THE EFFECTS OF INHOMOGENEITY ON ION ACOUSTIC SOLITARY WAVES IN NON UNIFORM PLASMAS

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Abstract: The propagation of ion-acoustic solitary waves in inhomogeneous plasma with spatial density gradient in ions has been investigated. The fluid equations for ions have been treated by reductive perturbation analysis technique. In this formulation process we have used a space–time stretched coordinates. The system of equations has been reduced to a modified Korteweg–de–Vries (mKdV) equation. The soliton solutions are found to be affected by density gradient in ions. The effective conditions for soliton propagation in inhomogeneous plasma have been analysed.

Keywords: Inhomogeneous Plasma, Ion–Acoustic Solitons, mKdV Equation, Solitary Waves.

Introduction: The study of nonlinear acoustic waves have a great deal of interest, both theoretically and experimentally, since the concept was first augmented by Washimi and Tanuity [1] through a nonlinear wave equation known as Korteweg- de-Vries(KdV) equation [**2**]. As the solitons are formed with the combined effect of nonlinearity and dispersion, which are very stable, create neither fission nor fusion in their own interactions and it describes the characteristics of the interaction between the waves and the plasmas, so propagation of solitons are also important for many scientific observations in laboratory plasmas as well as in many other astrophysical plasmas [**3, 4, 5, 6**]. However, most of the studies on the formation of solitons were limited to homogeneous (uniform) plasma. In practice, inhomogeneity exists widely in plasmas both in the laboratory as well as in space due to the density gradient or that of temperature or it could be due to the magnetic field in space. So propagation characteristics are influenced significantly by plasma inhomogeneities. Sakanaka [**7**] and Tappet [**8**] studied the propagation of ion acoustic waves in inhomogeneous (non uniform) plasma with warm adiabatic ions. The soliton propagation in weakly inhomogeneous plasma has been studied first by Asano [**9**] and then ion acoustic case by Nishikawa and Kaw [**10**] and Gell and Gomberoff [**11**]**.** These studies have an inconsistency due to the neglect of zeroth order quantities like ion-fluid velocity and electric field which are arise due to the presence of inhomogeneity. Later Rao and Verma [**12**] eliminated these shortcomings by using a right set of 'stretched coordinates' appropriate for the spatially inhomogeneous system. Since then, using these types of stretched coordinates; many researchers had studied different characteristic properties of soliton propagation analytically as well as in laboratory for different inhomogeneous plasma models [**13-23**] . Very recently Gogoi and Deka [24] have studied dust acoustic solitary waves in inhomogeneous plasma with dust charge fluctuations.

In this paper, we have derived a modified KdV equation in spatially inhomogeneous plasma with density gradient of ions. The reductive perturbation analysis of fluid equations is carried out by employing a set of 'stretched coordinates' appropriate for spatially inhomogeneous plasma.

Basic Equations: We have considered an unmagnetised spatially inhomogeneous and collisionless plasma having density gradient of the ions. In this plasma model we consider that the ions are cold with thermal electrons. The continuity and momentum equation for this plasma model with Poisson equation are as follows:

$$\frac{\partial n'}{\partial t'} + \frac{\partial}{\partial x'}\left(n'u'\right) = 0 \quad (2.1)$$

$$\frac{\partial u'}{\partial t'} + u' \frac{\partial u'}{\partial x'} + \frac{\partial \phi'}{\partial x'} = 0 \quad (2.2)$$

$$\frac{\partial^2 \phi'}{\partial x^2} - e^{\phi'} + n' = 0 \quad (2.3)$$

where n': ion density
u': ion fluid velocity

φ': electrostatic potential
and x and t are space and time variables.

We normalized the plasma parameters as

$$n' = \frac{n}{n_0}, \quad u' = u\sqrt{KT_e/m_i}, \quad \phi' = \phi\left[KT_e/e\right], \quad t' = t\sqrt{4\pi\, n_0 e^2/m_i}, \quad x' = x\sqrt{KT_e/4\pi n_0 e^2}$$

where n_0 is the ion density of the equilibrium state.
The normalized forms of the above equations are

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = 0 \tag{2.1a}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\partial \phi}{\partial x} = 0 \tag{2.2a}$$

$$\frac{\partial^2 \phi}{\partial x^2} - n_0 e^{\phi} + n = 0 \tag{2.3a}$$

Derivation and Solution of the Modified KdV Equation: In order to investigate the propagation characteristics of solitary waves for fast and slow modes, we derive a modified KdV (mKdV) equation for our present plasma model. For this, a set of spatial stretched coordinates [25], is used which is appropriate for specially inhomogeneous plasma, along with the zeroth order fluid velocities as

$$\xi = \varepsilon\left(\frac{x}{\lambda_0} - t\right), \qquad \tau = \varepsilon^2 x \tag{3.1}$$

where ε is expansion parameter and λ_0 is the phase velocity of the ion-acoustic wave which will be determined later in a self consistent manners.
Since n_0 and λ_0 are independent of t, we have

$$\frac{\partial n_0}{\partial \xi} = \frac{\partial \lambda_0}{\partial \xi} = 0 \tag{3.2}$$

Substituting equations (3.1) and (3.2) into equations (2.1) – (2.3) we get

$$-\frac{\partial n}{\partial \xi} + \frac{1}{\lambda_0}\frac{\partial}{\partial \xi}(nu) + \varepsilon\frac{\partial}{\partial \tau}(nu) = 0 \tag{3.3}$$

$$-\frac{\partial u}{\partial \xi} + \frac{u}{\lambda_0}\frac{\partial u}{\partial \xi} + \varepsilon u\frac{\partial u}{\partial \tau} + \frac{1}{\lambda_0}\frac{\partial \phi}{\partial \xi} + \varepsilon\frac{\partial \phi}{\partial \tau} = 0 \tag{3.4}$$

and

$$\frac{\varepsilon^2}{\lambda_0^{\,2}}\frac{\partial^2 \phi}{\partial \xi^2} + \frac{2\varepsilon^2}{\lambda_0}\frac{\partial^2 \phi}{\partial \xi \partial \tau} + \varepsilon^4\frac{\partial^2 \phi}{\partial \tau^2} - \frac{\varepsilon^3}{\lambda_0^{\,2}}\frac{\partial \lambda_0}{\partial \tau}\frac{\partial \phi}{\partial \xi} - g(\tau_0)e^{\phi} + n = 0 \tag{3.5}$$

To employ the reductive perturbation technique [1], the plasma parameters n, u and ϕ are expressed as a power series in ϵ as

$$f = f_0 + \varepsilon f_1 + \varepsilon^2 f_2 + \varepsilon^3 f_3 + \ \ldots \tag{3.6}$$

where $f \approx$ n, u and ϕ and n_0 u_0 and ϕ_0 are the plasma parameters in unperturbed state.

From the zeroth-order equations of (3.3) – (3.5) together with eqn. (3.2), we get $\frac{\partial u_0}{\partial \xi} = 0$ and ϕ_0 =0 (3.7)

Now using (3.6) into equations (3.3) – (3.5), the lowest order coefficients of ε gives

$$-\frac{\partial n_1}{\partial \xi}+\frac{n_0}{\lambda_0}\frac{\partial u_1}{\partial \xi}+\frac{n_0}{\lambda_0}\frac{\partial n_1}{\partial \xi}+\frac{\partial}{\partial \tau}\left(n_0 u_0\right)=0 \tag{3.8}$$

$$-\frac{\partial u_1}{\partial \xi}+\frac{u_0}{\lambda_0}\frac{\partial u_1}{\partial \xi}+u_0\frac{\partial u_0}{\partial \tau}+\frac{1}{\lambda_0}\frac{\partial \phi_1}{\partial \xi}+\frac{\partial \phi_0}{\partial \tau}=0 \tag{3.9}$$

$$-n_0\phi_1+n_1=0 \tag{3.10}$$

Integrating these equations and using boundary conditions $u_0,\phi_0 \to 0,\ n_1=u_1=\phi_1$ and $n_0,\lambda_0 \to 1$ as $|\xi| \to \infty$ yields

$$\left.\begin{aligned} u_1 &= Pn_1-\xi Q \\ \phi_1 &= u_1\left(\lambda_0-u_0\right)-\lambda_0\left(u_0\frac{\partial u_0}{\partial \tau}+\frac{\partial \phi_0}{\partial \tau}\right)\xi \\ &= \mathrm{n}_0 u_1 P-R\xi \\ n_1 &= n_0\phi_1 \end{aligned}\right\} \tag{3.11}$$

where

$$P=\frac{\lambda_0-u_0}{n_0},\ \mathrm{Q}=\frac{\lambda_0}{n_0}\frac{\partial}{\partial \tau}\left(n_0u_0\right),\qquad \mathrm{R}=\lambda_0\left(u_0\frac{\partial u_0}{\partial \tau}+\frac{\partial \phi_0}{\partial \tau}\right) \tag{3.12}$$

Using equation (3.9) with simple algebra, we get

$$\phi_1=\frac{Pn_0Q+R}{P^2{n_0}^2-1} \tag{3.13}$$

In equation (3.13) we see that the left hand side is a first order perturbation while the right hand side contains only zeroth-order quantities. Thus, in order to obtain nonsecular solution of φ_1, numerator and denominator of equation (3.13) must be equal to zero separately [3]. These yields

$$\left(\lambda_0-u_0\right)^2=1, \quad \Rightarrow \lambda_0=1\pm u_0 \tag{3.14}$$

and

$$\lambda_0^{\ 2}\frac{\partial u_0}{\partial \tau}+\frac{\lambda_0}{n_0}\left(\lambda_0-n_0\right)u_0\frac{\partial n_0}{\partial \tau}+\lambda_0\frac{\partial \phi_0}{\partial \tau}=0 \tag{3.15}$$

which is a self consistent relation between n_0 and u_0. Eq. (3.14) shows the existence of two types of phase velocities, fast and slow, corresponding to which two types of waves may be possible. The positive sign and negative signs in the right side give the fast mode and slow mode respectively.

For second order of ε, we obtain the following equations

$$-\frac{\partial n_2}{\partial \xi}+\frac{1}{\lambda_0}\frac{\partial}{\partial \xi}\left(n_0u_2+n_1u_1+n_2u_0\right)+\frac{\partial}{\partial \tau}\left(n_0u_1+n_1u_0\right)=0 \tag{3.16}$$

$$-\frac{\partial u_2}{\partial \xi}+\frac{u_0}{\lambda_0}\frac{\partial u_2}{\partial \xi}+\frac{u_1}{\lambda_0}\frac{\partial u_1}{\partial \xi}+u_1\frac{\partial u_0}{\partial \tau}+u_0\frac{\partial u_1}{\partial \tau}+\frac{1}{\lambda_0}\frac{\partial \phi_2}{\partial \xi}+\frac{\partial \phi_1}{\partial \tau}=0 \tag{3.17}$$

and

$$\frac{1}{\lambda_0^{\ 2}}\frac{\partial^2 \phi_1}{\partial \xi^2}-n_0\phi_2-\frac{1}{2}n_0{\phi_1}^2+n_2=0 \tag{3.18}$$

Using equation (3.14), we can eliminate all the second- order quantities from above three equations exactly. Substituting for n_1 and u_1 in terms of ϕ_1 from equation (3.11) into equations (3.16) - (3.18), we get the following modified KdV equation as

$$\frac{\partial\phi_1}{\partial\tau}+A\phi_1\frac{\partial\phi_1}{\partial\xi}+B\frac{\partial^3\phi_1}{\partial\xi^3}+C\phi_1=0 \qquad (3.19)$$

where

$$A=\frac{1}{2}\left(1+\lambda_0\right),\ \mathrm{B}=\frac{1}{2\mathrm{n}_0{\lambda_0}^4},\quad \mathrm{C}=\frac{1}{2}\left(\frac{1}{n_0}\frac{\partial n_0}{\partial\tau}+\frac{2}{\lambda_0}\frac{\partial\lambda_0}{\partial\tau}\right) \qquad (3.20)$$

Here A, B, and C are all functions of τ. Eq. (3.19) is a modified form of KdV(mKdV) equation as the term with coefficient C is an additional term (inhomogeneity) which arises due to the presence of density gradient in the plasma. In order to obtain the solitary wave solution of eq. (3.19) in inhomogeneous plasma, we use a variable transformation as

$$\phi_1=\psi\exp(-Cn_0) \qquad (3.21)$$

Using this variable transformation Eq. (3.19) transforms to a well known form of KdV equation as

$$\frac{\partial\psi}{\partial\tau}+A^*\psi\frac{\partial\psi}{\partial\xi}+B\frac{\partial^3\psi}{\partial\zeta^3}=0 \qquad (3.22)$$

where $A^*=A\exp(-Cn_0)$.

We have assumed that the nonlinear co-efficient functionally depends on the space of the plasma. For the sake of simplicity of mathematical calculations, the variations are assumed to be negligibly small as compared to the scale length and due to this it is assumed that all parameters could be locally constant. Under these situations, to obtain a steady state solution of the Eq. (3.22), we introduce a new variable $X=\xi-U\tau$ with respect to a frame moving with velocity U which transforms the pair variable $\left(\xi,\ \tau\right)$ to a single variable X. We have obtained the solution of this equation following the method of Kodama and Taniuty [26] as

$$\psi=\psi_m\,\mathrm{Sech}^2\left(\frac{X}{\Omega}\right), \qquad (3.23)$$

where the amplitude $\psi_m=\frac{3U}{A^*}$ and the width $\Omega=\left(\frac{4B}{U}\right)^{\frac{1}{2}}$.

Results and Discussion: We now investigate the influence of the inhomogeneity(density gradient) on the propagation of ion acoustic solitary waves in fast as well as in slow modes**.** In Figs. 1 & 2, the variations of soliton amplitude Ψ_m are shown against the ion number density n_0 for three different values of ion fluid velocity $u_0\left(=0.1,\ 0.15,\ 0.2\right)$. Fig. 1 shows the increase of soliton amplitude (fast mode) with nearly constant rate for increasing values of ion number density n_o and ion fluid velocity u_o. In case of slow mode solitary wave, amplitude decreases with nearly constant rate for increasing values of ion number density n_o and ion fluid velocity u_o.

The variations of soliton width against the ion number density n_0 for three different values of ion fluid velocity $u_0\left(=0.1,\ 0.15,\ 0.2\right)$ are shown in Figs. 3 & 4. It is shown that the soliton widths decrease with increasing values of ion number density n_o for fast as well as slow modes. In case of fast mode, for greater values of u_o, soliton widths are smaller (Fig. 3) which is opposite in case of slow modes (Fig. 4).

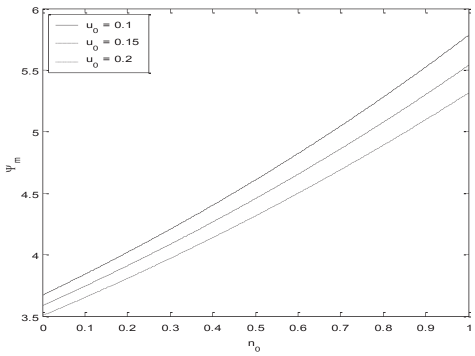

Fig.1 (Fast soliton): Variations of Amplitude ψ_m Against Density Gradient n_0 For Different Values of $u_0(=0.1,\ 0.15,\ 0.2)$

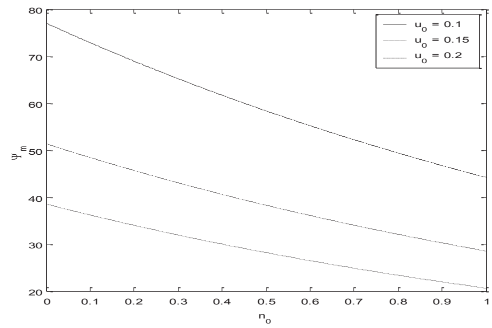

Fig.2 (Slow soliton): Variations Of Amplitude ψ_m Against Density Gradient n_0 For Different Values of $u_0(=0.1,\ 0.15,\ 0.2)$.

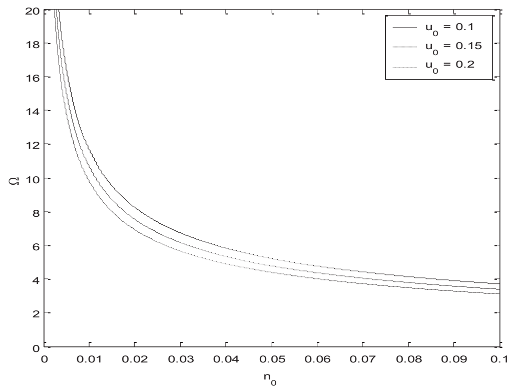

Fig.3 (Fast Soliton): Variations of Width Ω Against Density Gradient n_0 for Different Values of $u_0(=0.1,\ 0.15,\ 0.2)$

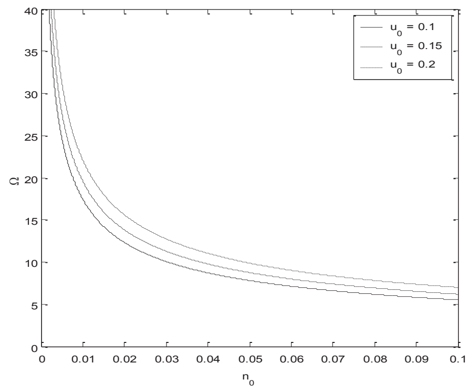

Fig.4 (Slow Soliton): Variations of Width Ω against Density Gradient n_0 for Different Values of $u_0(=0.1,\ 0.15,\ 0.2)$

Conclusion: In summary, a modified KdV (mKdV) equation is derived by employing a set of suitable stretched coordinates and reductive perturbation technique. A solitary wave solution of the mKdV equation is derived. The propagation characteristics in inhomogeneous plasmas are investigated for fast and slow modes phase velocities. The numerical results show that the inhomogeneity parameter has remarkable influence on the propagation characteristics of ion acoustic waves.

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