STUDY OF GRAPH COLORING - ITS TYPES AND APPLICATIONS

A. MUNEERA

Abstract: Graph coloring is one of the best known, popular subject in the field of graph theory, having many applications. In this paper we present a survey of graph coloring as an important subfield of graph theory, describing various methods of coloring, and a list of problems associated with them. Graph coloring and its generalizations are useful tools in modeling a wide variety of scheduling and assignment problems. In this paper we review several variants of Graph coloring, such as vertex coloring, edge coloring, list coloring, multicoloring, generalized circular coloring and coloring rectangular paths. We discuss their applications in the scheduling and transportation problems by graph coloring.

Key words: Vertex coloring, Edge coloring, List coloring, Multi-coloring, Circular coloring and about scheduling.

Introduction: Graph coloring is one of the most important, well known and studied subfields of graph theory. An evidence of this can be found in various papers and books, in which the coloring is studied, and the problems associated with this field of research are being described andsolved. Good examples of such works are [1] and [2]. In the following sections of this paper, we describe brief history of graph coloring.

Vertex coloring: A proper vertex coloring problems for a graph G is to color all the vertices of the graph with different colors in such a way that any two adjacent (having an edge connecting them) vertices of G have assigned different colors.

In terms of graph theory, a proper vertex coloring with k colors is a mapping

f: V(G) → N such that for all ∀ $(v_i, v_j \in V(G), i \neq j \exists (e_i, e_j) \Rightarrow f(i) \neq f(j)$.

Vertex coloring of graphs can represent a mathematical model of various resource assignments. For example the maths department is having difficulties scheduling courses A – G because of limited room availability. Make a graph with vertices A – G. Make an edge between vertices if the corresponding courses cannot be scheduled at the same time.

The chromatic number of a graph is the smallest number of colors needed to color the vertices of so that no two adjacent vertices share the same color. i.e., the smallest value of possible to obtain a k-coloring.

Fact: The vertex coloring problem is NP – complete.

So many types of vertex colorings are there like circular vertex coloring, equitable vertex coloring, Acyclic – vertex coloring.

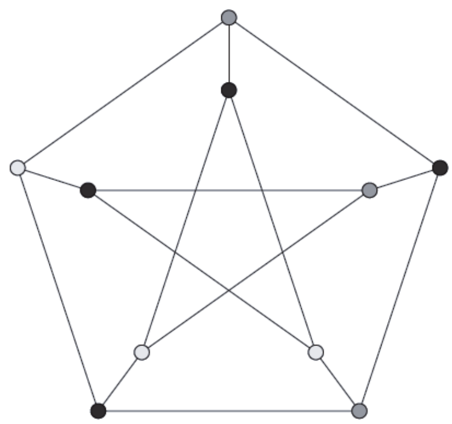


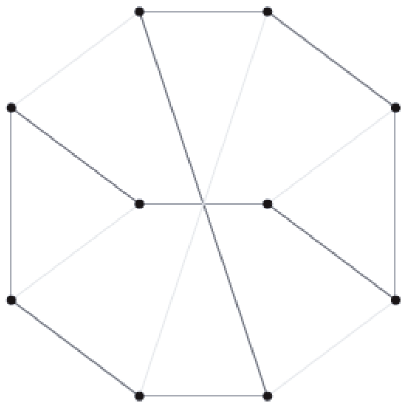
Figure 1 : A proper vertex coloring

Edge coloring: The other well-known and intensely studied type of graph coloring besides vertex coloring is the edge- coloring. The edge coloring of a graph G=(V, E) is a mapping, which assigns a color to every edge, satisfying condition that no two edges sharing a common vertex have the same color.

Mathematically, a proper edge coloring of a graph G is a mapping

f: E(G) → N such that $\forall (e_i, e_j \in E(G), i \neq j\ e_i, e_j$ are adjacent $\Rightarrow f(i) \neq f(j)$.

For example Time tabling problem involves factors such as teachers, classes and courses, various resources are rooms, time slots etc. Time tabling problem [6] is concerned with maximum utilization of the available resources subject to a set of constraints and can be classified [7] into three main classes namely school time tabling, course time tabling and examination time tabling.



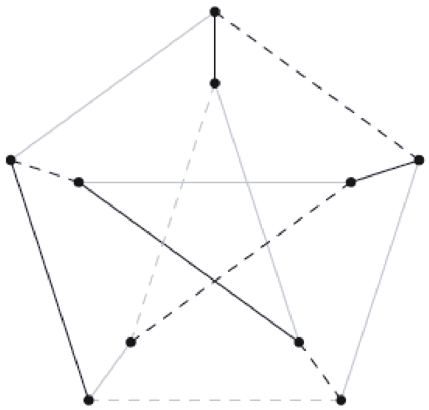


Figure 2 : Two examples of edge coloring.

List coloring: In terms of graph theory, a vertex list coloring is a mapping, which assigns every vertex a color from the vertex's list such that no two adjacent vertices have the same color. The edge list coloring can be defined analogically; in this case no two adjacent edges are painted with the same color. The list coloring of graphs can represent a mathematical model of job scheduling, where vertices represent employees, which will have assigns jobs, and the lists-describe possible jobs for a given vertex. The list - coloring and a detailed review of its properties shown in paper [9].

Multi coloring: In the multi-Coloring problem each vertex v has a demand x(v), and we have to assign a set of x(v) colors to each vertex v such that neighbors receive disjoint sets of colors. Multi-coloring can be used to model the scheduling of jobs with different time requirements, the set of colors assigned to vertex v corresponds to the x(v) time slots when we work on the job.

There are two main variants of multi-coloring, In non-Preemptive multi-coloring the set of colors assigned to a vertex has to be continuous interval of colors. This reflects the requirement that the jobs cannot be interrupted, they have to receive a continuous time window.

On the other hand, in preemptive multi-coloring we assume that the jobs can be interrupted, hence the set of colors assigned to a vertex can be arbitrary. It does not have to be continuous.

Like the graph coloring problem, the multi-coloring problem can model a number of applications. It is used in scheduling [11] where each node represents a job, edges represent jobs cannot be done simultaneously, and the colors represent time units. Each job requires multiple time units and can be scheduled preemptively. The minimum number of colors then represents the make span of the instance. Multi-colorings also arise in telecommunication channel assignment where the nodes represent transmitters, edges represent interference, and the transmitters send out signals on multiple wavelengths (the colors)[12]. It is due to this application in telecommunications that multi-coloring, as well as generalizations that further restrict feasible colorings, Aardal et al [10] provide an excellent survey on these problems.

The multi- coloring problem can be reduced to graph coloring by replacing each node by a clique of size equal to required number of colors. Edges are then replaced with complete bipartite graph between the corresponding cliques; such a transformation both increases the size of the graph and embeds an unwanted symmetry into the problem.

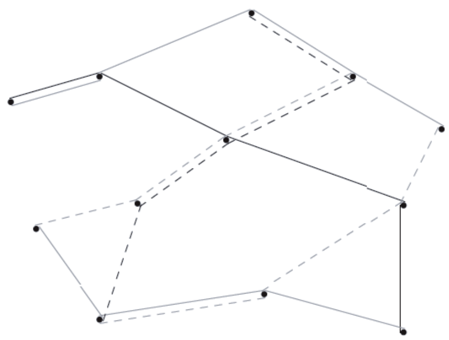


Figure 3 : Multi coloring of a graph

Generalized circular coloring: The study of circular chromatic number χ_c (G) of a graph G, which is a refinement of its chromatic number. The circular chromatic number of a graph can be defined in a few different but equivalent ways.

Circular – r- coloring: Suppose G: (V , E) is a graph and r ≥ 1 is real number. A circular-r-coloring of G is a mapping f : V → [o , r) such that for any edge xy of G, 1 ≤ $|f(x) - f(y)|$ ≤ r-1. We say a graph G is circular r-colorable if G has a circular r-coloring. The circular chromatic number χ_c (G) of G is defined as χ_c (G) = ing {r: G is circular r-colorable}

If r = k is an integer, then a k-coloring is a circular k-coloring. Conversely, if f is a circular k-coloring of G then g(v) = [f(v)] defines a k – coloring of G. So a graph G is circular k-colorable if and only if G is k-colorable. For this reason, a circular r-coloring of a graph G is usually simply called an r-coloring of a graph G instead of saying G is circular r-colorable, we usually simply say G is r- colorable.

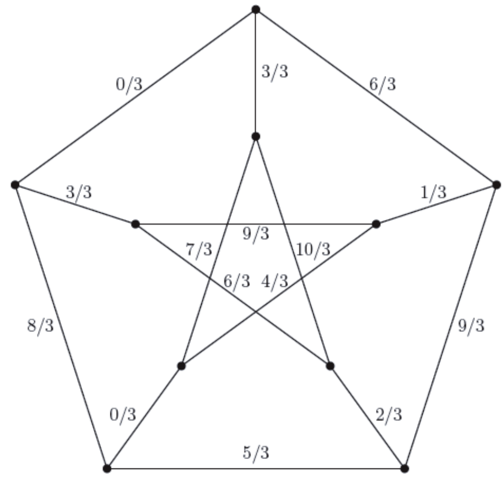


Figure 4: A circular edge coloring of Petersen graph.

(p, q) – coloring: Another useful definition of circular chromatic number uses the concept of (p,q)-coloring, where only finitely many colors are used. Suppose p ≥ q are positive integers. A (p, q) – coloring of a graph G =(V, E) is a mapping f : V → {0,1, 2,.....p-1} such that for any edge xy of G, q ≤ $|f(x) - f(y)|$ ≤ p – q. If f is a (p,q) – coloring of G, then the mapping g(x) = $\frac{f(x)}{q}$ defines a $\frac{p}{q}$ - coloring of G. conversely if g is a $\frac{p}{q}$ - coloring of G, then the mapping f(x) = [g(x) q] defines a (p,q) –coloring.

Therefore for any graph G, χ_c (G) = inf { $\frac{p}{q}$: G has a (p , q)- coloring }

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A. Muneera/ Department of Mathematics/
Andhra Loyola Institute of Engineering and Technology/ Vijayawada/