EFFECT OF MELTING ON NON-DARCY MHD MIXED CONVECTIVE FLOW FROM A VERTICAL PLATE EMBEDDED IN A SATURATED POROUS MEDIUM.

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Abstract: In the present work melting with thermal dispersion on non-Darcy, MHD mixed convective heat transfer from an infinite vertical plate embedded in a saturated porous medium is studied. Both aiding and opposing flows are examined. Forchheimer extension for the flow equations in steady state is considered. Similarity solution for the governing equations is obtained. The equations are numerically solved using Runge-kutta fourth order method coupled with shooting technique. The effects of melting (M), thermal dispersion (D) inertia (F) and mixed convection (Ra/Pe) on velocity distribution, temperature and Nusselt number are examined in the presence of magnetic parameter MH. It is observed that the Nusselt number decreases with increase in melting parameter and increases with the increase in thermal dispersion parameter.

Key words: Porous medium, Melting, Thermal dispersion, Mixed Convection.

Nomenclature:

a = mean absorption coefficient
B_o = magnetic field strength
C = inertial coefficient
c_f = specific heat of convective fluid
c_s = specific heat of solid phase
d = mean particle diameter
D = thermal dispersion parameter
f = dimensionless stream function
F = dimensionless inertia parameter
g = acceleration due to gravity
h = local heat transfer coefficient
h_{sf} = latent heat of melting of solid
k_d = dispersion thermal conductivity
k = effective thermal conductivity
K = permeability of the porous medium
M = melting parameter
MH = square of the Hartmann number
Nu = local Nusselt number
Pe_x = local Pecklet number
q_w = wall heat flux
Ra_x = local Rayleigh number
T = temperature in thermal boundary layer
T_o = temperature at the solid region
u = velocity in x-direction
u_{∞} = external flow velocity
v = velocity in y- direction
x = coordinate along the melting plate
y = coordinate normal to melting plate

Greek Symbols:

α = equivalent thermal diffusivity
β = coefficient of thermal expansion
η = dimensionless similarity variable
θ = dimensionless temperature
μ = dynamic viscosity of fluid
υ = kinematic viscosity of fluid
ρ = density of convective fluid
ψ = stream function

Subscripts:

m = melting point
∞ = condition at infinity

Introduction: Heat transfer, in porous media, is seen both in natural phenomena and in engineered processes. It is replete with the features that are influences of the thermal properties and volume fractions of the materials involved. These features are seen, of course, as responses to the causes that force the process into action. For instance, many biological materials, whose outermost skin is porous and pervious, saturated or semi saturated with fluids give-out and take- in heat from their surroundings. Industrial fluids in interaction with heat supply agencies and flowing over porous beds carry convective heat to different regions of their field of flow near and distant from the heat source.

In many processes associated with phase change, heat transfer occurs with melting. Studies of this phenomenon have applications such as in casting, welding and magma solidification .The melting of permafrost, the thawing of frozen grounds, and the preparation of semiconductor materials are some more areas involving heat transfer with melting.

In studies of heat transfer, associated with melting Bakier [1] studied the melting effect on mixed convection from a vertical plate of arbitrary wall temperature both in aiding and opposing flows in a fluid saturated porous medium. He observed that the melting phenomena decrease the local Nusselt number at solid-liquid Interface. The problem of mixed convection in melting from a vertical plate of uniform temperature in a saturated porous medium is studied by Gorla *et al.* [2].They found that melting process is analogous to mass injection and blowing near the boundary and thus, reduces the heat transfer through solid liquid interface. Recently, Cheng and Lin [3] studied the melting effect on mixed convective heat transfer form a solid porous vertical plate with uniform wall temperature embedded in the liquid saturated porous medium using Runge-kutta Gill method and Newton's iteration for similarity solutions .They established the criteria for $\left(G_r / R_e\right)$ values for forced mixed and free convection from an isothermal vertical plate in porous media with aiding and opposing external flows in melting process.

None of the above mentioned studies in porous media included the effect of thermal dispersion which is a necessary secondary effect due to local fluid flow in the tortuous paths that exist in the porous medium [4,5]. It would be, therefore, more realistic to consider the thermal dispersion with melting. Hong and Tien [6] examined analytically the effect of transverse thermal dispersion on natural convection from a vertical, heated plate in a porous medium. Their results show that due to the better mixing of the thermal dispersion effect, the heat transfer rate is increased. Plumb [7] modeled thermal dispersion effects over a vertical plate.

Hydromagnetic flow and heat transfer problems are of great importance because of their industrial applications. As a result, many researches have been conducted to study the effects of electrically conducting fluids such as liquid metals, water and others in the presence of magnetic fields on the flow and heat transfer. There has been a renewed interest in studying magnetohydrodynamic (MHD) flow and heat transfer in porous and non-porous media due to the effect of magnetic fields on the control and on the performance of many systems using electrically conducting fluids. Among those, Raptis et al. [8] have analyzed hydro magnetic free convection flow through a porous medium between two parallel plates. Aldoss et al. [9] have studied mixed convection from a vertical plate embedded in a porous medium in the presence of a magnetic field. Bian et al. [10] have reported on the effect of an electromagnetic field on natural convection in an inclined porous medium. Buoyancy-driven convection, in a rectangular enclosure with a transverse magnetic field, has been considered by Garandet et al [11].

In plasma physics, liquid metal flow, magneto hydrodynamic accelerators and power generation systems, there is a necessity of studying the MHD flow with thermal dispersion effect in porous medium. Understanding the development of hydrodynamic thermal boundary layers along with the heat transfer characteristics is basic requirement to further investigate the problem. From a search of the specialized literature, it appears that the only contribution found to melting phenomena accounting for a combination of melting parameter and buoyancy effect under the influence of applied magnetic field is a non – Darcy flow field (Forchheirmer model) adjacent to a vertical impermeable wall embedded in a porous medium investigation [12]

Mathematical Formulation: Consider a vertical melting front at the melting point T_m. The co-ordinate system x – y is attached to the melting front as shown in figure1.The porous medium flows to the right with melting velocity across x – axis. The melting front is modeled as a vertical plate. This plate constitutes the inter phase between the liquid phase and the solid phase during melting inside the porous matrix. The temperature of the solid region is considered less than the melting point, i.e., $T_0 < T_m$. On the right hand side of melting front, the liquid is super heated, i.e., $T_\infty > T_m$. A vertical boundary layer flow, on the liquid side, smoothes the transition from T_m to T_∞. The assisting external flow velocity is taken as u_∞. It is also assumed that the system exists in thermodynamic equilibrium everywhere. The fluid is assumed to be Newtonian and electrically conducting with constant properties except the density variation in the buoyancy term. Transverse magnetic field is applied to the plate and the effects of flow inertia and thermal dispersion are included. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected. In addition, there is no applied electric field and hence the Hall effect and Joule heating are also neglected.

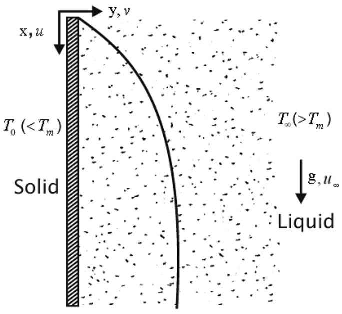

Fig.1. Schematic diagram of the problem

Taking into account the effect of thermal dispersion, the governing equations for steady non – Darcy flow in a porous medium can be written as follows.

The continuity equation is $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$ (1)

The momentum equation for a non - Darcy flow considering the magnetic effect is written as

$$\frac{\mu}{K}\frac{\partial u}{\partial y}+2\frac{\mu}{K}\left[\frac{C\sqrt{K}}{\upsilon}u\frac{\partial u}{\partial y}\right]+\sigma B_0^2\frac{\partial u}{\partial y}=-\rho_\infty g\beta\frac{\partial T}{\partial y} \quad (2)$$

The energy equation for the above condition is given by

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha\ \frac{\partial T}{\partial y}\right) \quad (3)$$

Thermal diffusivity $\alpha = \alpha_m + \alpha_d$, where α_m is the molecular diffusivity and α_d is the dispersion thermal diffusivity due to mechanical dispersion. As in the linear model proposed by Plumb [7], the dispersion thermal diffusivity α_d is proportional to the velocity component that is $\alpha_d = \gamma ud$, where γ is the dispersion coefficient and d is the mean particle diameter.

The boundary conditions for this model are

$$y=0,\ T=T_m,\ k\frac{\partial T}{\partial y}=\rho\left[h_{sf}+c_s\left(T_m-T_0\right)\right]v \quad (4)$$

and $y\to\infty,\ T\to T_\infty,\ u=u_\infty$ (5)

The stream function ψ with $u=\frac{\partial\psi}{\partial y}$, and $v=-\frac{\partial\psi}{\partial x}$ renders the continuity equation (1) satisfied and equations (2) and (3) transform to

$$\frac{\mu}{K}\frac{\partial^2\psi}{\partial y^2}+2\frac{\mu}{K}\left[\frac{C\sqrt{K}}{\upsilon}\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y^2}\right]+\sigma B_0^2\frac{\partial^2\psi}{\partial y^2}=-\rho_\infty g\beta\frac{\partial T}{\partial y} \quad (6)$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left[\left(\alpha_m+\gamma\frac{\partial\psi}{\partial y}d\right)\frac{\partial T}{\partial y}\right] \quad (7)$$

Introducing the similarity variables as

$$\psi=f(\eta)(\alpha_m u_\infty x)^{\frac{1}{2}},\quad \eta=\left(\frac{u_\infty x}{\alpha_m}\right)^{\frac{1}{2}}\left(\frac{y}{x}\right),\quad \theta(\eta)=\frac{T-T_m}{T_\infty-T_m},$$

equations (6) and (7) are reduced to

$$\left(1+MH+Ff'\right)f''+\frac{Ra_x}{Pe_x}\theta'=0 \quad (8)$$

$$(1+Df')\theta''+\left(\frac{1}{2}f+Df''\right)\theta'=0 \quad (9)$$

Where $Ra_x=\frac{g\beta K(T_\infty-T_m)x}{\upsilon\alpha_m}$ is the local Rayleigh number, $\frac{Ra_x}{Pe_x}$, is the mixed convection flow governing parameter (positive when the buoyancy is aiding the external flow and is negative when the buoyancy is opposing the external flow), $Pe_x=\frac{u_\infty x}{\alpha_m}$ is the local Pecklet number, $MH=\frac{\sigma B_0^2 K}{\rho\upsilon}$ is the magnetic parameter, $F=\frac{2C\sqrt{K}u_\infty}{\upsilon}$ is the flow inertia coefficient and $D=\frac{\gamma d u_\infty}{\alpha_m}$ is the thermal dispersion parameter.

Taking into consideration, the thermal dispersion effect together with melting the boundary conditions (4) and (5) take the form

$$\eta=0,\ \theta=0,\ f(0)+\{1+Df'(0)\}2M\theta'(0)=0 \quad (10)$$

And $\eta\to\infty,\ \theta=1,\ f'=1$ (11)

where $M=\frac{c_f\left(T_\infty-T_m\right)}{h_{sf}+c_s\left(T_m-T_0\right)}$ is the melting parameter. (12)

The local heat transfer rate from the surface of the plate is given by

$$q_w = -k\left[\frac{\partial T}{\partial y}\right]_{y=0}$$

(13)

The Nusselt number

$$Nu = \frac{hx}{k} = \frac{q_w x}{k(T_m - T_\infty)} \quad (14)$$

The modified Nusselt number is obtained as

$$\frac{Nu_x}{(Pe_x)^{\frac{1}{2}}} = [1 + Df'(0)]\theta'(0) \quad (15)$$

Solution Procedure: The dimensionless equations (8) and (9) together with the boundary conditions (10) and (11) are solved numerically by means of the fourth order Runge-kutta method coupled with the shooting technique. The solution, thus, obtained is matched with the given values of $f'(\infty)$ and θ(0). In addition, the boundary condition $\eta \to \infty$ is approximated by $\eta_{max} = 7$ which is found sufficiently large for the velocity and temperature to approach the relevant free stream properties. This choice of η_{max}, helps to compare the present results with these of earlier researchers.

Results and Discussion

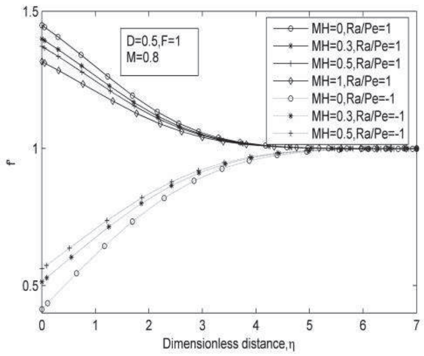

Fig.2. The velocity profiles for different values of (MH),in both aiding and opposing flows

Figure 2 shows that the effect of imposing a magnetic field and increasing its strength on the velocity profile at $\left|\frac{Ra}{Pe}\right|$=1, dispersion parameter D=0.5, flow inertia parameter F=1 when melting parameter M=0.8.The imposition of a magnetic field normal to the flow direction produces a resistive force that decelerates the motion of the fluid in the porous medium. It is obvious from the figure that the velocity decreases /increases in aiding/opposing flows as the magnetic parameter MH is increased in the presence of melting and dispersion effect.

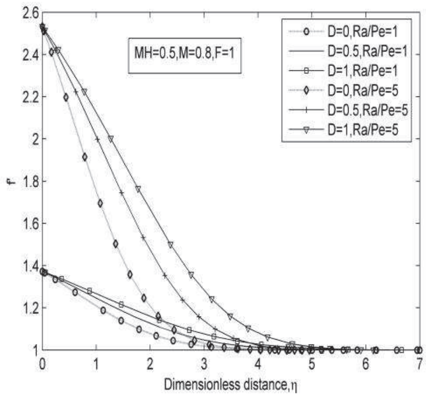

Fig.3. The velocity profiles for different dispersion parameters in aiding flow.

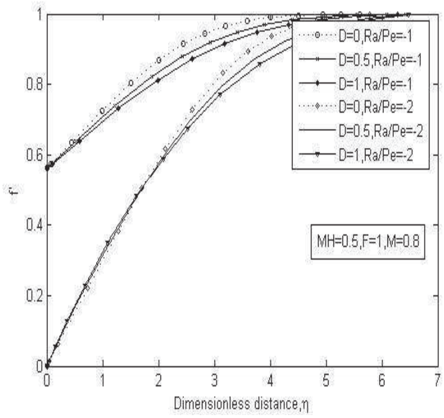

Fig.4. The velocity profiles for different dispersion parameters in opposing flow.

The dimensionless velocity component $f'(\eta)$ is presented in figures 3 and 4 in both aiding and opposing flows respectively, for the case of melting parameter M=0.8, magnetic parameter MH=0.5, flow inertia parameter F=1, and different buoyancy parameters Ra/Pe =1,5 when the buoyancy is aiding the flow and Ra/Pe=-1,-2 when the buoyancy is opposing the flow. In aiding flow (fig.3), the velocity increases with dispersion and the increment increases with the increase in the buoyancy parameter .In the opposing flow (fig.4.), the velocity decreases with increase in dispersion parameter.

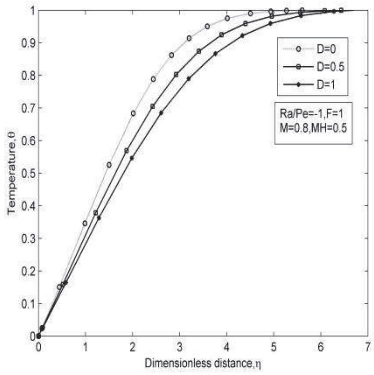

Fig.5. The temperature profiles for different dispersion parameters in aiding flow.

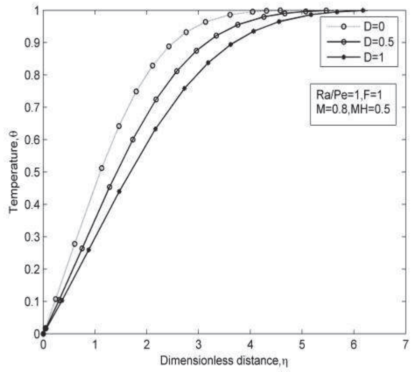

Fig.6. The temperature profiles for different dispersion parameters in opposing flow

The temperature profile θ(η) is presented in figures 5 and 6 in both aiding and opposing flows respectively , for the case of melting parameter M=0.8, magnetic parameter MH=0.5, flow inertia parameter F=1 , and the mixed convection parameter $\left|\frac{Ra}{Pe}\right|$=1 . The temperature decreases with dispersion both in aiding and opposing flows.

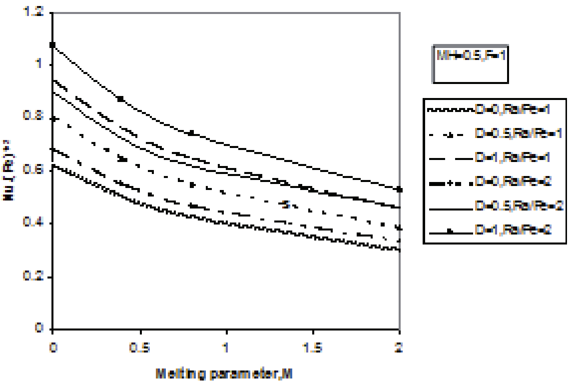

Fig.7. Nusselt number variation with M, for the case of D=0,0.5,1, and Ra/Pe=1,2 with MH=0.5 and F=1 in aiding flow.

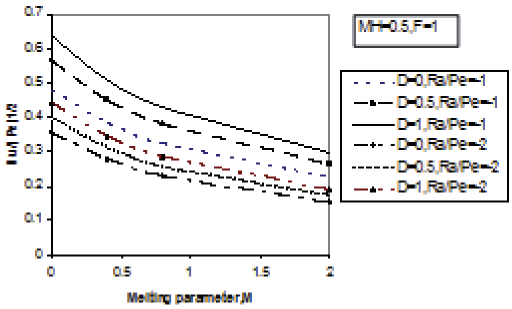

Fig.8. Nusselt number variation with M, for the case of D=0,0.5,1 ,and Ra/Pe=-1,-2 with MH=0.5 and F=1 in opposing flow.

Figures 7 and 8 show the effect of dispersion D and buoyancy parameter Ra/Pe on the average heat transfer coefficients as a function of melting parameter M in aiding and opposing flows respectively .The heat transfer increases/ decreases with increase in dispersion/ melting both in aiding and opposing flows for a fixed Ra/Pe value. In the presence of dispersion, this increment is more than in its absence.

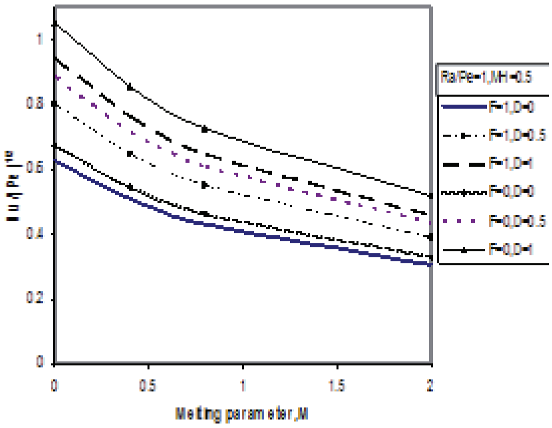

Fig.9. Nusselt number variation with M in Darcy and non-Darcy regionsfor different D with MH=0.5,Ra/Pe=1 in aiding flow.

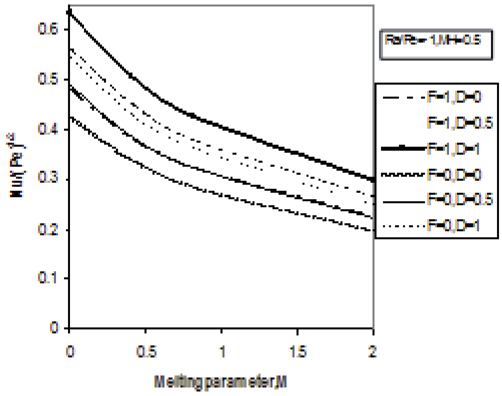

Fig. 10. Nusselt number variation with M in Darcy and non-Darcy regions for different D with MH=0.5,Ra/Pe=-1 in opposing flow.

In aiding /opposing flows, the Nusselt number decreases/increases from Darcy to non-Darcy regions both in the presence and in the absence of thermal dispersion effect. From figures 11, and 12, it can be seen that the magnetic field strength increases, the heat transfer rates decrease/increase in aiding/opposing flows due to the fact that the fluid velocity becomes lower and the thermal boundary layer increases.

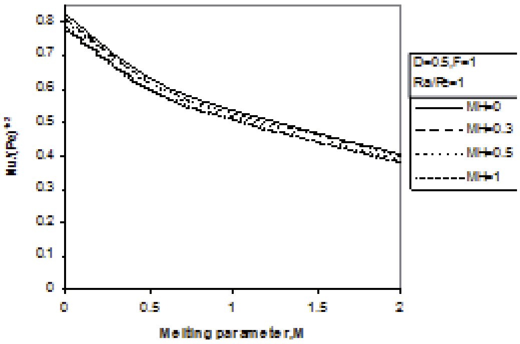

Fig 11.Nusselt number variation with M for different MH values with D=0.5,F=1, and fixed Ra/Pe=1 in aiding flow.

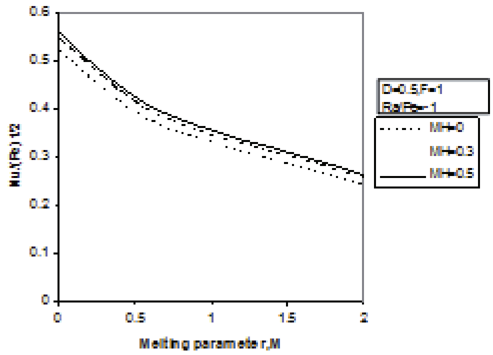

Fig .12.Nusselt number variation with M for different MH values with D=0.5,F=1, and fixed Ra/Pe=-1 in opposing flow.

Conclusions: The melting phenomenon has been analyzed with mixed convection flow and heat transfer in a fluid saturated porous medium under the influence of applied magnetic field with thermal dispersion effect by considering Forchheimer extension in the flow equations. These effects are analyzed for both aiding and opposing flows. This study shows that the velocity decreases/ increases in aiding/opposing flows as the magnetic parameter MH is increased in the presence of melting and dispersion effects. It is also seen that the velocity increases with increase in aiding buoyancy in the presence of dispersion and melting under the influence of applied magnetic field, while the velocity decreases as the opposing buoyancy becomes larger. Also, it is observed that the thermal dispersion effect in the presence of melting and applied magnetic field tends to increase/decrease the velocity within the boundary in aiding/ opposing flows and it is found that the temperature decreases as the dispersion parameter increases in both aiding and opposing flows. Heat transfer rate decreases with increase in melting parameter and increases with increase in dispersion parameter.

References:

1. A.David Christopher, M. Davamani Christober, on Number of Restricted Partitions With A Congruence; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Issue 2 (2014), Pg 780-783
2. Bakier, A.Y., Transport in Porous Media, 29, 127-139, 1997.
3. S.Vaithyasubramanian, A.Christy, A Practice to Create User Friendly Secured Password; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 3 Issue 1 (2014), Pg 272-274
4. Gorla, R.S.R., Mansour, Bakier, A.Y., Transport in Porous Media, 36,245- 254, 1999.
5. A.Praveen Prakash, V.S.Shrimathi, J.Esther Jerlin, A Study on the Issues/Problems Pertaining to Single Parents; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Issue 2 (2014), Pg 788-791
6. Cheng, W.T., and Lin, C.H., Int.J.Heat and Mass transfer, 50, 3026-3034, 2007.
7. Amiri, A., and Vafai, K., Int. J. Heat Mass Transfer, 37,936-954, 1994.
8. Chris Monica. M, Santhakumar. S, Partition Dimension Of Pyramid Network; Mathematical Sciences International Research Journal : ISSN 2278-8697Volume 4 Issue 2 (2015), Pg 446-449
9. Nield, D.A.,and Bejan,A., "Convection in Porous Media", Springer-verlag, NewYork,1992
10. Hong, J.T., and Tien,C.L., Int.J.Heat Mass Transfer, 30,143-150.,1987.
11. Dhanesh Garg, Bharat Goyal, Addtion of Two Dimensional Random Variables Using Continuous Distributions; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 4 Issue 1 (2015), Pg 233-236
12. Plumb, O.A., Proceedings of 1st ASME/JSME. Thermal Engg. Joint Conference, 2, 17-21,1983.
13. Raptis, A., Massias, C., and Tzivanidis, G., Phys. Lett.90A, 288-299, 1982.
14. Aldoss, T.K., Al-Nimr, and Al-Sha'er, B.J., Num. Heat Transfer, 28A, 635-645, 1995.
15. Bian, W., Vasseur, P., and Meng, F., Int.J. Heat Fluid Flow, 17, 36-44, 1996.
16. M. Rangamma, P. Rama Bhadra Murthy, Common Fixed Point Theorem Using Comparision Function In Cone B-Metric Spaces; Mathematical Sciences International Research Journal : ISSN 2278-8697Volume 4 Issue 2 (2015), Pg 461
17. Garandet, J.P., Moreau, R., Int.J.Heat Mass Transfer, 34, pp.741-748, 1992.
18. Tashtoush, B, Energy conversion and Management. 46, 2566-2577, 2005

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