ORDERING IN BOOLEAN NEAR-RINGS AND GENERALIZED D.G. NEAR RINGS

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Abstract: In this paper we discuss about ordering in Boolean near rings and generalized distributively generated near rings. Some examples which shows that a Boolean near ring is l-directed need not be a Boolean ring. Some results which are needed in proving the sub direct product representation of Boolean near rings. As d.g. Boolean near rings are Boolean rings, a meaningful subdirect product representation for Boolean near rings requires a weaker form of d.g.near rings. For this purpose we take the concept of g.d.g. near rings.

Key words: IFB, Up directed Set(μ-directed),down directed set(l-directed), g.d.g and a.a

Introduction: Throughout this paper N stands for a near ring. Some basic notations are given. We denote $d_r= \{n \in N/ dn=0\}$ and we define IFP (intersection of factors property),l-directed set, g.d.g (generalised distributively generated) near ring, abstract affine (a.a). As d.g Boolean near rings are Boolean rings, a meaningful sub direct product representation for Boolean near rings requires a weaker form of d.g near rings. For this purpose we introduce the concept of g.d.g near rings.

Ordering In Boolean Near-Rings:

Defnition 1.1: N is said to fulfill the intersection of factors property(IFP) provided that $ab=0 \Rightarrow anb=0$ forall $a,b,n \in N$.

Defnition 1.2: Let N be a Boolean near-ring. Define a relation ≤ on N by $a \leq b$ if a=ba.

Theorem 1.3: If N is a Boolean near-ring, then (N,≤) is partially ordered set.

Proof: since $a^2 =a$, we have that a≤a for all aєN. If a≤b and b≤a then a=ba and b=ab. Therefore a=ba=aba=aab=ab=b, since N is weak-commutative. Finally, suppose that a≤b and b≤c. Then a=ba and b=cb. Hence a=ba=cba=ca and this implies that a≤c. Thus ≤ is a partial order on N and hence(N,≤) is a poset.

Theorem 1.4: let N be a boolean near-ring.if a,bεN have an upper bound x in N, then ab=ba.In particular if a and b are comparable then ab=ba,

Proof: we have that a≤x and b≤x. Then a=xa and b=xb. By the weak-commutativity of N, ab=xab=xba=ba.

Corollary 1.5: If a Boolean near-ring N has the greatest element 1, then N is a boolean ring.

Proof: If N has the greatest element 1,then 1 is an upper bound for any two elements a,b of N. Hence by the theorem 1.4., ab=ba. By a known result , N is a Boolean ring.

Definition 1.6: A partially ordered set P is called an up directed (u-directed) set if any two elements of p have an upper bound in P. Similarly, one can define a down-directed (l-directed) set.

Remark 1.7: If a Boolean near-ring N is u-directed, then by theorem 1.4,N is a Boolean ring. Further, it is easy to prove that N has the least element if and only if N is zero-symmetric. Also, observe that no two elements of n_c are comparable, for if $n,n' \in N_c$ are comparable, then nn'=n'n and this implies that n=nn'=n'n=n'

Following example shows that a Boolean near ring, which is l-directed, need not be a Boolean ring.

Example 1.8: let Γ be an additively return group with zero 0.Define multiplication on Γ by ab=a if b≠0; ab=0 if b=0

Clearly Γ is a Boolean near ring such that for any a≠b ∈ Γ, 0 is the g.l.b. of a and b.

Notatoin1.9: let d be a distributive element in a near ring N.

We denote the set $\{n \in N/dn=0\}$ by d_r. Clearly d_r is a right ideal of N.

Lemma1.10: If N is a weak commutative near ring and if d is a distributive element in N then d_r is an ideal of N.

Proof: since d_r is an right ideal of N, it suffices if we prove that d_r is a left ideal. Let n,n'εN, iεd_r, since N is a weak-commutative, we have that

d[n(n'+i)-nn']=d(n(n'+i))-d(nn')=d(n'+i)n-dnn'=(dn'+di)n-dn'n=dn'n-dn'n=0.

Thus n(n'+i)-nn'εd_r. Hence d_r is an ideal of N.

Notation 1.11: let N be near-ring and nεN. Let(0:n)={n'εN/n'n=0}.

We need the following result.

Preposition 1.12: A near ring N has the IFP property if and only if (o:n) is an ideal for all nεN.

Definition 1.13: A near ring N is called sub directly irreducible if the intersection of all non zero ideals of N is non zero.

We prove the following result which is needed in proving the subdirect product representation of Boolean near rings.

Theorem 1.14: let N be a sub directly irreducible Boolean near-ring containing a non-zero distributive element. Then N is a two-element feild.

Proof: let d be a non-zero distributive element in N. Then $d\varepsilon N_o$ and hence $N_o \neq (o)$. Let J={nεN/(o:n)≠(o)}. Since(o:o)= N_o, we have that oεJ.hence J≠Φ. Now {(o:n)/nεJ} is a non-empty family of non-zero ideals. Since N is sub directly irreducible $\bigcap_{n\in J}$ (o:n)≠(o).

Let o≠eε $\bigcap_{n\in J}$ (o:n).

Since e does not belongs to (o:e), it follows that e does not belongs to J,

i.e. (o:e)=(o) for all aεN, (a-ae) e=ae-ae=o and hence a-aeε(o:e)=(o). Thua a=ae for aεN. So e is a right identity of N. If jεJ, then eε(o:j) and hence ej=o.

Now $j=je=(je)^2=(je)(je)=jo$. Therefore $j\varepsilon N_c$. Thus we have that j subset N_c. If b ∉J and by the above argument, we get that b is a right identity of N.

Let d_r ={n∈N/dn=o}. Then d_r is an ideal of N. If $n_c \in N_c$ then $dn_c=dn_co=don_c=do=o$. Therefore $n_c\varepsilon d_r$. Hence N_c is subset of d_r. We claim that $N_o \cap d_r=(o)$. Let $a\varepsilon N_o \cap d_r$. If a≠o, then a is a right identity, since a doesn't belongs to N_c. Thus d=da=o, this is a contradiction. Hence a=o. Thus we have proved our claim, namely $N_o \cap d_r=(o)$. Since N is a sub directly irreducible, we have that $d_r=(o)$ thus $N_c=(o)$ since N_c is subset of d_r. So, $N=N_o$ if $o\neq x\varepsilon N=N_o$ then d(d+x)=d+dx=d+d=o, since x is aright identity. Thus $d+x\varepsilon d_r=(o)$ and this implies that x=-d=d. Hence N={o,d} is a two element field.

Generalized D.G. Near Rings: Let N be a near ring and let N_o be the zero symmetric part of N. Then D(N) is sub set of N_o, where D(N) is the set of all distributive elements in N.

Definition2.1: A near ring N is called a genaralized distributively generated (g.d.g) near ring if D(N) generates(N_o,+).

It is clear that every ring is a g.d.g near ring and every constant near ring is a g.d.g near ring. Further every d.g near ring is also a g.d.g near ring. But the trivial near ring given at EXAMPLE 2.7 is not a g.d.g near ring, since $\Gamma_o=\Gamma$ and D(Γ)={o}.

Definition 2.2: A near ring N is called an abelian near ring if (N,+) is an abelian group.

Definition 2.3: A near ring N is called an abstract affine (a.a) near ring if (i) N is abelian, and (ii) N_o=D(N).

Clearly every a.a. near ring is a g.d.g near ring. Converse is true for abelian near-rings

Proof: To prove the theorem it suffices to show that N_o=(N). Clearly D(N) subset of N_o.

Let $n\varepsilon N_o$,n',n''εN.

Then

$n_o=\sum(\sigma_i d_i n'+\sigma_i d_i n'')=(\sum\sigma_i d_i n')+(\sum\sigma_i d_i n'')=n_on'+n_on''$.

Therefore $n_o\varepsilon D(N)$. Thus N_o subset D(N). Hence N_o=D(N). This completes the proof.

Corollary 2.4: let N be an abelian g.d.g. near –ring. Then N_o is a ring and N_c is an ideal of N.

Proof: since (N,+) is abelian, (N_o,+) is also abelian. By THEOREM 3.4, N_o =D(N).

Thus every element of N_o is distributive. Therefore N_o is a ring. Since N is abelian, N_c is a right ideal of a N. Let n,n'εN,$n''_c\varepsilon N_c$. Then n can be written as $n=n_o+n_c$ where $n_o\varepsilon N_o$, $n_c\varepsilon N_c$. Hence

$n(n'+n''_c)-nn'=(n_o+n_c)(n'+n_c)-(n_o+n_c)n'=n_o(n'+n''_c)+n_c(n'+n''_c)-(n_on'+n_cn')=n_on'+n_on''_c+n_c-n_on'-n_c$(since N_o=D(N))$=n_on''_c\varepsilon N_c$

Hence N_c is an ideal of N

Example 2.5: let v be a vector space over a feild F. A mapping v->v is called an affine map if it is the sum of a linear and a constant one. The set $M_{aff}(V)$ of all affine maps is a g.d.g. near ring under point wise addition and composition of mappings.

Theorem 2.6: If N' is a homomorphic image of a g.d.g near ring N then either N' contains a non zero distributive element or N' is a constant near ring.

Proof: let h:N->N' be an epimorphism. Then h(D(N)) subset of D(N')

If D(N')≠{o}, then clearly N' contains a non zero distributive element. Otherwise, h(D(N))={o} and hence D(N) subset of ker h since D(N) generates (N_o,+), we have that N_o subset ker h. Let n',m'εN'. Since h is an epimorphism, there exists m,nεN such that h(n)=n' and h(m)=m'.

suppose $n=n_o+n_c$ and $m=m_o+m_c$, where $n_o,m_o\varepsilon N_o$ and $n_c,m_c\varepsilon N_c$. Then

$n'm'=h(n)h(m)=h(n_o+n_c)h(m_o+m_c)=h(n_c)h(m_c)=h(n_c)$

=n'. Thus n'm'=n' for all n',m'εN'. Hence N' is a constant near ring.

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