PROBABILISTIC NANO APPROXIMATION SPACE

Dr. M. Lellis Thivagar
School of Mathematics, Madurai Kamaraj University, Madurai- 625 021, Tamilnadu, India
I.Geetha Antoinette
Jayaraj Annapackiam College for Women,Periyakulam, Theni - 625 601, Tamilnadu, India

Abstract: The primary aim of this paper is to introduce a new type of approximation space nano topologized stochastic approximation space. The concept of nano open sets in weak form and strong form are used to define the lower and upper probability of an event.Different types are defined and compared using nano measure.We provide results, examples in digraph.

Keywords: Nano topology, Nano topologized approximation space, Nano topologized stochastic approximation space

2010 AMS Subject Classification : 54C05.

Introduction: The most powerful notion in analysis is the concept of topological structures and their generations. Topology is a branch of mathematics,it's concepts exist in almost all branches and also in real life problems. Lellis Thivagar et al [1] introduced nano topological space with respect to a subset X of an finite universe which is defined interms of lower and upper approximations of X. The elements of the nano topological space are called nano open sets.In this paper we have introduced a new space nano topologized stochastic approximation space from the approximation space. We define the upper and lower probability of an event interms of the lower and upper approximation of the event using the nano open set,nano α - open set, nano semi- open set, nano regular open set and nano generalised open set and the results are compared. Nano closure space in digraph is also introduced and the upper and lower probability of a subgraph of a digraph is defined and studied.

Preliminaries:

Definition 2.1 :[6] Let U be a non-empty finite set of objects called the universe and R be an equivalance relation on U named as the indiscernibility relation. Elements belonging to the same equivalance class are said to indiscernible with one another.The pair (U,R) is said to be the approximation space.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by ($\mathsf{U}, \tau_R(X)$). That is

$L_R(X) = \cup\{R(x) : R(x) \subseteq X\}$ where R(x)denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is

$U_R(X) = \cup\{R(x) : R(x) \cap X \neq \emptyset\}$

(iii) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$.That is

$B_R(X) = \mathsf{U}_R(X) - L_R(X)$.

Definition 2.2 *:* [6] Let U be the universe, R be an equivalance relation on U and $\tau_R(X) = \{\mathsf{U}, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where X $\subseteq \mathsf{U}$. Then $\tau_R(X)$ satisfies the following axioms

(i) U and $\emptyset \in \tau_R(X)$.

(ii) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is $\tau_R(X)$ is a topology on U called the nanotopology on U with repect to X. We call ($\mathsf{U}, \tau_R(X)$) as the nano topological space. The elements of $\tau_R(x)$ are called as nano open sets.

Definition 2.3: :[6] If (U, $\tau_R(X)$) is a nano topological space with respect to X where X $\subseteq$ U and if A $\subseteq$ U,then the nano interior of A is defined as the union of all nano-open subsets of A and it is denoted by Nint(A). That is Nint(A)is the largest nano-open subset of A. The nano closure of A is defined as the intersection of all nanoclosed sets containing A and it is denoted by Ncl(A). That is Ncl(A)is the smallest nano closed set containing A.

Definition 2.4 :[6] Let (U, $\tau_R(X)$) be a nano topological space and A$\subseteq U$. Then A is said to be

(i) nano semi open if A $\subseteq \mathrm{N}cl(\mathrm{N}int(A))$

(ii) nano pre-open if A $\subseteq \mathrm{N}int(\mathrm{N}cl(A))$

(iii) nano α -open if A $\subseteq \mathrm{N}int(\mathrm{N}cl(\mathrm{N}int(A))$

(iv) nano regular open if A = $\mathrm{N}int(\mathrm{N}cl(A))$

NSO(U,X), NPO(U,X), Nα O(U,X) respectively denote the families of all nano semi-open, nano pre-open and nano α -open subset of U.

Definition 2.5 *:* Let (U, $\tau_R(X)$) be a nano topological space and A$\subseteq U$. Then A is said to be nano generalised closed set (briefly, $\mathrm{N}g-closed$) if $\mathrm{N}cl(A) \subseteq G$ whenever $A \subseteq G$ where G is nano open in (U, $\tau_R(X)$). Complement of a nano generalised closed set is called nano generalised open set.

Definition 2.6:[2] A graph G is an ordered pair of disjoint sets (V, E), where V is nonempty and E is a subset of unordered pairs of V. The vertices and edges of a graph G are the elements of V=V(G) and E=E(G) respectively. We say that a graph G is finite (resp. infinite) if the set V(G) is finite (resp.finite). The degree of a vert ex u$\in$ V(G)is the number of edge in a graph contains a vertex u. u is called an isolated point if the degree of u is zero. An edge which has the same vertex to ends is called a loop and the edge with distinct ends is called a link.

Definition 2.7:[2] A graph is simple if it has no loops and no two of its links join the same same pair of vertices. A graph which has no edge called a null graph. A graph which has no vertices is called a empty graph.

Definition 2.8: [2] If G(V, E) is a directed graph and u,v $\in$ V, then

(i) u is invertex of v if $\overleftarrow{uv} \in E(G)$.

(ii) u is outvertex of v if $\overleftarrow{vu} \in E(G)$.

(iii) The indegree of a vertex 'v' is the number of vertices 'u' such that $\overleftarrow{uv} \in E(G)$.

(iv) The outdegree of a vertex 'v' is the number of vertices 'u' such that $\overleftarrow{vu} \in E(G)$.

Probability in Nano Topological Spaces: Here we introduce nano topologized approximation space, nano topologized stochastic approximation space, nano measure and discuss their properties.

Definition 3.1 *:* Let U be a non-empty finite set of objects called the universe, R be an equivalence relation on U then (U,R) is called the approximation space. Let $\tau_R(A)$ is the nano topology associated with a subset A of U then the triple $(U, R, \tau_R(A))$ is called the nano-topological approximation space.

Definition 3.2 *:* Let (U,R) be the approximation space with the equivalence relation R and $\tau_R(A)$ is the nano topology associated with a subset A of U. Let p be the probability measure with the following properties $p(\emptyset)$ = 0, $p(U)$= 1 and if B = $\cup X_i$ then $p(B)$= $\sum p(X_i)$. Then $(U, R, p, \tau_R(A))$ is called the nano-topologized stochastic approximation space.

Definition 3.3 *:* Let B be an event in the nano-topologized stochastic approximation space $(U, R, p, \tau_R(A))$ then the nano lower and nano upper probability of B is given by

$_{N}\underline{p}(B) = p(Nint(B))$

$_{N}\overline{p}(B) = p(Ncl(B))$

Definition 3.4 *:* Let B be an event in the nano-topologized stochastic approximation space $(U, R, p, \tau_R(A))$ then the nano generalised lower and upper probability of B is given by

$_{Ng}\underline{p}(B) = p(Ngint(B))$

$_{Ng}\overline{p}(B) = p(Ngcl(B))$

Definition 3.5 *:* Let B be an event in the nano-topologized stochastic approximation space $(U, R, p, \tau_R(A))$ then

(i) The nano measure of B is given by $\mu^*(B) = {}_{N}\overline{p}(B) - {}_{N}\underline{p}(B)$

(ii) The nano generalised measure of B is given by $\mu_g^*(B) = {}_{Ng}\overline{p}(B) - {}_{Ng}\underline{p}(B)$

Proposition 3.6 *:* Let X and Y are events in the nano-topologized stochastic approximation space $(U, R, p, \tau_R(A))$ then the nano generalised lower and upper probability of B satisfy the following properties

(i) $_{Ng}\underline{p}(\emptyset) = {}_{Ng}\overline{p}(\emptyset) = 0$

(ii) $_{Ng}\underline{p}(U) = {}_{Ng}\overline{p}(U) = 1$

(iii) $_{Ng}\underline{p}(X) \leq p(X) \leq {}_{Ng}\overline{p}(X)$

(iv) $_{Ng}\underline{p}(X \cup Y) \geq {}_{Ng}\underline{p}(X) + {}_{Ng}\underline{p}(Y)$

(v) $_{Ng}\overline{p}(x \cup Y) = {}_{Ng}\overline{p}(X) + {}_{Ng}\overline{p}(Y)$

(vi) $_{Ng}\underline{p}(X \cap Y) = {}_{Ng}\underline{p}(X) \;.\; {}_{Ng}\underline{p}(Y)$

(vii) $_{Ng}\overline{p}(X \cap Y) \leq {}_{Ng}\overline{p}(X) \;.\; {}_{Ng}\overline{p}(Y)$

Proof:

(i) Since $Ngint(\emptyset) = Ngcl(\emptyset) = \emptyset$.

Therefore p($Ngint(\emptyset)$) = p($Ngcl(\emptyset)$) = p($\emptyset$) = 0. By the definition we get the result.

(ii) Since $Ngint(U) = Ngcl(U) = U$.

Hence p($Ngint(U)$) = p($Ngcl(U)$) = p(U) = 1.we get the result.

(iii) Since $Ngint(X) \subseteq X \subseteq Ngcl(X)$.

Therefore p($Ngint(X)$) ≤ p(X) ≤ p($Ngcl(X)$).

From the definition we get the required result.

(iv) We know that $Ngint(X \cup Y) \supseteq Ngint(X) \cup Ngint(Y)$.

Therefore p($Ngint(X \cup Y)$ ≥ p($Ngint(X)$) + p($Ngint(Y)$). Hence the result.

(v) Since $Ngcl(X \cup Y) = Ngcl(X) \cup Ngcl(Y)$.

Hence p($Ngcl(X \cup Y)$) = p($Ngcl(X)$) + p($Ngcl(Y)$).We get the result.

(vi) Since $Ngint(X \cap Y) = Ngint(X) \cap Ngint(Y)$.

Therefore p($Ngint(X \cap Y)$) =p($Ngint(X)$) . p($Ngint(Y)$). We get the result.

(vii) Since $Ngcl(X \cap Y) \subseteq Ngcl(X) \cap Ngcl(Y)$.

Therefore p($Ngcl(X \cap Y)$) ≤ p($Ngcl(X)$) . p($Ngcl(Y)$). We get the result.

Proposition 3.7 *:* Let X and Y are events in the nano-topologized stochastic approximation space $(U, R, p, \tau_R(A))$ then the nano generalised measure of X and Y satisfy the following properties

(i) $\mu_g^*(X \cup Y) \leq \mu_g^*(X) + \mu_g^*(Y)$

(ii) $\mu_g^*(X \cap Y) \leq \mu_g^*(X) \,.\, \mu_g^*(Y)$

Proof :

(i) $\mu_g^*(X \cup Y) = {}_{Ng}\overline{p}(X \cup Y) - {}_{Ng}\underline{p}(X \cup Y)$. By the proposition (v) and (vi) we get $\mu_g^*(X \cup Y) \leq {}_{Ng}\overline{p}(X) + {}_{Ng}\overline{p}(Y) - {}_{Ng}\underline{p}(X) - {}_{Ng}\underline{p}(Y) \leq \mu_g^*(X) + \mu_g^*(Y)$

(ii) $\mu_g^*(X \cap Y) \leq {}_{Ng}\overline{p}(X \cap Y) - {}_{Ng}\underline{p}(X \cap Y)$. By the proposition (vii) and (viii) we get $\mu_g^*(X \cap Y) \leq {}_{Ng}\overline{p}(X) \,.\, {}_{Ng}\overline{p}(Y) - {}_{Ng}\underline{p}(X) \,.\, {}_{Ng}\underline{p}(Y) \leq \mu_g^*(X) \,.\, \mu_g^*(Y)$

Consider the experiment of choosing one card from four cards numbered from one to four.The collection of four elements forms the outcome space $U = \{1,2,3,4\}$. Let R be the equivalence relation on U such that $U/R = \{\{1\},\{2\},\{3,4\}\}$. Let A= $\{2,3\}$ thus

$\tau_R(A) = \{U, \emptyset, \{2\}, \{2,3,4\}, \{3,4\}\}$. Define the variable X to be the number on the chosen card. The following table gives the nano lower and upper probabilities of the random variable X.

X	1	2	3	4
${}_N\underline{p}(X = x)$	0	1/4	0	0
${}_N\overline{p}(X = x)$	1/4	1/2	¾	3/4

The following table gives the nano generalised lower and upper probabilities of the random variable X

X	1	2	3	4
${}_{Ng}\underline{p}(X = x)$	0	1/4	1/4	1/4
${}_{Ng}\overline{p}(X = x)$	1/4	1/2	½	1/2

To find the measure : Consider the event X = $\{3\}$

$$\mu^*(X) = {}_N\overline{p}(X) - {}_N\underline{p}(X) = \frac{3}{4} - 0 = \frac{3}{4}$$

$$\mu_g^*(X) = {}_{Ng}\overline{p}(X) - {}_{Ng}\underline{p}(X) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

Therefore,we get $0 \leq \mu_g^*(X) \leq \mu^*(X) \leq 1$.

Remark 3.9 *:*

a) Neither the sum of the nano lower probabilities nor the sum of nano upper probabilities equal to one.
b) Neither the sum of the nano generalised lower probabilities nor the sum of nano generalised upper probabilities equal to one.
c) The nano generalised measure of an event is smaller than the nano measure of an event.

Near Probability in Nano Topological Spaces: Here we find some rules to define nano j-lower probability and nano j-upper probability of an event B where j represent the near open sets in nano toplogy

Definition 4.1 *:* Let B be an event in the nano-topologized stochastic approximation space $(U, R, p, \tau_R(A))$ then the nano j-lower probability and nano j-upper probability of B is given by

${}_{Nj}\underline{p}(B) = p(Njint(B))$

${}_{Nj}\overline{p}(B) = p(Njcl(B))$

where j $\in \{\alpha, s, r\}$

Proposition 4.2 *:* Let B be an event in the nano-topologized stochastic approximation space $(U, R, p, \tau_R(A))$

then the implication between the nano j- lower probability is given by the following diagram for all j $\in \{\alpha, s, r, g\}$

${}_{N}\underline{p}(B) \leq {}_{NS}\underline{p}(B) \leq {}_{N\alpha}\underline{p}(B) \leq {}_{Nr}\underline{p}(B) \leq {}_{Ng}\underline{p}(B)$

Proof : The proof is obvious.

Proposition 4.3: Let B be an event in the nano-topologized stochastic approximation space $(U, R, p, \tau_R(A))$ then the implication between the nano j- upper probability is given by the following diagram for all j $\in \{\alpha, s, r, g\}$

${}_{Ng}\overline{p}(B) \leq {}_{Ns}\overline{p}(B) \leq {}_{N\alpha}\overline{p}(B) \leq {}_{N}\overline{p}(B) \leq {}_{Nr}\overline{p}(B)$

Proof : The proof is obvious.

Probability in Digraph via Nano g-open sets: Here we introduced nano closure space and nano generalised lower and upper probabilities on a digraph.

Definition 5.1 *:* Let G=[V(G), E(G)] be a digraph and $Cl_G: P[V(G)] \to P[V(G)]$ an operator such that

(i) It is Gm-closure operator if $Cl_{G_m}[V(H)] = Cl_G[Cl_G(.....(Cl_G(V(H))]$ m times, forevery subgraph $H \subseteq G$.

(ii) It is called G_m- topological closure operator if $Cl_{G_{m+1}}[V(H)] = Cl_{G_m}[V(H)]$ for all $H \subseteq G$.

Definition 5.2 *:* Let $G_m = (G, Cl_{G_m})$ be an approximation space where G be a nonempty finite universe graph and Cl_{G_m} be the closure general relation on G and τ_{CG_m} is the G_m-topological space associated with the G_m. Then the triple G_m = [G, Cl_G, τ_{CG_m}] is called as G_m topological closure approximation space.

Definition 5.3: *Let G_m = [G, Cl_G, τ_{CG_m}] be a G_m topological closure approximation space and H be any subgraph of G then $[V(G), \tau_{G_m}(V(H)]$ is called the $nanoClG_m$ topological space.*

Definition 5.4 *:Let $[V(G), \tau_{G_m}(V(H)]$ be the $nanoClG_m$ topological space where G_m = [G, Cl_G, τ_{CG_m}] be the G_m topological closure approximation space and H be any subgraph of G.Then $[V(G), \tau_{G_m}(V(H), p]$ is called the $nanoClG_m$ topological stochastic approximation space.*

Definition 5.5: *Let K be an event in the $nanoClG_m$ topological stochastic approximation space $[V(G), \tau_{G_m}(V(H), p]$ then the nano generalised lower and upper probability is given by*

${}_{Ng}\underline{p}(K)$ = p($Ngint(K)$)

${}_{Ng}\overline{p}(K)$ = p($Ngcl(K)$)

Definition 5.6 *:* Let K be an event in the $nanoClG_m$ topological stochastic approximation space $[V(G), \tau_{G_m}(V(H), p]$ then the nano generalised measure of K is given by $\mu^*(K) = {}_{Ng}\overline{p}(K) - {}_{Ng}\underline{p}(K)$

Example 5.7 *:* Consider the following graph G=[V(G),E(G)] whereV(G)= $\{v_1, v_2, v_3, v_4, v_5\}$ and E(G)= $\{(v_1, v_2), (v_1, v_3), (v_3, v_4)(v_2, v_4), (v_2, v_5), (v_4, v_5)\}$

$\tau_{CG_m} = \{G, \varnothing, \{v_1\}, \{v_1, v_2\}, \{v_1, v_3\}, \{v_1, v_2, v_3\}, \{v_1, v_2, v_3, v_4\}\}$

Let H= $\{v_1, v_5\}$ and τ_{G_m} = $\{V(G), \varnothing, \{v_1\}, \{v_1, v_4\}, \{v_2, v_3, v_4, v_5\}\}$

v	v_1	v_2	v_3	v_4	v_5
${}_{Ng}\underline{p}(V = v)$	1/5	1/5	1/5	0	1/5
${}_{Ng}\overline{p}(V = v)$	2/5	1/5	2/5	1/5	1/5

Proposition 5.8 *:* Let K be an event in the $nanoClG_m$ topological stochastic approximation space $[V(G), \tau_{G_m}(V(H), p]$ then the nano generalised lower and upper probability of K satisfy the following properties

(i) ${}_{Ng}\underline{p}(\varnothing)$ = ${}_{Ng}\overline{p}(\varnothing)$ =0

(ii) ${}_{Ng}\underline{p}(G)$ = ${}_{Ng}\overline{p}(G)$ =1

(iv) ${}_{Ng}\underline{p}(K)$ ≤ p(K) ≤ ${}_{Ng}\overline{p}(K)$

(v) ${}_{Ng}\underline{p}(K^c)$ = 1- ${}_{Ng}\overline{p}(K)$

(vi) ${}_{Ng}\overline{p}(K^c)$ = 1- ${}_{Ng}\underline{p}(K)$

Proof :

(i) Since N$gint(\varnothing)$ = $\varnothing$ and N$gcl(\varnothing)$ = $\varnothing$ therfore p(N$gint(\varnothing)$)= 0 and p(N$gcl(\varnothing)$)=0. Hence the result.

(ii) Since N$gint(G)$ = G and N$gcl(G)$ = G therfore p(N$gint(G)$)= 1 and p(N$gcl(G)$)=1.Hence the result.

(iii) Since N$gint(K)$ $\subseteq$ K $\subseteq$ N$gcl(K)$. Therefore p(N$gint(K)$) ≤ p(K) ≤ p(N$gcl(K)$).Therefore we get the required result.

(iv) ${}_{Ng}\underline{p}(K^c)$ = p(N$gint(K^c)$)= p(G)- N$gcl(K)$) = p(G)-p(N$gcl(K)$) = 1- ${}_{Ng}\overline{p}(K)$

(v) As similar to the above case.

Since N$gint(\varnothing)$ = $\varnothing$ and N$gcl(\varnothing)$ = $\varnothing$ therefore p(N$gint(\varnothing)$)= 0 and p(N$gcl(K)$)=0.Hence the result.

Proposition:5.9 Let K and T are events in the $nanoClG_m$ topological stochastic approximation space $[V(G), \tau_{G_m}(V(H), p]$ then the nano generalised lower and upper probability of K and T satisfy the following properties

(i) ${}_{Ng}\underline{p}(K \cup T)$ ≥ ${}_{Ng}\underline{p}(K)$ + ${}_{Ng}\underline{p}(T)$

(ii) ${}_{Ng}\overline{p}(K \cup T)$ = ${}_{Ng}\overline{p}(K)$ + ${}_{Ng}\overline{p}(T)$

(iii) ${}_{Ng}\underline{p}(K \cap T)$ = ${}_{Ng}\underline{p}(K)$. ${}_{Ng}\underline{p}(T)$

(iv) ${}_{Ng}\overline{p}(K \cap T)$ ≤ ${}_{Ng}\overline{p}(K)$. ${}_{Ng}\overline{p}(T)$

Proof : Since the following condition are true for the graphs K and T the proof is obvious.

(i) N$gint(K \cup T)$ $\supseteq$ N$gint(K)$ $\cup$ N$gint(T)$.

(ii) N$gcl(K \cup T)$ = N$gcl(K)$ $\cup$ N$gcl(T)$

(iii) N$gint(K \cap T)$ = N$gint(K)$ $\cap$ N$gint(T)$

(iv) N$gcl(K \cap T)$ $\subseteq$ N$gcl(K)$ $\cap$ N$gcl(T)$

Conclusion: Here we intoduce some new type of probabiliy measures in nano topological space and explain it's advantages with the example. In digraph we introduce the closure stochastic approximation space and study its properties. This can be further extended to matrices and real life problems.

References:

1. Abd El-Monsef.M.E,Shokry.M,Yousif.Y.Y,Near approximations in G_m closure spaces,ISRN Applied Mathematics,vol-1,1-23(2012).
2. Bondy.J., Murty.D.S., Graph theory with Applications, North-Holland., (1992).
3. Satish Malhotra, Proving theories of Relativity Wrong: Held-Up For Want of Mathematical Support; Mathematical Sciences international Research Journal Issn 2278 - 8697 Vol 6 Issue 1 (2017), Pg 151-155
4. Iqelan.M.J, On Topological Structures and Uncertainty, Ph.D Thesis, Tanta University,(2010)
5. Lashin E.F., Kozae A.M., Abo Khadra A.A.and Medhat T.,Rough set theory for topological spaces,International Journal of Approximate Reasoning, 40/1-2, 35-43(2005).
6. Lashin E.F.and Medhat T.,Topological reduction of information systems, Chaos, Solitons and Fractals,25, 277-286,(2005).
7. Rajesh Kumar Mishra, Reduction in Co2 Emission By the Use of Blended Cement Using Mathematical Modeling; Mathematical Sciences international Research Journal Issn 2278 - 8697 Vol 6 Issue 1 (2017), Pg 156-170
8. Lellis Thivagar .M and Carmel Richard,On nano continuity, Mathematical Theory and Modeling, Vol.3, No.7(2013)
9. Lellis Thivagar.M and Carmel Richard,On Nano Forms Of Weakly Open Sets,International Journal of Mathematics and Statistics Invention (IJMSI), Vol.1, No.1,31-37(2013)
10. Lellis Thivagar.M and Sutha Devi.V, A detection for patent infringement suit via nano topology induced by graph, Cogent Mathematics, Vol.3,DOI-10.1080/1161129(2016).
11. Pawlak.Z.,Rough probability,Bulletin of the polish Academy of sciences, Mathematics, 32,607-612(1984).
12. Dharamender Singh, Bani Mukherjee, Production inventory Model Controllable Deterioration Rate and Multiple -Market Demand With Preservation Technique; Mathematical Sciences international Research Journal Issn 2278 - 8697 Vol 6 Issue 1 (2017), Pg 171-174

***