A CHARACTERIZATION OF CONGRUENCE PERMUTABLE IN PRE A*-ALGEBRA

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Abstract: In this paper we define an ideal of Pre A*-algebra A and discuss certain examples. We study two types of fundamental congruences on a Pre A*-algebra A and discuss various properties of these, it is proved that for any element a of Pre A*-algebra $\theta_a = \{(x, y) \in A \times A /\ a \wedge x = a \wedge y\}$ then $\theta_a = \beta_{a^{\sim}} = \{(x, y) \in A \times A /\ a^{\sim} \vee x = a^{\sim} \vee y\}$. We prove let A be a Pre A*-algebra and a, b$\in$A then $\theta_a \, o \, \theta_b \subseteq \theta_{a \wedge b}$ also we remark that the converse of the above theorem is not true, which the congruences need not permute. We give sufficient conditions for two congruences on a Pre A*-algebra A to be permutable.

Keywords: Boolean algebra, Pre A*-algebra, Ideal, congruence, permutable.

Introduction: In 1994, P. Koteswara Rao [2] first introduced the concept A*-Algebra $(A, \wedge, \vee, *, (-)^{\sim}, (-)_{\pi}, 0, 1, 2)$ not only studied the equivalence with Ada, C-algebra, Ada's connection with 3- Ring, the If-Then-Else structure over A*-algebra and Ideal of A*-algebra. In 2000, J. Venkateswara Rao [5] introduced the concept of Pre A*-algebra $(A, \wedge, \vee, (-)^{\sim})$ as the variety generated by the 3-element algebra A = {0, 1, 2} which is an algebraic form of three valued conditional logic. In [8] Satyanarayana et al. generated Semilattice structure on Pre A*-Algebras .In [6], Venkateswara Rao.K.and Srinivasa Rao.K defined a partial ordering on a Pre A*-algebra A and the properties of A as a poset are studied. In [9] Satyanarayana.A, et.all derive necessary and sufficient conditions for pre A*-algebra A to become a Boolean algebra in terms of the partial ordering.

In this paper we define an ideal of Pre A*-algebra A and discuss certain examples. We proved that for any element a of Pre A*-algebra then $\theta_a = \beta_{a^{\sim}}$. We also prove let A be a Pre A*-algebra and a, b$\in$A then $\theta_a \, o \, \theta_b \subseteq \theta_{a \wedge b}$ also we remark that the converse of the above theorem is not true, which the congruences need not permute. We give sufficient conditions for two congruences on a Pre A*-algebra A to be permutable.

Preliminaries: In this section we concentrate on the algebraic structure of Pre A*-algebra and state some results which will be used in the later text.

Definition: An algebra $(A, \wedge, \vee, (-)^{\sim})$ where A is non-empty set with 1, $\wedge$, $\vee$ are binary operations and $(-)^{\sim}$ is a unary operation satisfying

(a) $x^{\sim\sim} = x \quad \forall x \in A$

(b) $x \wedge x = x, \quad \forall x \in A$

(c) $x \wedge y = y \wedge x, \quad \forall x, y \in A$

(d) $(x \wedge y)^{\sim} = x^{\sim} \vee y^{\sim} \quad \forall x, y \in A$

(e) $x \wedge (y \wedge z) = (x \wedge y) \wedge z, \quad \forall x, y, z \in A$

(f) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z), \quad \forall x, y, z \in A$

(g) $x \wedge y = x \wedge (x^{\sim} \vee y), \quad \forall x, y \in A$ is called a Pre A*-algebra.

Example:3 = {0, 1, 2} with operations $\wedge, \vee, (-)^{\sim}$ defined below is a Pre A*-algebra.

$\wedge$	0	1	2
0	0	0	2
1	0	1	2
2	2	2	2

$\vee$	0	1	2
0	0	1	2
1	1	1	2
2	2	2	2

x	$x^{\sim}$
0	1
1	0
2	2

Note: The elements 0, 1, 2 in the above example satisfy the following laws:

(a) $2^{\sim} = 2$

(b) $1 \wedge x = x$ for all $x \in \mathbf{3}$

(c) $0 \vee x = x$ for all $x \in \mathbf{3}$

(d) $2 \wedge x = 2 \vee x = 2$ for all $x \in \mathbf{3}$.

Example: $\mathbf{2} = \{0, 1\}$ with operations $\wedge, \vee, (-)^{\sim}$ defined below is a Pre A*-algebra.

$\wedge$	0	1
0	0	0
1	0	1

$\vee$	0	1
0	0	1
1	1	1

x	$x^{\sim}$
0	1
1	0

Note:

1. $\left(2, \vee, \wedge, (-)^{\sim}\right)$ is a Boolean algebra. So every Boolean algebra is a Pre A* algebra.
2. The identities 1.1(a) and 1.1(d) imply that the varieties of Pre A*-algebras satisfies all the dual statements of 1.1(b) to 1.1(g).

Definition: Let A be a Pre A*-algebra. An element $x \in A$ is called central element of A if $x \vee x^{\sim} = 1$ and the set$\{x \in A / x \vee x^{\sim} = 1\}$ of all central elements of A is called the centre of A and it is denoted by B (A).

Theorem: [6] Let A be a Pre A*-algebra with 1, then B (A) is a Boolean algebra with the induced operations $\wedge, \vee, (-)^{\sim}$

Ideals and Congruences on Pre A*-algebra

Definition: A nonempty subset U of a Pre A*-algebra A is said to be an ideal of A if the following hold

(i) $a, b \in U \Rightarrow a \vee b \in U$

(ii) $a \in U \Rightarrow x \wedge a \in U$ for each $x \in A$

Example: All the ideals of Pre A*-algebra A={0, 1, 2} are $I_1 = \{2\}$ $I_2 = \{0, 2\}$ and A itself.Now we give some examples of Pre A*-algebras and collect all the ideals of these.

Example: Let $G = \{a_1, a_2, a_3, a_4, a_5\}$ where $a_1 = (1, 2)$, $a_2 = (0, 2)$, $a_3 = (2, 1)$, $a_4 = (2, 0)$, $a_5 = (2, 2)$

Then G is a Pre A*-algebra (a sub algebra of $A \times A$) under the point wise operations given in the following tables

x	$x^{\sim}$
a_1	a_2
a_2	a_1
a_3	a_4
a_4	a_3
a_5	a_5

$\wedge$	a_1	a_2	a_3	a_4	a_5
a_1	a_1	a_2	a_5	a_5	a_5
a_2	a_2	a_2	a_5	a_5	a_5
a_3	a_5	a_5	a_3	a_4	a_5
a_4	a_5	a_5	a_4	a_4	a_5
a_5	a_5	a_5	a_5	a_5	a_5

$\vee$	a_1	a_2	a_3	a_4	a_5
a_1	a_1	a_1	a_5	a_5	a_5
a_2	a_1	a_2	a_5	a_5	a_5
a_3	a_5	a_5	a_3	a_3	a_5
a_4	a_5	a_5	a_3	a_4	a_5
a_5	a_5	a_5	a_5	a_5	a_5

This algebra (G, $\wedge, \vee, (-)^{\sim}$) is a Pre A*-algebra without 1.All the ideals of G are

$I_1=\{a_5\}$, $I_2=\{a_2, a_5\}$, $I_3=\{a_4, a_5\}$, $I_4=\{a_1, a_2, a_5\}$, $I_5=\{a_2, a_4, a_5\}$, $I_6=\{a_3, a_4, a_5\}$, $I_7=\{a_1, a_2, a_4, a_5\}$, $I_8=\{a_2, a_3, a_4, a_5\}$, $I_9= G$

Example: Let $H = \{b_1, b_2, b_3, b_4, b_5, b_6, b_7\}$ where $b_1 = (1, 2)$, $b_2 = (0, 2)$, $b_3 = (2, 1)$, $b_4 = (2, 0)$, $b_5 = (2, 2)$, $b_6 = (1, 1)$, $b_7 = (0, 0)$. Then H is a Pre A*-algebra (a sub algebra of $A \times A$) under the point wise operations given in the following tables

x	$x^{\sim}$
b_1	b_2
b_2	b_1
b_3	b_4
b_4	b_3

$\wedge$	b_1	b_2	b_3	b_4	b_5	b_6	b_7
b_1	b_1	b_2	b_5	b_5	b_5	b_1	b_2
b_2	b_2	b_2	b_5	b_5	b_5	b_2	b_2
b_3	b_5	b_5	b_3	b_4	b_5	b_3	b_4
b_4	b_5	b_5	b_4	b_4	b_5	b_4	b_4

$\vee$	b_1	b_2	b_3	b_4	b_5	b_6	b_7
b_1	b_1	b_1	b_5	b_5	b_5	b_1	b_2
b_2	b_1	b_2	b_5	b_5	b_5	b_1	b_2
b_3	b_5	b_5	b_3	b_3	b_5	b_3	b_3
b_4	b_5	b_5	b_3	b_4	b_5	b_3	b_4

b_5	b_5
b_6	b_7
b_7	b_6

b_5	b_5	b_5	b_5	b_5	b_5	b_5	b_5
b_6	b_1	b_2	b_3	b_4	b_5	b_6	b_7
b_7	b_2	b_2	b_4	b_4	b_5	b_7	b_7

b_5	b_5	b_5	b_5	b_5	b_5	b_5	b_5
b_6	b_1	b_1	b_3	b_3	b_5	b_6	b_6
b_7	b_1	b_2	b_3	b_4	b_5	b_6	b_7

This algebra (H , ∧ , ∨, $(-)^{\sim}$) is a Pre A*-algebra with 1 and 1= b_6 . All the ideals of H are
$I_1=\{b_5\}$, $I_2=\{b_2, b_5\}$, $I_3=\{b_4, b_5\}$, $I_4=\{b_1, b_2, b_5\}$, $I_5=\{b_3, b_4, b_5\}$, $I_6=\{b_1, b_2, b_4, b_5\}$, $I_7=\{b_2, b_3, b_4, b_5\}$, $I_8=\{b_2, b_4, b_5, b_7\}$, $I_9=\{b_1, b_2, b_3, b_4, b_5\}$, $I_{10}=\{b_1, b_2, b_4, b_5, b_7\}$, $I_{11}=\{b_1, b_2, b_3, b_4, b_5, b_7\}$, $I_{12}=\{b_2, b_3, b_4, b_5, b_7\}$, $I_{13}=$ H

Definition: Let A be a Pre A*-algebra and θ be binary relation on A. Then θ is said to be an equivalence relation on A if θ satisfies the following:
(i) Reflexive: $(x, x) \in \theta$, for all $x \in$ A
(ii)Symmetric: $(x, y) \in \theta \Rightarrow (y, x) \in \theta$,for all $x, y \in$ A
(iii)Transitive: $(x, y) \in \theta$ and $(y, z) \in \theta \Rightarrow (x, z) \in \theta$,for all $x, y, z \in$ A .
we write $x\,\theta\,$y to indicate $(x, y) \in \theta$

Definition: A relation θ on a Pre A*- algebra $(A, \wedge, \vee, (-)^{\sim})$ is called congruence relation if
(i) θ is an equivalence relation
(ii) θ is closed under $\wedge, \vee, (-)^{\sim}$.

Lemma: Let $(A, \wedge, \vee, (-)^{\sim})$ be a Pre A*-algebra and let $a \in A$. Then the relation $\theta_a = \{(x, y) \in A \times A /\ a \wedge x = a \wedge y\}$ is a congruence relation.

Proof: Since $a \wedge x = a \wedge x$ then $(x, x) \in \theta_a$, the relation is reflexive.
Let $(x, y) \in \theta_a$ then $a \wedge x = a \wedge y$ $\Rightarrow a \wedge y = a \wedge x \Rightarrow (y, x) \in \theta_a$, the relation is symmetric
Let $(x, y) \in \theta_a$ and $(y, z) \in \theta_a$ then $a \wedge x = a \wedge y$ and $a \wedge y = a \wedge z \Rightarrow a \wedge x = a \wedge z \Rightarrow (x, z) \in \theta_a$, the relation is transitive.
Hence the relation θ_a is equivalence relation.
Let $x, y, z, t \in L$ such that $(x, y) \in \theta_a$ and $(z, t) \in \theta_a$ then $a \wedge x = a \wedge y$, $a \wedge z = a \wedge t$
Now $a \wedge (x \vee z) = (a \wedge x) \vee (a \wedge z)$
$= (a \wedge y) \vee (a \wedge t)$ $= a \wedge (y \vee t)$
This shows that $(x \vee z, y \vee t) \in \theta_a$
Hence θ_a is closed under ∨.
Also $a \wedge (x \wedge z) = (a \wedge x) \wedge (a \wedge z)$
$= (a \wedge y) \wedge (a \wedge t)$ $= a \wedge (y \wedge t)$
This shows that $(x \wedge z, y \wedge t) \in \theta_a$
Hence θ_a is closed under ∧.
If $x, y \in A$ satisfy $(x, y) \in \theta_a$, then $a \wedge x = a \wedge y$
$\Rightarrow a^{\sim} \vee x^{\sim} = a^{\sim} \vee y^{\sim}$
$\Rightarrow a \wedge (a^{\sim} \vee x^{\sim}) = a \wedge (a^{\sim} \vee y^{\sim})$
$\Rightarrow a \wedge x^{\sim} = a \wedge y^{\sim}$
$\Rightarrow (x^{\sim}, y^{\sim}) \in \theta_a$
Therefore θ_a is closed under $(-)^{\sim}$
Therefore θ_a is a congruence relation on A .

Lemma: Let $(A, \wedge, \vee, (-)^{\sim})$ be a Pre A*-algebra and let $a \in A$ Then the relation $\beta_a = \{(x, y) \in A \times A /\ a \vee x = a \vee y\}$ is a congruence relation.

Lemma: For any element a of Pre A*-algebra then $\theta_a = \beta_{a^{\sim}}$

Proof: Let a, $x, y \in$ A.
Let $(x, y) \in \theta_a \Rightarrow$ a∧ x =a∧ y
$\Rightarrow$ $a^{\sim} \vee (a \wedge x) = a^{\sim} \vee (a \wedge y)$
$\Rightarrow$ $a^{\sim} \vee x = a^{\sim} \vee y$ (from Definition (g))
$\Rightarrow (x, y) \in \beta_{a^{\sim}}$
There fore $\theta_a \subseteq \beta_{a^{\sim}}$
Let $(x, y) \in \beta_{a^{\sim}} \Rightarrow a^{\sim} \vee x = a^{\sim} \vee y$
$\Rightarrow$ $a \wedge (a^{\sim} \vee x) = a \wedge (a^{\sim} \vee y)$
$\Rightarrow$ a∧ x =a∧ y (from 1.1 Definition (g))
$\Rightarrow (x, y) \in \theta_a$
Therefore $\beta_{a^{\sim}} \subseteq \theta_a$

Hence $\theta_a = \beta_{a^\sim}$

Definition: If A is Pre A*-algebra then the congruences A×A and $\{(x, x) / x \in A\}$ are denoted by ∇_A and Δ_A respectively.

Definition: Let A be a Pre A*-algebra and α, β be binary relations on A. Then we define $\alpha \, o \, \beta = \{(x, y) \in A\times A \,/\, (x, z) \in \beta$ and $(z, y) \in \alpha$ for some $z \in A\}$.

Definition: Let A be a Pre A*-algebra and $\alpha, \beta \in$ Con(A).Then α, β are said to be permutable if $\alpha \, o \, \beta = \beta \, o \, \alpha$.

Lemma: Let A be a Pre A*-algebra and a, b ∈ A then $\theta_a \, o \, \theta_b \subseteq \theta_{a\wedge b}$

Proof: Let $(x, y) \in \theta_a \, o \, \theta_b$. Then there exist $z \in A$ such that $(x, z) \in \theta_b$ and $(z, y) \in \theta_a$.

Thus $b \wedge x = b \wedge z$ and $a \wedge z = a \wedge y$.

Now $a \wedge b \wedge x = a \wedge b \wedge z$

$= a \wedge b \wedge a \wedge z$

$= a \wedge b \wedge a \wedge y$

$= a \wedge b \wedge y$

Therefore $(x, y) \in \theta_{a\wedge b}$, Hence $\theta_a \, o \, \theta_b \subseteq \theta_{a\wedge b}$.

We remark that the converse of the above theorem is not true, that is $\theta_{a\wedge b} \subseteq \theta_a \, o \, \theta_b$ is not true and which the congruence need not permute.

Example: Let **G**={a_1, a_2, a_3, a_4, a_5} where $a_1 = (1, 2)$, $a_2 = (0, 2)$, $a_3 = (2, 1)$, $a_4 = (2, 0)$, $a_5 = (2, 2)$. Then G is a Pre A*-algebra (a sub algebra of A×A) under the point wise operations given in the Example. Then we have the following.

$\theta a_1 = \{(x, y) / a_1 \wedge x = a_1 \wedge y\}$

$= \Delta_A \cup \{(a_3, a_4), (a_4, a_3), (a_4, a_5), (a_5, a_4), (a_5, a_3), (a_3, a_5)\}$

$\theta a_3 = \{(x, y) / a_3 \wedge x = a_3 \wedge y\}$

$= \Delta_A \cup \{(a_1, a_2), (a_2, a_1), (a_2, a_5), (a_5, a_2), (a_5, a_1), (a_1, a_5)\}$

Now $\theta a_1 \, o \, \theta a_3 = \Delta_A \cup \theta a_1 \cup \theta a_3 \cup \{(a_2, a_4), (a_2, a_3), (a_1, a_4), (a_1, a_3)\}$

$\theta a_3 \, o \, \theta a_1 = \Delta_A \cup \theta a_1 \cup \theta a_3 \cup \{(a_4, a_2), (a_3, a_2), (a_4, a_1), (a_3, a_1)\}$

Also $\theta a_1 \wedge a_3 = \theta a_5 = A\times A$.

Therefore $\theta a_1 \wedge a_3 \not\subseteq \theta a_1 \, o \, \theta a_3$ and $\theta a_1 \, o \, \theta a_3 \neq \theta a_3 \, o \, \theta a_1$.

Example: Let H = {$b_1, b_2, b_3, b_4, b_5, b_6, b_7$} where $b_1 = (1, 2)$, $b_2 = (0, 2)$, $b_3 = (2, 1)$, $b_4 = (2, 0)$, $b_5 = (2, 2)$, $b_6 = (1, 1)$, $b_7 = (0, 0)$. Then H is a Pre A*-algebra (a sub algebra of A×A) under the point wise operations given in the Example.

$\theta a_1 = \{(x, y) / a_1 \wedge x = a_1 \wedge y\} = \Delta_A \cup \{(b_1, b_6), (b_6, b_1), (b_3, b_4), (b_4, b_3), (b_4, b_5), (b_5, b_4) (b_3, b_5), (b_5, b_3), (b_2, b_7), (b_7, b_2)\}$

$\theta a_3 = \{(x, y) / a_3 \wedge x = a_3 \wedge y\} = \Delta_A \cup \{(b_1, b_2), (b_2, b_1), (b_3, b_6), (b_6, b_3), (b_1, b_5), (b_5, b_1) (b_2, b_5), (b_5, b_2), (b_4, b_7), (b_7, b_4)\}$

Now $\theta a_1 \, o \, \theta a_3 = \Delta_A \cup \theta a_1 \cup \theta a_3 \cup \{(b_1, b_7), (b_2, b_6), (b_3, b_1), (b_6, b_4), (b_6, b_5), (b_1, b_3), (b_1, b_4), (b_5, b_6), (b_2, b_4), (b_2, b_3), (b_5, b_7), (b_4, b_2), (b_7, b_5), (b_7, b_3)\}$

$\theta a_3 \, o \, \theta a_1 = \Delta_A \cup \theta a_1 \cup \theta a_3 \cup \{(b_1, b_3), (b_6, b_2), (b_6, b_5), (b_3, b_7), (b_4, b_6), (b_4, b_1), (b_4, b_2), (b_5, b_7), (b_5, b_6), (b_3, b_1), (b_3, b_2), (b_7, b_5), (b_2, b_4), (b_7, b_1)\}$

Also $\theta a_1 \wedge a_3 = \theta a_5 = A\times A$.

Therefore $\theta a_1 \wedge a_3 \not\subseteq \theta a_1 \, o \, \theta a_3$ and $\theta a_1 \, o \, \theta a_3 \neq \theta a_3 \, o \, \theta a_1$.

Theorem: Let A be a Pre A*-algebra with 1 and a, b ∈ B(A) then θ_a, θ_b are permute and $\theta_a \, o \, \theta_b = \theta_{a\wedge b}$

Proof: Let A be a Pre A*-algebra with 1 and a, b ∈ B(A).

By lemma 7.19 we have $\theta_a \, o \, \theta_b \subseteq \theta_{a\wedge b}$

Let $(p, q) \in \theta_{a\wedge b} \Rightarrow a \wedge b \wedge p = a \wedge b \wedge q$ ------ (i)

Consider $r = (b \wedge p) \vee (b^\sim \wedge q)$

Now $b \wedge r = b \wedge ((b \wedge p) \vee (b^\sim \wedge q))$

$= (b \wedge b \wedge p) \vee (b \wedge b^\sim \wedge q)$

$= (b \wedge p) \vee (0 \wedge q)$ (Since b ∈ B (A))

$= (b \wedge p) \vee 0$ (provided $q \neq 2$)

$= b \wedge p$

If $q = 2$ then $r = (b \wedge p) \vee (b^\sim \wedge q) = 2$ and also $b \wedge r = 2$. For $b \wedge p = 2$, p should be 2.

Therefore $b \wedge r = b \wedge p$

$\Rightarrow (p, r) \in \theta_b$

Now $a \wedge r = a \wedge ((b \wedge p) \vee (b^\sim \wedge q))$

$= (a \wedge b \wedge p) \vee (a \wedge b^\sim \wedge q)$

$= (a \wedge b \wedge q) \vee (a \wedge b^\sim \wedge q)$ (from (i))

$= a \wedge ((b \vee b^\sim) \wedge q)$

$= b \wedge (1 \wedge q)$ (Since $b \in B(A)$)

$= b \wedge q$

Therefore (r, q) $\in \theta_b$

We have (p, r) $\in \theta_b$ and (r, q) $\in \theta_a$ hence (p, q) $\in \theta_a$ $o\ \theta_b$, which implies that $\theta_{a \wedge b} \subseteq \theta_a\ o\ \theta_b$

Hence $\theta_a\ o\ \theta_b = \theta_{a \wedge b}$.

We have $\theta_a\ o\ \theta_b = \theta_{a \wedge b} = \theta_{b \wedge a} = \theta_b\ o\ \theta_a$.

Hence θ_a, θ_b are permute congruences.

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