

CERTAIN FILTERS IN TERNARY SEMIGROUPS

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**Abstract:** The notion of a ternary semigroup was introduced by Lehmer in 1932. In this paper the concepts of ternary filter, ternary filter of a ternary semigroup T generated by A, principal ternary filter are introduced and studied. We generalize the results in using the concepts of 3- system and 3\*- system. It is proved that (1) The nonempty intersection of two ternary filters of a ternary semigroup T is also a ternary filter and (2) The nonempty intersection of a family of ternary filters of a ternary semigroup T is also a ternary filter. It is proved that a nonempty subset F of a ternary semigroup T is a ternary filter if and only if T\F is a completely prime ideal of T or empty. It is also proved that if F is a ternary filter of a ternary semigroup T, then T\F is (i) a prime ideal, (ii) a completely semiprime ideal, (iii) a semiprime ideal of T or empty and finally (iv) a nonempty subset F of a commutative ternary semigroup T is a ternary filter if and only if T\F is a prime ideal of T or empty. Further it is proved that every ternary filter F of a ternary semigroup T is (1) a 3-system of T (2) a 3-system of T and (3) a 3\*-system of T.

**Key Words:** Ternary semigroup, Ternary ideal, Principal ideal, Ternary filter, 3\*-system and Principal ternary filter.

**Introduction:** The theory of ternary algebraic systems was introduced by LEHMER [7] in 1932, but earlier such structures was studied by KERNER [5] who give the idea of n-ary algebras. Ternary semigroups are universal algebras with one associative ternary operative. ANJANEYULU.A [1] initiated the study of ideals in semigroups. S.KAR and B.K.MAITY [4] initiated the study of some ideals of ternary semigroups. SIOSON. F.M [11] studied about ideal theory in ternary semigroups. SANTIAGO (1990) studied regular ternary semigroups. DIXIT and DEWAN (1995) studied quasi – ideals and bi-ideals in ternary semigroups. PETRICH. M. (1973) made a study on filters in general semigroups. LEE. S. K and LEE. S. S. [6], introduced the notion of a left (right) filter in a po-semigroup and gave a characterization of the left (right)-filter of S in term of the right (left) prime ideals. N. KEHAYOPULU [10], gave the characterization of the filter of S in terms of the prime ideals in ordered semigroups.

**Preliminaries:**

**Definition 2.1:** Let T be a non-empty set. Then T is said to be a Ternary semigroup if there exist a mapping from  $T \times T \times T$  to T which maps  $(x_1, x_2, x_3) \rightarrow [x_1 x_2 x_3]$  satisfying the condition :  $[[x_1 x_2 x_3] x_4 x_5] = [x_1 [x_2 x_3 x_4] x_5] = [x_1 x_2 [x_3 x_4 x_5]]$   $\forall x_i \in T, 1 \leq i \leq 5$ .

**Note 2.2:** Let T be a ternary semigroup. If A, B and C are three subsets of T, we shall denote the set  $ABC = \{abc : a \in A, b \in B, c \in C\}$ .

**Note 2.3:** Any semigroup can be reduced to a ternary semigroup.

**Definition 2.4:** A ternary semigroup T is said to be commutative provided  $abc = bca = cab = bac = cba = acb$  for all  $a, b, c \in T$ .

**Definition 2.5:** A ternary semigroup T is said to be quasi commutative provided for each  $a, b, c \in T$ , there exists a odd natural number n such that  $abc = b^n ac = bca = c^n ba = cab = a^n cb$ .

**Definition 2.6:** (IAMPAN. A [3]) : Let T be ternary semigroup. A non empty subset 'S' is said to be a ternary subsemigroup of T if  $abc \in S$  for all  $a, b, c \in S$ .

**Note 2.7:** A non empty subset S of a ternary semigroup T is a ternary subsemigroup if and only if  $SSS \subseteq S$ .

**Definition 2.8:** (SARITHA DEWAN [14]) : A nonempty subset A of a ternary semigroup T is said to be left (lateral, right) ternary ideal or left (lateral, right) ideal of T if  $b, c \in T, a \in A$  implies  $bca \in A$  ( $bac \in A, abc \in A$ ).

**Note 2.9:** A nonempty subset A of a ternary semigroup T is a left (lateral, right) ideal of T if and only if  $TTA \subseteq A$  ( $TAT \subseteq A, ATT \subseteq A$ ).

**Definition 2.10:** A nonempty subset  $A$  of a ternary semigroup  $T$  is *ternary ideal* or *ideal* of  $T$  if  $b, c \in T, a \in A$  implies  $bca \in A, bac \in A, abc \in A$ .

**Definition 2.11:** An ideal  $A$  of a ternary semigroup  $T$  is said to be a *principal ideal generated by  $a$*  provided  $A$  is an ideal generated by  $\{a\}$  for some  $a \in T$ . It is denoted by  $J(a)$  (or)  $\langle a \rangle$ .

**Definition 2.12:** An ideal  $A$  of a ternary semigroup  $T$  is said to be a *completely prime ideal* of  $T$  provided  $x, y, z \in T$  and  $xyz \in A$  implies either  $x \in A$  or  $y \in A$  or  $z \in A$ .

**Example 2.13:** In the commutative ternary semigroup  $Z^-$  of all negative integers, the ideal  $P = \{3k : k \in Z^-\}$  is a completely prime ideal. For  $x, y, z \in Z^-$ ,  $xyz \in P \Leftrightarrow xyz$  is divisible by 3  $\Leftrightarrow x$  is divisible by 3 or  $y$  is divisible by 3 or  $z$  is divisible by 3  $\Leftrightarrow x = 3k_1$  or  $y = 3k_2$  or  $z = 3k_3$  for  $k_1, k_2, k_3 \in Z^- \Leftrightarrow x \in P$  or  $y \in P$  or  $z \in P$ .

**Example 2.14:** In example 2.13,  $P$  is a completely prime ideal. But the ideal  $Q = \{30k : k \in Z^-\}$  is not a prime ideal of  $Z^-$ , since  $(-2)(-3)(-5) = -30 \in Q$  but  $(-2) \notin Q, (-3) \notin Q$  and  $(-5) \notin Q$ .

**Definition 2.15:** (Muhammad Shabir And Mehar Bano [10]) : An ideal  $A$  of a ternary semigroup  $T$  is said to be a *prime ideal* of  $T$  provided  $X, Y, Z$  are ideals of  $T$  and  $XYZ \subseteq A \Rightarrow X \subseteq A$  or  $Y \subseteq A$  or  $Z \subseteq A$ .

**Theorem 2.16:** Every completely prime ideal of a ternary semigroup  $T$  is a prime ideal of  $T$ .

**Proof:** Suppose that  $A$  is a completely prime ideal of a ternary semigroup  $T$ . Let  $a, b, c \in T$  and  $\langle a \rangle \langle b \rangle \langle c \rangle \subseteq A$ . Then  $abc \in A$ . Since  $A$  is a completely prime, either  $a \in A$  or  $b \in A$  or  $c \in A$ . Therefore  $A$  is a prime ideal of  $T$ .

**Theorem 2.17:** Let  $T$  be a commutative ternary semigroup. An ideal  $P$  of  $T$  is a prime ideal if and only if  $P$  is a completely prime ideal.

**Definition 2.18:** A nonempty subset  $A$  of a ternary semigroup  $T$  is said to be a *3-system* provided for any  $a, b, c \in A$  implies that  $T^1aT^1bT^1c \cap T^1A \neq \emptyset$ .

**Theorem 2.19:** An ideal  $A$  of a ternary semigroup  $T$  is a prime ideal of  $T$  if and only if  $T \setminus A$  is a 3-system of  $T$  or empty.

**Proof :** Suppose that  $A$  is a prime ideal of a ternary semigroup  $T$  and  $T \setminus A \neq \emptyset$ .

Let  $a, b, c \in T \setminus A$ . Then  $a \notin A, b \notin A$  and  $c \notin A$ . Suppose if possible  $T^1aT^1bT^1c \cap T \setminus A = \emptyset$ .

$T^1aT^1bT^1c \cap T \setminus A = \emptyset \Rightarrow T^1aT^1bT^1c \cap T^1A \subseteq A$ .

Since  $A$  is prime, either  $a \in A$  or  $b \in A$  or  $c \in A$ .

It is a contradiction. Therefore  $T^1aT^1bT^1c \cap T \setminus A \neq \emptyset$ .

Hence  $T \setminus A$  is a 3-system.

Conversely suppose that  $T \setminus A$  is either a 3-system of  $T$  or  $T \setminus A = \emptyset$ .

If  $T \setminus A = \emptyset$ , then  $T = A$  and hence  $A$  is a prime ideal of  $T$ .

Assume that  $T \setminus A$  is a 3-system of  $T$ . Let  $a, b, c \in T$  and  $\langle a \rangle \langle b \rangle \langle c \rangle \subseteq A$ .

Suppose if possible  $a \notin A, b \notin A$  and  $c \notin A$ . Then  $a, b, c \in T \setminus A$ . Since  $T \setminus A$  is a 3-system,

$\Rightarrow T^1aT^1bT^1c \cap T \setminus A \neq \emptyset \Rightarrow T^1aT^1bT^1c \cap T^1A \neq \emptyset$

$\Rightarrow \langle a \rangle \langle b \rangle \langle c \rangle \not\subseteq A$ . It is a contradiction.

Therefore  $a \in A$  or  $b \in A$  or  $c \in A$ . Hence  $A$  is a prime ideal of  $T$ .

**Definition 2.20:** An ideal  $A$  of a ternary semigroup  $T$  is said to be a *completely semiprime ideal* provided  $x \in T, x^n \in A$  for some odd natural number  $n > 1$  implies  $x \in A$ .

**Example 2.21:** In commutative ternary semigroup  $Z^-$  of all negative integers, the ideal  $Q = \{6k : k \in Z^-\}$  is a semiprime ideal. For  $x \in Z^-, x^3 \in Q \Leftrightarrow x^3$  is divisible by 6  $\Leftrightarrow x$  is divisible by 6  $\Leftrightarrow x = 6k_1$  for  $k_1 \in Z^- \Leftrightarrow x \in Q$ .

**Corollary 2.22:** If an ideal  $A$  of a ternary semigroup  $T$  is completely semiprime then  $x, y, z \in T, xyz \in A \Rightarrow \langle x \rangle \langle y \rangle \langle z \rangle \subseteq A$ .

**Theorem 2.23:** Every completely prime ideal of a ternary semigroup  $T$  is a completely semiprime ideal of  $T$ .

**Proof:** Let  $A$  be a completely prime ideal of a ternary semigroup  $T$ . Suppose that  $x \in T$  and  $x^3 \in A$ . Since  $A$  is a completely prime ideal of  $T, x \in A$ .

Therefore  $A$  is a completely semiprime ideal.

**Theorem 2.24:** Let  $A$  be a prime ideal of a ternary semigroup  $T$ . If  $A$  is completely semiprime ideal of  $T$  then  $A$  is completely prime.

**Proof:** Let  $x, y, z \in T$  and  $xyz \in A$ . Since  $A$  is completely semiprime, by corollary 2.22,  $xyz \in A \Rightarrow \langle x \rangle \langle y \rangle \langle z \rangle \subseteq A \Rightarrow x \in A$  or  $y \in A$  or  $z \in A$  and hence  $A$  is completely prime.

**Theorem 2.25:** The nonempty intersection of any family of a completely prime ideal of a ternary semigroup  $T$  is a completely semiprime ideal of  $T$ .

**Definition 2.26:** Let  $T$  be a ternary semigroup. A non-empty subset  $A$  of  $T$  is said to be a  $d$ -system of  $T$  if  $a \in A \Rightarrow a^3 \in A$ .

**Theorem 2.27:** An ideal  $A$  of a ternary semigroup  $T$  is completely semiprime if and only if  $T \setminus A$  is a  $d$ -system of  $T$  or empty.

**Proof:** Suppose that  $A$  is a completely semiprime ideal of  $T$  and  $T \setminus A \neq \emptyset$ .

Let  $a \in T \setminus A$ . Then  $a \notin A$ . Suppose if possible  $a^3 \notin T \setminus A$ .

Then  $a^3 \in A$ . Since  $A$  is a completely semiprime ideal then  $a \in A$ .

It is a contradiction. Therefore  $a^3 \in T \setminus A$  and hence  $T \setminus A$  is a  $d$ -system.

Conversely suppose that  $T \setminus A$  is a  $d$ -system of  $T$  or  $T \setminus A$  is empty.

If  $T \setminus A$  is empty then  $T = A$  and hence  $A$  is completely semiprime.

Assume that  $T \setminus A$  is a  $d$ -system of  $T$ . Let  $a \in T$  and  $a^3 \in A$ .

Suppose if possible  $a \notin A$ . Then  $a \in T \setminus A$ .

Since  $T \setminus A$  is a  $d$ -system,  $a^3 \in T \setminus A$ . It is a contradiction. Hence  $a \in A$ .

Thus  $A$  is a completely semiprime ideal of  $T$ .

**Definition 2.28:** An ideal  $A$  of a ternary semigroup  $T$  is said to be *semiprime ideal* provided  $X$  is an ideal of  $T$  and  $X^n \subseteq A$  for some odd natural number  $n$  implies  $X \subseteq A$ .

**Theorem 2.29:** Every completely semiprime ideal of a ternary semigroup  $T$  is a semiprime ideal of  $T$ .

**Proof:** Suppose that  $A$  is a completely semiprime ideal of a ternary semigroup  $T$ .

Let  $a \in T$  and  $\langle a \rangle^n \subseteq A$  for some odd natural number  $n$ .

Now  $aaa \dots a(n \text{ odd terms}) \in \langle a \rangle^n \subseteq \langle a \rangle \subseteq A \Rightarrow a^n \in A \Rightarrow a \in A \Rightarrow \langle a \rangle \subseteq A$ .

Therefore  $A$  is a semiprime ideal of  $T$ .

**Definition 2.30:** A non-empty subset  $A$  of a ternary semigroup  $T$  is said to be a  $3^*$ -system provided for any  $a \in A$  implies that  $T^i T^j a T^k T^l a T^m T^n \cap A \neq \emptyset$ .

**Theorem 2.31:** Every  $3^*$ -system in a ternary semigroup  $T$  is an  $3^*$ -system.

**Proof:** Let  $A$  be  $3^*$ -system of a ternary semigroup  $T$ . Let  $a \in A$ . Since  $A$  is  $3^*$ -system,  $a \in A$ ,  $T^i T^j a T^k T^l a T^m T^n \cap A \neq \emptyset$ . Therefore  $A$  is  $3^*$ -system of  $T$ .

**Theorem 2.32:** An ideal  $Q$  of a ternary semigroup  $T$  is a semiprime ideal if and only if  $T \setminus Q$  is a  $3^*$ -system of  $T$  (or) empty.

**Proof:** Suppose that  $A$  is a semiprime ideal of a ternary semigroup  $T$  and  $T \setminus A \neq \emptyset$ .

Let  $a \in T \setminus A$ . Then  $a \notin A$ .

Suppose if possible  $T^i T^j a T^k T^l a T^m T^n \cap T \setminus A = \emptyset$ .

$T^i T^j a T^k T^l a T^m T^n \cap T \setminus A = \emptyset \Rightarrow T^i T^j a T^k T^l a T^m T^n \subseteq A$ .

Since  $A$  is semiprime,  $a \in A$ .

It is a contradiction. Therefore  $T^i T^j a T^k T^l a T^m T^n \cap T \setminus A \neq \emptyset$ .

Hence  $T \setminus A$  is a  $3^*$ -system.

Conversely suppose that  $T \setminus A$  is either a  $3^*$ -system or  $T \setminus A = \emptyset$ .

If  $T \setminus A = \emptyset$  then  $T = A$  and hence  $A$  is a semiprime ideal.

Assume that  $T \setminus A$  is a  $3^*$ -system of  $T$ . Let  $a \in T$  and  $\langle a \rangle \subseteq A$ .

Let  $a \in T \setminus A$ ,  $T \setminus A$  is a  $3^*$ -system of  $T \Rightarrow T^i T^j a T^k T^l a T^m T^n \cap T \setminus A \neq \emptyset$ .

Suppose if possible  $a \notin A$ . Then  $a \in T \setminus A$ . Since  $T \setminus A$  is a  $3^*$ -system.

Then  $T^i T^j a T^k T^l a T^m T^n \subseteq T \setminus A$

$\Rightarrow T^i T^j a T^k T^l a T^m T^n \notin A \Rightarrow \langle a \rangle \not\subseteq A$ .

It is a contradiction. Therefore  $a \in A$ . Hence  $A$  is a semiprime ideal of  $T$ .

**Main Results In Ternary Filters:**

**Definition 3.1.1:** A Ternary Subsemigroup  $F$  is said to be *ternary filter* of  $T$

if  $a, b, c \in T$  and  $abc \in F$  implies  $a, b, c \in F$ .

**Example 3.1.2:** Let  $T = \{a, b, c, d\}$  with the ternary multiplication defined by

$$xyz = \begin{cases} b & \text{if } x = y = z = b \\ c & \text{if } x = y = z = c \\ d & \text{if } x = y = z = d \\ a & \text{otherwise} \end{cases}$$

Then  $T$  is a ternary semigroup and  $\{a, b, c, d\}, \{b\}, \{c\}, \{d\}$  are all filters of  $T$ .

**Theorem 3.1.3:** The nonempty intersection of two ternary filters of a ternary semigroup  $T$  is also a ternary semigroup of  $T$ .

**Proof:** Let  $A, B$  be two ternary filters of  $T$ .

Let  $a, b, c \in T$  and  $abc \in A \cap B$ .

$abc \in A \cap B \Rightarrow abc \in A$  and  $abc \in B$ .

$a, b, c \in T, abc \in A$  and  $A$  is a ternary filter of  $T \Rightarrow a, b, c \in A$ .

$a, b, c \in T$  ,  $abc \in B$  and  $B$  is a ternary filter of  $T \Rightarrow a, b, c \in B$ .

$a, b, c \in A$  and  $a, b, c \in B \Rightarrow a, b, c \in A \cap B$ .

Therefore  $A \cap B$  is a ternary filter of  $T$ .

**Theorem 3.1.4:** The nonempty intersection of a family of ternary filters of a ternary semigroup  $T$  is also a ternary filter of  $T$ .

**Proof:** Let  $\{F_\alpha\}_{\alpha \in \Delta}$  be a family of ternary filters of  $T$  and let  $F = \bigcap_{\alpha \in \Delta} F_\alpha$ .

Let  $a, b, c \in T$ ,  $abc \in F$ . Now  $abc \in F \Rightarrow abc \in \bigcap_{\alpha \in \Delta} F_\alpha \Rightarrow$

$abc \in F_\alpha$  for each  $\alpha \in \Delta$ .

$abc \in F_\alpha$ ,  $F_\alpha$  is a ternary filter of  $T \Rightarrow a, b, c \in F_\alpha$ .

$\Rightarrow a, b, c \in \bigcap_{\alpha \in \Delta} F_\alpha \Rightarrow a, b, c \in F$ . Therefore  $F$  is a ternary

filter of  $T$ .

**Note 3.1.5:** In general, the union of two ternary filters is not a ternary filter.

**Example 3.1.6:** As in the example 3.1.2,  $T$  is a ternary semigroup and  $\{b\}, \{c\}, \{d\}$ , are ternary filters, but  $\{b\} \cup \{c\} \cup \{d\}$  is not a ternary filter of  $T$ , because  $bcd = a$  is not in  $\{b\} \cup \{c\} \cup \{d\}$ .

**Theorem 3.1.7:** A nonempty subset  $F$  of a ternary semigroup  $T$  is a ternary filter if and only if  $T \setminus F$  is a completely prime ideal of  $T$  or empty.

**Proof:** Assume that  $T \setminus F \neq \emptyset$ . Let  $x, y, z \in T \setminus F$ .

Suppose that  $xyz \notin T \setminus F$ , then  $xyz \in F$ . Since  $F$  is ternary filter and hence  $x, y, z \in F$

It is a contradiction. Thus  $xyz \in T \setminus F$ , and so  $TT(T \setminus F) \subseteq T \setminus F$

Therefore  $T \setminus F$  is an ideal of  $T$ .

Now we shall prove that  $T \setminus F$  is completely prime. Let  $xyz \in T \setminus F$ , for  $x, y, z \in T$

Suppose that  $x \notin T \setminus F, y \notin T \setminus F$  and  $z \notin T \setminus F$ . then  $x \in F, y \in F$  and  $z \in F$ .

Since  $F$  is a ternary subsemigroup of  $T$ ,  $xyz \in F$

It is a contradiction. Thus  $x \in T \setminus F$  or  $y \in T \setminus F$  or  $z \in T \setminus F$ .

Hence  $T \setminus F$  is completely prime and hence  $T \setminus F$  is completely prime ideal of  $T$ .

Conversely suppose that,  $T \setminus F$  is completely prime ideal of  $T$  or empty.

If  $T \setminus F$  is empty, Then  $F = T$ . Thus  $F$  is ternary filter of  $T$ .

Assume that  $T \setminus F$  is completely prime ideal of  $T$ .

Suppose that for  $x, y, z \in F, xyz \notin F$ . Then  $xyz \in T \setminus F$ , for  $x, y, z \in F$ .

Since  $T \setminus F$  is completely prime,  $x \in T \setminus F$  or  $y \in T \setminus F$  or  $z \in T \setminus F$ .

It is contradiction. Thus  $xyz \in F$  and hence  $F$  is ternary subsemigroup of  $T$ .

Let  $x, y, z \in T$  and hence  $xyz \in F$ . If  $x, y, z \notin F$ . Then  $x, y, z \in T \setminus F$ .

Since  $T \setminus F$  is completely prime ideal of  $T$ , then  $xyz \in T \setminus F$ . It is a contradiction.

Thus  $x, y, z \in F$ . Therefore  $F$  is a ternary filter of  $T$ .

**Corollary 3.1.8:** Let  $T$  be a ternary semigroup. If  $F$  is a ternary filter, then  $T \setminus F$  is a prime ideal of  $T$  or empty.

**Proof:** Since  $F$  is a filter of  $T$ . By theorem 3.1.7,  $T \setminus F$  is a completely prime ideal of  $T$  or empty. By theorem 2.16,  $T \setminus F$  is a prime ideal of  $T$  or empty.

**Corollary 3.1.9:** A nonempty subset  $F$  of a commutative ternary semigroup  $T$  is a ternary filter if and only if  $T \setminus F$  is a prime ideal of  $T$  or empty.

**Proof:** Suppose that  $T \setminus F$  is a filter of commutative ternary semigroup  $T$ . By corollary 3.1.8,  $T \setminus F$  is prime ideal of  $T$  or empty.

Conversely suppose that  $T \setminus F$  is a prime ideal of  $T$  or empty. If  $T \setminus F = \emptyset$ , then  $F = T$ . Thus  $F$  is a filter of  $T$ . Assume that  $T \setminus F$  is a prime ideal of  $T$ . By theorem 2.17,  $T \setminus F$  is a completely prime ideal of  $T$  or empty. By theorem 3.1.7,  $F$  is a ternary filter of  $T$ .

**Theorem 3.1.10:** Every ternary filter  $F$  of a ternary semigroup  $T$  is a 3-system of  $T$ .

**Proof:** Suppose that  $F$  is a ternary filter of a ternary semigroup  $T$ . By corollary 3.1.8,  $T \setminus F$  is a prime ideal of  $T$ . By theorem 2.19,  $T \setminus (T \setminus F) = F$  is a 3-system of  $T$  or empty.

**Theorem 3.1.11:** Let  $T$  be a ternary semigroup. If  $F$  is a ternary filter, then  $T \setminus F$  is a completely semi prime ideal of  $T$ .

**Proof:** Since  $F$  is a ternary filter of a ternary semigroup  $T$ , by theorem 3.1.7,  $T \setminus F$  is a completely prime ideal of  $T$ . By theorem 2.25,  $T \setminus F$  is a completely semiprime ideal of  $T$ .

**Theorem 3.1.12:** Every ternary filter  $F$  of a ternary semigroup  $T$  is a  $d$ -system of  $T$ .

**Proof:** Since  $F$  is a ternary filter of a ternary semigroup  $T$ , by theorem 3.1.11,  $T \setminus F$  is a completely semiprime ideal of  $T$ . By theorem 2.27,  $T \setminus (T \setminus F) = F$  is a  $d$ -system of  $T$  or empty.

**Theorem 3.1.13:** Let  $T$  be a ternary semigroup. If  $F$  is a filter of  $T$ , then  $T \setminus F$  is a semiprime ideal of  $T$ .

**Proof:** Since  $F$  is a ternary filter of a ternary semigroup  $T$ . By theorem 3.1.7,  $T \setminus F$  is a completely prime ideal of  $T$ . By theorem 2.24,  $T \setminus F$  is a completely semiprime ideal of  $T$ . By theorem 2.29,  $T \setminus F$  is a semiprime ideal of  $T$ .

**Theorem 3.1.14:** Every ternary filter  $F$  of a ternary semigroup  $T$  is a  $3^*$ -system of  $T$ .

**Proof:** Since  $F$  is a ternary filter of a ternary semigroup  $T$ . By theorem 3.1.13,  $T \setminus F$  is a semiprime ideal of  $T$ . By theorem 2.32,  $T \setminus (T \setminus F) = F$  is a  $3^*$ -system of  $T$ .

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