_o(A) \Rightarrow (y^2)^n \in A$ for some natural number $n \Rightarrow y^{2n} \in A \Rightarrow y \in N_o(A)$. It is a contradicition. Therefore $x \in N_1(A)$. Hence $N_1(A)$ is a semiprime ideal of S containing A.

(3) Let $x \in N_2(A)$. Then there exists an A-potent ideal B such that $x \in B$. B is A-potent ideal implies there exists $n \in$ N such that $B^n \subseteq A \Rightarrow <x>^n \subseteq B^n \subseteq A$ for some $n \in N \Rightarrow x \in A_4$.

Therefore $N_2(A) \subseteq A_4$. Let $x \in A_4 \Rightarrow <x>^n \subseteq A$ for some $n \in$ N. So $<x>$ is an A-potent ideal in S and hence $<x> \subseteq N_2(A) \Rightarrow x \in N_2(A)$. Therefore $A_4 \subseteq N_2(A)$. Hence $N_2(A) = A_4$.

Note 3.6: It is natural to ask whether $N_1(A) = A_3$. This is not true.

Example 3.7: In the free semigroup S over the alphabet a,b,c. For the ideal $A = Sa^2S$, $N_o(A) = \{a\} \cup S^1a^2S^1$ and $N_1(A) = \{a^3\} \cup Sa^2S = S^1a^2S^1$. But Sa^2S is a prime ideal, let A,B are two ideals of S such that $AB \subseteq Sa^2S$, implies all words containing $a^2 \in A$ or all words containing $a^2 \in B \Rightarrow A \subseteq Sa^2S$ or $B \subseteq Sa^2S$. Therefore Sa^2S is a prime ideal.

We have $A_3 = Sa^2S$, so $N_1(A) \neq A_3$. Therefore we can remark that the inclusions in $A_3 \subseteq N_1(A) \subseteq N_o(A) = A_2$ may be proper in an arbitrary semigroup.

Theorem 3.8: If A is a semipseudo symmetric ideal in a semigroup S, then $N_o(A) = N_1(A) = N_2(A)$.

Proof: Suppose A is a semi pseudo symmetric ideal in a semigroup S. By theorem 3.5, $N_o(A) = A_2$ and $N_2(A) = A_4$. Also by theorem 2.31, we have $A_2 = A_4$. Hence $N_o(A) = N_2(A)$. By the theorem 3.4, $A \subseteq N_2(A) \subseteq N_1(A) \subseteq N_0(A)$.We have $N_2(A) \subseteq N_1(A)$. Now let $x \in N_1(A) \Rightarrow x \in N_o(A) \Rightarrow x \in N_2(A)$.Therefore $N_1(A) \subseteq N_2(A)$. Hence $N_1(A) = N_2(A)$. Therefore $N_o(A) = N_1(A) = N_2(A)$.

Theorem 3.9: For any semipseudo symmetric ideal A in a semigroup S, a nontrivial A-potent element $x (x \notin A)$ cannot be semisimple.

Proof: Since x is a nontrivial A-potent element, there exists a natural number n such that $x^n \in A$. Since A is semipseudo symmetric ideal, we have $<x>^n \subseteq A$. If x is semisimple, then $<x> = <x>^2$ and hence $<x> = <x>^n \subseteq A$, this is a contradiction. Thus x is not semisimple.

Theorem 3.10: If A is an ideal in a semigroup S, such that $N_o(A) = A$, then A is a completely semiprime ideal and A is a pseudo symmetric ideal.

Proof: Let $x \in S$ and $x^2 \in A$. Since $N_o(A) = A$, $x^2 \in N_o(A)$.. Thus there exists a natural number n such that $(x^2)^n \in A \Rightarrow x^{2n} \in A \Rightarrow x \in N_o(A) = A$. Therefore A is a completely semiprime ideal. By theorem 2.21, A is pseudo symmetric ideal. Hence A is completely semiprime and pseudo symmetric ideal.

Definition 3.11: Let A be an ideal in a semigroup S. An element $x \in S$ is said to be a *left A-divisor* provided there exists an element $y \in S \backslash A$ such that $xy \in A$.

Definition 3.12: Let A be an ideal in a semigroup S. An element $x \in S$ is said to be a *right A-divisor* provided there exists an element $y \in S \backslash A$ such that $yx \in A$.

Definition 3.13: Let A be an ideal in a semigroup S. An element $x \in S$ is said to be an *A-divisor* if x is both a left A-divisor and a right A-divisor element.

Definition 3.14: Let A be an ideal in a semigroup S. An ideal B in S is said to be a *left A-divisor ideal* provided every element of B is a left A-divisor element.

Definition 3.15: Let A be an ideal in a semigroup S. An ideal B in S is said to be a *right A-divisor ideal* provided every element of B is a right A-divisor element.

Definition 3.16: Let A be an ideal in a semigroup S. An ideal B in S is said to be an *A-divisor ideal* provided if it is both a left A-divisor ideal and a right A-divisor ideal of a semigroup S.

Notation 3.17: $R_l(A)$ = The union of all left A-divisor ideals in S.

$R_r(A)$ = The union of all right A-divisor ideals in S.

$R(A) = R_l(A) \cap R_r(A)$. We call R(A), the divisor radical of A.

Theorem 3.18: If A is any ideal of a semigroup S, then $N_1(A) \subseteq R(A)$.

Proof: Let $x \in N_1(A)$. Since $N_1(A) \subseteq N_0(A)$, we have $x \in N_0(A) \Rightarrow x^n \in A$ for some $n \in N$.

Let n be the least natural number such that $x^n \in A$. If $n = 1$ then $x \in A$ and hence $x \in R(A)$.

If $n > 1$, then $x^n = x^{n-1}.x \in A$, where $x^{n-1} \in S \backslash A$. Hence x is an A-divisor element.

Thus $x \in R(A)$. Therefore $N_1(A) \subseteq R(A)$.

Theorem 3.19: If A is an ideal in a semigroup S, then R(A) is the union of all A-divisor ideals in S.

Proof: Suppose A is an ideal in a semigroup S. Let B be an A –divisor ideal in S. Then B is both a left A-divisor and a right A-divisor ideal in S. Thus $B \subseteq R_l(A)$ and $B \subseteq R_r(A)$

$\Rightarrow B \subseteq R_l(A) \cap R_r(A) = R(A) \Rightarrow B \subseteq R(A)$.

Therefore R(A) contains the union of all

A-divisor ideals in S. Let $x \in R(A)$. Then $x \in R_l(A) \cap R_r(A)$. So $<x> \subseteq R_l(A) \cap R_r(A)$.

Hence $<x>$ is an A-divisor ideal. So R(A) is contained in the union of all divisor ideals in S. Thus R(A) is the union of all divisor ideals of S.

Corollary 3.20: If A is a pseudo symmetric ideal in a semigroup S, then R(A) is the set of all A-divisor elements in S.

Proof: Suppose A is a pseudo symmetric ideal in S. Let x be an A-divisor element in S. Then $xy \in A$, where $y \in S \backslash A$. $xy \in A$, A is pseudo symmetric $\Rightarrow <x><y> \subseteq A \Rightarrow <x>$ is an A-divisor ideal $\Rightarrow <x> \subseteq R(A) \Rightarrow x \in R(A)$. Hence R(A) is the set of all A-divisor elements in S.

Definition 3.21: Let A be an ideal in a semigroup S. S is said to be a *N(A)-semigroup* provided every A-divisor is A-potent.

Notation 3.22: Let S be a semigroup with zero. If A={0}, then we write R for R(A) and N for $N_0(A)$ and N-semigroup for N(A)-semigroup.

Theorem 3.23: If S is an N(A)-semigroup, then R(A) = $N_1(A)$.

Proof: Suppose S is an N(A)-semigroup. By theorem 3.18, $N_1(A) \subseteq R(A)$.

Let $x \in R(A) \Rightarrow x$ is an A-divisor $\Rightarrow x$ is an A-potent $\Rightarrow x \in N_1(A)$. $\therefore R(A) \subseteq N_1(A)$.

Hence $N_1(A) = R(A)$.

Theorem 3.24: Let A be a semipseudo symmetric ideal in a semigroup S. Then S is an N(A)-semigroup iff $R(A) = N_0(A)$.

Proof: Since A is a semipseudo symmetric ideal, by the theorem 3.8, $N_0(A) = N_1(A) = N_2(A)$. If S an N(A)-semigroup, then by theorem 3.22, $R(A) = N_1(A)$. Hence $R(A) = N_0(A)$. Conversely suppose that $R(A) = N_0(A)$. Then clearly every A-divisor element is an A-potent element. Hence S is an N(A)-semigroup.

Corollary 3.25: Let S be a semigroup with 0 and $< 0 >$ is a pseudo symmetric ideal. Then R = N iff S is an N-semigroup.

Corollary 3.26: Let S be a semigroup with 0 and with 1 as an *r*-element. Then R = N iff S is an N-semigroup.

Proof: Suppose S is a semigroup with 0 and with 1 as an *r*-element. Let $xy \in < 0 >$. $xy \in < 0 > \Rightarrow xy = 0$. Let $s \in S$, since 1 is an *r*-element in S, we have $xs = bsx$ for some $b \in S$.

Now $xsy = bsxy = 0 \in < 0 >$. $\therefore < 0 >$ is a pseudo symmetric ideal.

Then by corollary 3.25, R = N iff S is an N – semigroup.

Theorem 3.27: If M is a maximal ideal in a semigroup S containing a pseudo symmetric ideal A, then M contains all A-potent elements in S or S\M is singleton which is A-potent.

Proof: Suppose M does not contain all A-potent elements. Let a∈S\M be any A-potent element and b be any element in S\M. Since M is a maximal ideal, M∪< a > = S = M∪< b >
⇒ < a > = < b >. Since b ∉ M, we have b∈< a >. Let n be the least positive integer such that a^n∈A. Since A is a pseudo symmetric ideal then A is a semipseudo symmetric ideal and hence $<a>^n$ ⊆A. Therefore b^n∈A and hence b is A-potent. Similarly we can show that if m is the least positive integer such that b^m∈A, then a^m∈A. Therefore there exists a natural number p such that a^P∈A and a^{P-1}∉A for all a∈S\M. Let a, b∈S\M. Since M is maximal ideal, we have $a = sbt$ for some s, t ∈S^1. Now since A is a pseudo symmetric ideal,
we have $(ab)^{p-1}= (ab)^{p-2}ab = (ab)^{p-2}\, sbtb \in A \Rightarrow ab \notin$ S\M. ∴ ab ∈M. Suppose a ≠ b. Then one of s, t is not an empty symbol say s. If s∈M then a∈M. If s∈S\M then sb∈M and hence a∈M. In both the cases we have a contradiction. Hence $a = b$.

Corollary 3.28: If M is a nontrivial maximal ideal in a semigroup S containing a pseudo symmetric ideal A. Then $N_o(A)$ ⊆M.

Proof: Suppose in $N_o(A)$ ⊄M. Then by above theorem 3.27, M is trivial ideal. It is a contradiction. Therefore $N_o(A)$ ⊆M.

Corollary 3.29: If M is a maximal ideal in a semisimple semigroup S containing a semipseudo symmetric ideal A. Then $N_o(A)$ ⊆M.

Proof: By the theorem 2.32, A is pseudo symmetric ideal. If a ∈ S\M is A-potent, then a cannot be semisimple. It is a contradiction. Therefore $N_o(A)$ ⊆ M.

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