
EFFECT OF THERMAL RADIATION CONVECTIVE HEAT TRANSFER FLOW OF A ROTATING NANO-FLUID IN A VERTICAL CHANNEL

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Abstract: We investigate the effect of thermal radiation on steady convective heat transfer flow of a nanofluid in a vertical channel in the presence of heat generating sources. Analytical closed form solutions are obtained for both the momentum and the energy equations. Graphs are used to illustrate the significance of key parameters on the nanofluid velocity and temperature distributions.

Keywords: Thermal Radiation, Heat transfer, Nanofluid, Vertical Plate.

Introduction: Nanofluids are solid-liquid composite materials consisting of solid nanoparticles or nanofibers, with sizes typically on the order of 1–100 nm, suspended in a liquid. Nanofluids are characterized by an enrichment of a base fluid like water, toluene, ethylene glycol or oil with nanoparticles in variety of types like Metals, Oxides, Carbides, Carbon, Nitrides, etc. Today nanofluid are sought to have wide range of applications in medical application, biomedical industry, detergency, power generation in nuclear reactors and more specifically in any heat removal involved industrial applications. The ongoing research ever since then has extended to utilization of nanofluids in microelectronics, fuel cells, pharmaceutical processes, hybrid-powered engines, engine cooling, vehicle thermal management, domestic refrigerator, chillers, heat exchanger, nuclear reactor coolant, grinding, machining, space technology, Defense and ships, and boiler flue gas temperature reduction [Agarwal et al. (2011)]. Indisputably, the nanofluids are more stable and have acceptable viscosity and better wetting, spreading, and dispersion properties on a solid surface [Akbarinia et al. (2011), Nguyen et al. (2007)]. Several reviews [Ghadimi et al.(2011), Mahabudul et al. (2012)] on nanofluids with respect to thermal and rheological properties have been reported.

Thus, nanofluids have an ample collection of potential applications in electronics, pharmaceutical processes, hybrid-powered engines, automotive and nuclear applications where enhanced heat transfer or resourceful heat dissipation is required. In view of these, [Kiblinki et al. (2002)] suggested four possible explanations for the anomalous increase in the thermal conductivity of nanofluids. These are nanoparticles clustering, Brownian motion of the particles, molecular level layering of the liquid/particles interface and ballistic heat transfer in the nanoparticles. Despite a vast amount of literature on the flow of nanofluid model proposed by [Buongiorno (2006)], we are referring to a few recent studies [Alsaedi et al.(2012), Hajipour and Dekhordi (2012)]. However, we are following the nanofluid model proposed by [Tiwari and Das (2007)], which is being used by many current researchers [Hamad and Ferdows (2012), Hamad and Pop (2011), Norifah et al. (2012)] on various flow fields.

The study of MHD flow and heat transfer due to the effect of a magnetic field in a rotating frame of reference has attracted the interest of many investigators in view of its applications in many industrial, astrophysical (dealing with the sunspot development, the solar cycle and the structure of a rotating magnetic stars), technological and engineering applications (MHD generators, ion propulsion, MHD pumps, etc.) and many other practical applications, such as in biomechanical problems (e.g., blood, flow in the pulmonary alveolar sheet). Many authors have studied the flow and heat transfer in a rotating system with various geometrical situations [Hickman(1957), Hide (1960), Mazumder (2012)]. [Hamad (2011)] investigated the effect of a transverse magnetic field on free convection flow of a nanofluid past a vertical semi-infinite flat plate. Recently, [Satya Narayana et al. (2013)] studied the Hall current and radiation absorption effects on MHD micropolar fluid in a rotating system. Some other related works can also be found in recent papers [Kameswaran et al. (2012), Kesavaiah et al. (2011), Rushi kumar and Sivaraj (2013), Srinivas et al. (2012)].

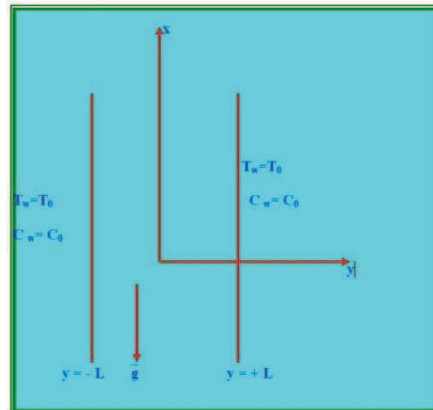
Thermal radiation is important in some applications because of the manner in which radiant emission depends

on temperature and nanoparticles volume fraction. The thermal radiation effect on mixed convection heat transfer in porous media has many important applications such as the sensible heat storage bed, the nuclear reactor cooling system, space technology, and underground nuclear waste disposal. To the best of the author's knowledge (from the literature), no studies have been communicated thus far with regard to the study of flow and heat transfer distinctiveness of a nanofluid past a vertical plate with thermal radiation in a rotating frame of reference

Recently [Satyanarayana et al (2011)] have studied the effect of radiation on the convective heat transfer flow of a rotating nanofluid past a porous vertical plate with oscillatory velocity.

In this paper we investigate the effect of thermal radiation on steady convective heat transfer flow of a rotating nanofluid in a vertical channel in the presence of heat generating sources. Analytical closed form solutions are obtained for both the momentum and the energy equations. Graphs are used to illustrate the significance of key parameters on the nanofluid velocity and temperature distributions.

Formulation of the Problem: We consider the steady, three dimensional flow of a nanofluid consisting of a base fluid and small nanoparticles in a vertical porous channel with thermal radiation. A uniform magnetic field of strength H_0 is applied normal to the plate. It is assumed that there is no applied voltage which implies the absence of an electric field. The flow is assumed to be in the x-direction which is taken along the plane in an upward direction and z-axis is normal to the plate. Also it is assumed that the whole system is rotating with a constant angular velocity vector $\bar{\Omega}$ about the z-axis. The fluid is assumed to be gray, absorbing emitting but not scattering medium. The radiation heat flux in the x-direction is considered negligible in comparison with that in the z-direction. As the flow is fully developed, the flow variables are functions of z and t only. Figure. 1 shows that the problem under consideration and the co-ordinate system.



Under the above mentioned assumptions, the equation of momentum and thermal energy respectively, can be written in dimensional form as :

$$\frac{\partial w}{\partial z} = 0 \tag{2.1}$$

$$w \frac{\partial u}{\partial z} - 2\Omega v = \frac{1}{\rho_{nf}} \left(\mu_{nf} \frac{\partial^2 u}{\partial z^2} + (\rho \beta_{nf}) g (T - T_\infty) - (\sigma \mu_e^2 H_0^2) u \right) \tag{2.2}$$

$$w \frac{\partial v}{\partial z} + 2\Omega u = \frac{1}{\rho_{nf}} \left(\mu_{nf} \frac{\partial^2 v}{\partial z^2} - (\sigma \mu_e^2 H_0^2) v \right) \tag{2.3}$$

$$w \frac{\partial T}{\partial z} = k_{nf} \frac{\partial^2 T}{\partial z^2} - \frac{1}{(\rho C_p)_{nf}} \frac{\partial (q_r)}{\partial z} - \frac{Q_H}{(\rho C_p)_{nf}} (T - T_\infty) \tag{2.4}$$

The boundary conditions are (see ref.(42&43)):

$$\begin{aligned} u(\pm L) = 0, \quad v(\pm L) = 0, \\ T(-L) = T_1, \quad T(+L) = T_2 \end{aligned} \tag{2.5}$$

The properties of the nanofluids are defined as follows (see ref.(44-46)):

$$\left. \begin{aligned} \mu_{nf} &= \mu_f / (1 - \phi)^{2.5} & \alpha_{nf} &= \frac{k_{nf}}{(\rho C_p)_{nf}} & \rho_{nf} &= (1 - \phi)\rho_f + \phi\rho_s \\ (\rho C_p)_{nf} &= (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s & (\rho\beta)_{nf} &= (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s \end{aligned} \right\} \quad (2.6)$$

$$k_{nf} = \frac{k_f(k_s + 2k_f - 2\phi(k_f - k_s))}{(k_s + 2k_f + 2\phi(k_f - k_s))}$$

We consider the solution of equation(2.1) as:

$$w = -w_0 \tag{2.7}$$

The radiation heat term(Brewster(47))by using The Rosseland approximation is given by

$$q_r = -\frac{4\sigma^*}{3\beta_R} \frac{\partial T'^4}{\partial z} \tag{2.8}$$

$$T'^4 \cong 4TT_\infty^3 - 3T_\infty^4 \tag{2.9}$$

$$\frac{\partial q_R}{\partial z} = -\frac{16\sigma^*T_\infty^3}{3\beta_R} \frac{\partial^2 T}{\partial z^2} \tag{2.10}$$

We introduce the following dimensionless variables:

$$\left. \begin{aligned} \eta &= \frac{z}{L}, u' = \frac{u}{w_0}, v' = \frac{v}{w_0}, \theta = \frac{T - T_1}{T_2 - T_1}, G = \frac{\beta g(T_2 - T_1)L^2}{\mu_f w_0} \\ S &= \frac{w_0 L}{\mu_f}, M = \frac{\sigma \mu_e^2 H_0^2 L^2}{\rho_f \mu_f}, Q = \frac{Q_H L^2}{k_f}, F = \frac{4\sigma^* T_\infty^3}{\beta_R k_f} \end{aligned} \right\} \quad (2.11)$$

Equations(2.2)-(2.4) in the non-dimensional form are

$$-S \frac{\partial u}{\partial \eta} - 2Rv = \frac{1}{A_1 A_3} \frac{\partial^2 u}{\partial \eta^2} + \frac{A_4}{A_3} G\theta - \frac{M^2}{A_3} u \tag{2.12}$$

$$-S \frac{\partial v}{\partial \eta} + 2Ru + \frac{1}{A_1 A_3} \frac{\partial^2 v}{\partial \eta^2} - \frac{M^2}{A_3} v \tag{2.13}$$

$$-S \frac{\partial \theta}{\partial \eta} = \frac{1}{P_r} \left(\frac{A_2}{A_5} \left(1 + \frac{4F}{3} \right) \frac{\partial^2 \theta}{\partial \eta^2} - \frac{1}{A_5} Q\theta \right) \tag{2.14}$$

The boundary conditions (2.5) reduce to

$$u(\pm 1) = 0, v(\pm 1) = 0, \theta(-1) = 0, \theta(+1) = 1 \tag{2.15}$$

Using (2.12) the velocity characteristic Uo is defined as:

In view of the fluid velocity in the component form:

$$V(z, t) = u(z, t) + iv(z, t)$$

The equations (2.12) and (2.13) reduce to

$$-S \frac{\partial V}{\partial \eta} - 2iRV = \frac{1}{A_1 A_3} \frac{\partial^2 V}{\partial \eta^2} + \frac{A_4}{A_3} G\theta - \frac{M^2}{A_2} V \tag{2.16}$$

The boundary conditions(2.15) reduce to

$$V(\pm 1) = 0 \quad \theta(-1) = 0, \theta(+1) = 0, \tag{2.17}$$

Method Solution: Solving the equations (2.14) and (2.16) we get

$$V(z) = \exp\left(-\frac{b_9\eta}{2}\right)(B_5\text{Cosh}(m_3\eta) + B_6\text{Sinh}(m_3\eta) + b_{14} \exp((m_1 - b_1)\eta) + b_{15} \exp(-(m_1 + b_1)\eta))$$

$$\theta(z) = \exp(-b_1\eta)\left(\frac{\text{Cosh}(m_1\eta)}{\text{Cosh}(m_1)}\text{Sinh}(b_1) + \frac{\text{Sinh}(m_1\eta)}{\text{Sinh}(m_1)}\text{Cosh}(b_1)\right)$$

The physical quantities of interest are skin friction and Nusselt number which are, respectively, defined as:

$$C_f = \frac{\tau_w}{\rho_f U_o^2}$$

$$Nu = \frac{xq_w}{k_f(T_w - T_\infty)}$$

Where τ_w and q_w are the wall shear and the wall heat flux from the plate respectively, which are given by

$$\tau_w = \mu_{nf} \left(\frac{\partial u}{\partial z}\right)_{z=0} \quad \text{and} \quad q_w = -k_{nf} \left(\frac{\partial T}{\partial z}\right)_{z=0}$$

In view of Equation (2.11) we obtain

$$C_f = \frac{1}{A_1} V'(\pm 1) = b_{18} \text{Cosh}(m_3) + b_{19} \text{Sinh}(m_3) + b_{20} \text{Exp}(m_1 - b_1) + b_{21} \text{Exp}(-(m_1 + b_1))$$

$$= \frac{1}{A_1} V'(-1) = b_{22} \text{Cosh}(m_3) + b_{23} \text{Sinh}(m_3) + b_{20} \exp(-(m_1 - b_1)) + b_{21} \text{Exp}(-(m_1 + b_1))$$

$$Nu = -\frac{k_{nf}}{k_f} \theta'(\pm 1) = -A_2 \theta'(\pm 1)$$

$$Nu(+1) = -b_1 \exp(-b_1)(\text{Sinh}(b_1) + \text{Cosh}(b_1)) + m_1 \exp(-b_1)(\text{Sinh}(b_1)\text{Tanh}(m_1) + \text{Cosh}(b_1)\text{Coth}(m_1))$$

$$Nu(-1) = -b_1 \exp(b_1)(\text{Sinh}(b_1) - \text{Cosh}(b_1)) + m_1 \exp(b_1)(-\text{Sinh}(b_1)\text{Tanh}(m_1) + \text{Cosh}(b_1)\text{Coth}(m_1))$$

Discussion of the Numerical Results: A mathematical assessment for the analytical solution of this problem is performed, and the outcomes are illustrated graphically in Figures 1- 4. They explain the fascinating features of important parameters on the nanofluid velocity, temperature, skin friction and Nusselt number distributions in a rotating system for three different types of water based nanofluids. As in [Oztop and Abu-Nada (2008)], we take the values of the nanofluid volume fraction ϕ in the range of $0 \leq \phi \leq 0.08$. We considered for the convective flow in a lid driven cavity, the value of the nanofluid volume fraction in the range $0 \leq \phi \leq 0.08$. If the concentration exceeds the maximum level of 0.08, sedimentation could take place. We have chosen here $n = 10$, $nt = \pi/2$, $\varepsilon = 0.02$, $Pr = 6.2$ while M, R, S, ϕ, Q, F are varied over a range, which are listed in the Figure legends.

Fig.1a represents the effect of magnetic field on the nanofluid velocity profile. It is found that the nanofluid velocity field enhances with increase of magnetic field parameter M along the surface. These effects are much significant near the surface of the plate. This shows that the fluid velocity is enhanced by increasing the magnetic field and confirms the fact that the application of the magnetic field to an electrically conducting fluid produces a drag like force which causes an enhancement in the fluid velocity. From Fig.1b, it is interesting to note that the effect of magnetic parameter on the secondary velocity is to enhance the magnitude of v . It is seen that the velocity rapidly increases attaining maximum at $y=3$ and then reduces to attain the prescribed value zero far away from the boundary.

Fig.2a represents the nanofluid velocity profiles for different values of rotational parameter R . This result displays that the nanofluid velocity reduces with an increase in R , as noted in reference [Hamad et al. (2011)]. Fig.2b depicts the variation of the secondary velocity with rotation parameter (R). It is observed from the profiles that the magnitude of v enhances with increase in R .

Figs.3a&3b depict the behavior of the primary and secondary velocities with heat source parameter Q . It is found that the both components of velocities exhibit a increasing tendency with increase in the strength of the heat source. This is due to the fact that when heat is generated, the buoyancy forces increases which enhances the flow rate and there by gives rise to an enhancement in the velocity component profiles. An increase in the strength of the heat absorbing source ,the velocity components reduces as the heat is absorbed in the boundary layer. Fig.3c represents the temperature (T) with Heat source parameter Q . It is observed from the profiles that an increase in Q reduces the temperature and hence the thickness of the thermal boundary layer reduces with increase in Q in CuO-water nanofluids.

Figs.4a & 4b exhibit the nanofluid primary and secondary velocity components for various values of radiation parameter F . An increase in F leads to a depreciation in the primary and secondary velocity distributions across the boundary layer. The effect of thermal radiation is to reduce heat transfer because of the fact that thermal boundary layer thickness decreases with an increase in the thermal radiation. Fig.4c represents T with radiation parameter F . It can observe that an increase in F results in a decrease in T . This is attributed to the fact that an enhancement of the radiation parameter results in a decrease of thickness of the thermal boundary layer. Thus it is pointed out that the radiation should be minimized to have the cooling process at a faster rate.

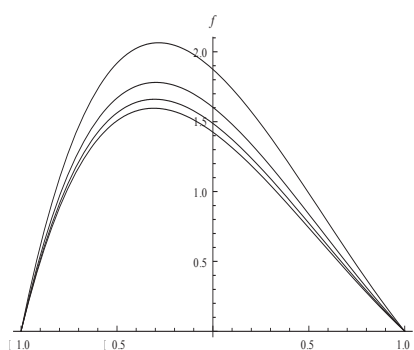


Fig. 1a

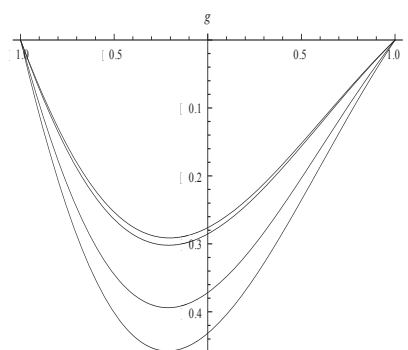


Fig. 1b

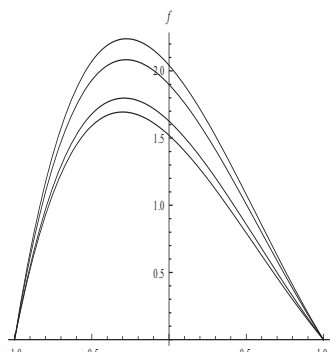


Fig 2 a

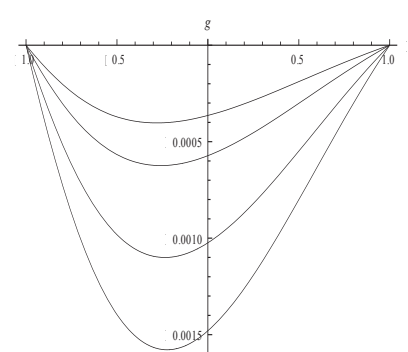


Fig 2 b

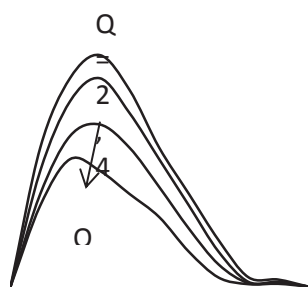


Fig 3 a

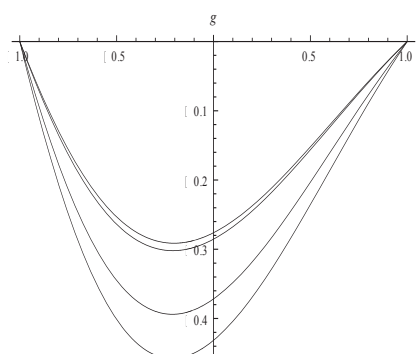


Fig 3 b

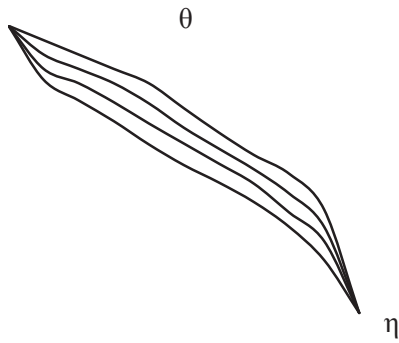


Fig 3 c

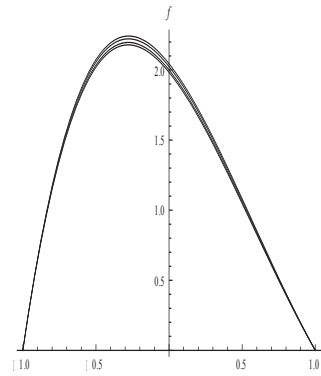


Fig 4 a

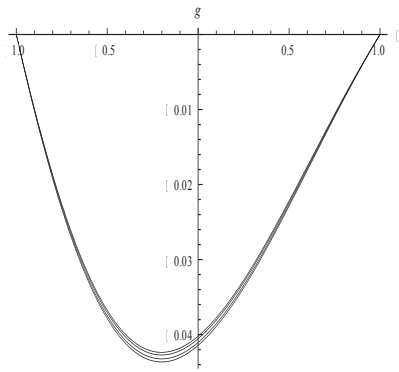


Fig 4 b

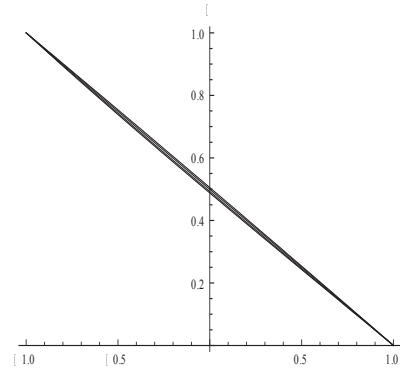


Fig 4 c

Table.1 displays the behavior of local skin friction component τ_x and Nusselt number Nu at the plates $\eta = \pm 1$. It is found that an increase in the Hartmann number M reduces τ_x at $\eta = -1$ and increases it at $\eta = +1$ while an increase in the rotation parameter R reduces τ_x at both the walls.. Also τ_x reduces with increase in the suction parameter S and the radiation parameter F at $\eta = \pm 1$. An increase in $Q > 0$ enhances τ_x at $\eta = \pm 1$ and reduces with increase in $Q < 0$ at both the walls. An increase in the nanoparticle volume fraction ϕ reduces τ_x for Cu-water nanofluid. Lesser the thermal diffusivity smaller the skin friction component at $\eta = \pm 1$. The local Nusselt number (Nu) at $\eta = +1$ is found to reduce with increase in S or Q or ϕ or F or Prandtl number Pr while at $\eta = -1$, it enhances with increase in F or $Q > 0$ or S or ϕ or Pr and reduces with $Q < 0$ in Cu-water nanofluid .

Table - 1: Skin friction(τ), Nusselt Number(Nu at $\eta = \pm 1$)

M	R	S	Q	F	ϕ	Pr	$\tau_x(+1)$	$\tau_x(-1)$	Nu(+1)	Nu(-1)
1	0.5	0.2	2	0.5	0.05	6.2	-0.046315	0.67143	0.49466	0.51096
2	0.5	0.2	2	0.5	0.05	6.2	-0.054031	0.57668	-----	-----
3	0.5	0.2	2	0.5	0.05	6.2	-0.068279	0.57712	-----	-----
1	1.5	0.2	2	0.5	0.05	6.2	-0.040801	0.52665	-----	-----
1	2.0	0.2	2	0.5	0.05	6.2	-0.038121	0.50846	-----	-----
1	0.5	0.4	2	0.5	0.05	6.2	-0.039312	-0.07086	0.49352	0.51344
1	0.5	0.2	4	0.5	0.05	6.2	-0.168533	-0.66876	0.48924	0.51342
1	0.5	0.2	-2	0.5	0.05	6.2	-0.167884	0.67001	0.398765	0.45678
1	0.5	0.2	-4	0.5	0.05	6.2	-0.166466	0.66706	0.356743	0.43563
1	0.5	0.2	2	1.5	0.05	6.2	-0.046299	0.671287	0.49352	0.51489
1	0.5	0.2	2	5.0	0.05	6.2	-0.046183	0.670992	0.49271	0.51969
1	0.5	0.2	2	0.5	0.1	6.2	-0.013495	0.36641	0.49476	0.51117
1	0.5	0.2	2	0.5	0.3	6.2	-0.004588	0.13857	0.49466	0.51139
1	0.5	0.2	2	0.5	0.05	0.71	-0.046574	0.67189	0.49640	0.50665
1	0.5	0.2	2	0.5	0.05	2.00	-0.064654	0.67129	0.49485	0.50822

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