ON FUZZY SEMI-REGULAR WEAKLY CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

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Abstract: In this article, we introduced the new class of fuzzy closed sets in fuzzy topological spaces namely fuzzy semi-regular weakly closed sets in fuzzy topological spaces. This class is mainly lies between fuzzy semi-closed sets, fuzzy α-regular weakly closed sets and fuzzy gs-closed sets in fuzzy topological spaces. Also we introduced the fundamental results of the new class.

Keywords: Fuzzy Semi-Open Sets, Fuzzy **α**rw-Closed Sets, Fuzzy Srw-Closed Sets, Fuzzy Gs-Closed Sets.

Introduction: The concept of fuzzy sets was introduced in 1965 by Zadeh[12]. Subsequently many researchers have been worked in this field and its related fields which have applications in different fields of mathematics and engineering. The theory of fuzzy topological spaces was developed by several authors by considering the basic concepts of quasi coincidences and q-neighborhoods by Pu and Liu[8]. In 1968 C. L. Chang[6] introduced and studied fuzzy topological spaces as a generalization of topological spaces. K. K. Azad[1,2], R. S. Wali et. al.[9] and R. K. Saraf[10] introduced the concept of fuzzy semi-open sets, fuzzy αrw-closed sets and fuzzy gs-closed sets in fuzzy topological spaces resp. In this article we introduced the new class of fuzzy closed sets i.e. fuzzy semi-regular weakly closed sets in fuzzy topological spaces, which lies between class of fuzzy semi-closed sets, fuzzy αrw-closed sets and fuzzy gs-closed sets in fuzzy topological spaces. Also, we introduced the some fundamental results of new fuzzy closed sets.

Preliminaries: We give few definitions and results which are required for our study. Throughout this article simply X (instead of (X,T)) represents fuzzy topological space on which no separation axioms are assumed unless otherwise mentioned. For a fuzzy subset *A* of fuzzy topological space X, cl(A), int(A) and A^c denote the closure, interior and complement of A resp. these can be defined as cl(A)=$\wedge\{\mu: \mu \geq A, 1-\mu \in T\}$, int(A)=$\vee\{\mu: \mu \leq A, 1-\mu \in T\}$ and A^c=1-A.

Definition 2.1: A fuzzy subset A of a fuzzy topological space is said to be;
1. Fuzzy regular open set [2], if int(cl(A))=A.
2. Fuzzy semi-open set [2] if and only if there exists a fuzzy open set G in X such that G≤A≤cl(G).
3. Fuzzy semi-closed set [2] if and only if there exists a fuzzy open set F in X such that int(F)≤A≤F.
4. Fuzzy regular weakly (briefly, fuzzy rw) closed set [4], if cl(A)≤G whenever A≤G and G is fuzzy regular semi open set in X.
5. Fuzzy α-open set [5] if A≤int(cl(int(A))).
6. Fuzzy α-regular weakly (briefly, fuzzy αrw) closed set [9], if αcl(A) ≤G whenever A≤G and G is fuzzy regular weakly open set in X.
7. Fuzzy generalized semi (briefly, fuzzy gs) closed set [11], if scl(A) ≤G whenever A≤G and G is fuzzy open set in X.
8. Fuzzy generalized (briefly, fuzzy g) closed set [3], if cl(A) ≤G whenever A≤G and G is fuzzy open set in X.

The compliment of the above all fuzzy closed sets becomes corresponding fuzzy open sets in the same fuzzy topological spaces.

Remark 2.2: [2]:
1. Every fuzzy regular open set is a fuzzy open set but not conversely.
2. Every fuzzy regular closed set is a fuzzy closed set but not conversely.

Lemma 2.3: Let A be a fuzzy subset of X, then the following implications are holds good [7];

1. spcl(A) ≤scl(A) ≤ αcl(A) ≤cl(A).
2. spcl(A) ≤pcl(A) ≤ αcl(A).

Fuzzy Semi-Regular Weakly Closed sets (briefly, fuzzy srw-closed sets) in Fuzzy Topological Spaces:
In this section we introduced and studied the fuzzy srw-closed set and its properties.

Definition 3.1: Let X be any fuzzy topological space. A fuzzy subset *A* of X is called a fuzzy semi-regular weakly closed set (briefly, fuzzy srw-closed set), if scl(A)≤G whenever A≤G and G is fuzzy rw-open set in X. We denote the class of fuzzy srw-closed sets in X by FSRWC(X).

Example 3.2: Let X={a, b, c, d} and fuzzy subsets are 0_X={(a,0), (b,0), (c,0), (d,0)}=0, α_1={(a,1), (b,0), (c,0), (d,0)}, α_2={(a,0), (b,1), (c,1), (d,0)}, α_3={(a,1), (b,1), (c,1), (d,0)} and α_X={(a,1), (b,1), (c,1), (d,1)}=1. Then fuzzy topology of X is T={0_X, α_1, α_2, α_3, α_X}. The fuzzy srw-closed sets are, 0_X={(a,0), (b,0), (c,0), (d,0)}=0, β_1={(a,1), (b,0), (c,0), (d,0)},β_2={(a,0), (b,0), (c,0), (d,1)}, β_3={(a,0), (b,1), (c,1), (d,0)}, β_4={(a,1), (b,0), (c,0), (d,1)}, β_5={(a,0), (b,1), (c,1), (d,1)}, β_6={(a,1), (b,0), (c,1), (d,1)}, β_7={(a,1), (b,1), (c,0), (d,1)} and β_X={(a,1), (b,1), (c,1), (d,1)}=1. i.e. FSRWC(X)= {0_X, β_1, β_2, β_3, β_4, β_5, β_6, β_7, β_X}.

Theorem 3.3: Every fuzzy closed set is a fuzzy srw-closed set in fuzzy topological space.

Proof: Let A be a fuzzy closed subset of X. Let G be any fuzzy rw-open set in X such that A≤G. Since A is fuzzy closed set, cl(A)=A, but scl(A)≤cl(A)=A implies that scl(A)≤A≤G. Therefore scl(A)≤G. Hence A is fuzzy srw-closed set in X.

The converse of Theorem 3.3 need not be true in general, which can be shown from following example.

Example 3.5: Let X={a, b, c, d} and fuzzy subsets are 0_X={(a,0), (b,0), (c,0), (d,0)}=0, α_1={(a,1), (b,0), (c,0), (d,0)}, α_2={(a,0), (b,1), (c,1), (d,0)}, α_3={(a,1), (b,1), (c,1), (d,0)} and α_X={(a,1), (b,1), (c,1), (d,1)}=1. Then fuzzy topology of X is T={0_X, α_1, α_2, α_3, α_X}. Let β= {(a,0), (b,1), (c,1), (d,0)} be a fuzzy srw-closed set but not fuzzy closed set in X.

Corollary 3.6: By Remark 2.2(2), it has been proved that every fuzzy regular closed set is a fuzzy closed set but not conversely. By Theorem 3.3, every fuzzy closed set is a fuzzy srw-closed set in X but not conversely. Hence every fuzzy regular closed set is a fuzzy srw-closed set in X, but not conversely.

Example 3.7: Let X={a, b, c, d} and fuzzy subsets are 0_X={(a,0), (b,0), (c,0), (d,0)}=0, α_1={(a,1), (b,0), (c,0), (d,0)}, α_2={(a,0), (b,1), (c,1), (d,0)}, α_3={(a,1), (b,1), (c,1), (d,0)} and α_X={(a,1), (b,1), (c,1), (d,1)}=1. Then fuzzy topology of X is T={0_X, α_1, α_2, α_3, α_X}. Let β={(a,1), (b,0), (c,1), (d,1)} be a fuzzy srw-closed set but not fuzzy regular closed set in X.

Theorem 3.8: Every fuzzy semi-closed set is a fuzzy srw-closed set in fuzzy topological space.

Proof: Let A be a fuzzy semi-closed set in X and G be any fuzzy rw-open set in X such that A≤G. Since A is fuzzy semi-closed set, scl(A)=A implies that scl(A)=A≤G. Hence A is fuzzy srw-closed set in X.

The converse of Theorem 3.8 need not be true in general, which can be shown from following example.

Example 3.9: Let X={a, b, c, d} and fuzzy subsets are 0_X={(a,0), (b,0), (c,0), (d,0)}=0, α_1={(a,1), (b,0), (c,0), (d,0)}, α_2={(a,0), (b,1), (c,1), (d,0)}, α_3={(a,1), (b,1), (c,1), (d,0)} and α_X={(a,1), (b,1), (c,1), (d,1)}=1. Then fuzzy topology of X is T={0_X, α_1, α_2, α_3, α_X}. Let β={(a,1), (b,0), (c,1), (d,1)} be a fuzzy srw-closed set but not fuzzy semi closed set in X.

Theorem 3.10: Every fuzzy αrw-closed set is a fuzzy srw-closed set in fuzzy topological space.
Proof: Let A be a fuzzy αrw-closed set in X and G be any fuzzy rw-open set in X such that A≤G. Since A is fuzzy αrw-closed set, αcl(A)≤G whenever A≤G and G is fuzzy rw-open set in X. but scl(A)≤ αcl(A)≤G implies that scl(A)≤G whenever A≤G and G is fuzzy rw-open set in X. Hence A is fuzzy srw-closed set in X.

The converse of Theorem 3.10 need not be true in general, which can be shown from following example.

Example 3.11: Let X={a, b, c, d} and fuzzy subsets are 0_X={(a,0), (b,0), (c,0), (d,0)}=0, α_1={(a,1), (b,0), (c,0), (d,0)}, α_2={(a,0), (b,1), (c,0), (d,0)}, α_3={(a,1), (b,1), (c,0), (d,0)}, α_4={(a,1), (b,1), (c,1), (d,0)} and α_X={(a,1), (b,1), (c,1), (d,1)}=1. Then fuzzy topology of X is T={0_X, α_1, α_2, α_3, α_4, α_X}. Let β={(a,1), (b,0), (c,0), (d,0)} be a fuzzy srw-closed set but not fuzzy αrw-closed set in X.

Theorem 3.12: Every fuzzy srw-closed set is a fuzzy gs-closed set in fuzzy topological space.
Proof: Let A be a fuzzy srw-closed set in X and G be any fuzzy open set in X such that A≤G. Since A is fuzzy srw-closed set, scl(A)≤G whenever A≤G and G is fuzzy rw-open set in X. since every fuzzy open set is fuzzy rw-open set in X. Hence we have scl(A)≤G whenever A≤G and G is fuzzy open set in X. Hence A is fuzzy gs-closed set in X.

The converse of Theorem 3.12 need not be true in general, which can be shown from following example.

Example 3.13: Let X={a, b, c, d} and fuzzy subsets are 0_X={(a,0), (b,0), (c,0), (d,0)}=0, α_1={(a,1), (b,0), (c,0), (d,0)}, α_2={(a,0), (b,1), (c,1), (d,0)}, α_3={(a,1), (b,1), (c,1), (d,0)} and α_X={(a,1), (b,1), (c,1), (d,1)}=1. Then fuzzy topology of X is T={0_X, α_1, α_2, α_3, α_X}. Let β={(a,0), (b,1), (c,0), (d,0)} be a fuzzy gs-closed set but not fuzzy srw-closed set in X.

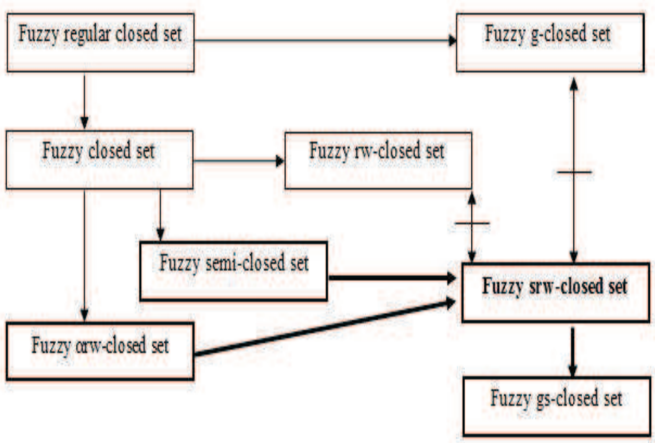
Remark 3.14:
1. Fuzzy srw-closed sets and fuzzy rw-closed sets are independent.
2. Fuzzy srw-closed sets and fuzzy g-closed sets are independent.
3. Fuzzy srw-closed sets and fuzzy w-closed sets are independent.

The above Remark 3.14 can be shown from following examples.

Example 3.15: Let X={a, b, c, d} and fuzzy subsets are 0_X={(a,0), (b,0), (c,0), (d,0)}=0, α_1={(a,1), (b,0), (c,0), (d,0)}, α_2={(a,0), (b,1), (c,1), (d,0)}, α_3={(a,1), (b,1), (c,1), (d,0)} and α_X={(a,1), (b,1), (c,1), (d,1)}=1. Then fuzzy topology of X is T={0_X, α_1, α_2, α_3, α_X}.

1) Let β={(a,1), (b,0), (c,0), (d,0)} be a fuzzy srw-closed set but not fuzzy rw-closed set in X and γ={(a,1), (b,1), (c,0), (d,0)} be fuzzy rw-closed set but not fuzzy srw-closed set in X.
2) Let β={(a,1), (b,0), (c,0), (d,0)} be a fuzzy srw-closed set but not fuzzy g-closed set in X and γ={(a,0), (b,1), (c,0), (d,1)} be a fuzzy g-closed set but not fuzzy srw-closed set in X.
3) Let β={(a,1), (b,0), (c,0), (d,0)} be a fuzzy srw-closed set but not fuzzy w-closed set in X and γ={(a,0), (b,0), (c,1), (d,1)} be a fuzzy w-closed set but not fuzzy srw-closed set in X.

From the above example results and discussion, we have the following implications:
In the diagram,
A ⟶ B means A implies B but not conversely and A ⟺̸ B means A and B are independent.



Remark 3.16: Union and intersection of any two fuzzy srw-closed sets is not necessarily fuzzy srw-closed, which can be illustrated in the following example.

Example 3.17: Let X={a, b, c, d} and fuzzy subsets are 0_X={(a,0), (b,0), (c,0), (d,0)}=0, α_1={(a,1), (b,0), (c,0), (d,0)}, α_2={(a,0), (b,1), (c,1), (d,0)}, α_3={(a,1), (b,1), (c,1), (d,0)} and α_X={(a,1), (b,1), (c,1), (d,1)}=1. Then fuzzy topology of X is T={0_X, α_1, α_2, α_3, α_X}. Let β={(a,0), (b,1), (c,0), (d,0)} and γ={(a,0), (b,1), (c,1), (d,0)}. But β ∨ γ ={(a,1), (b,1), (c,1), (d,0)} which is not a fuzzy srw-closed set in X and let β*={(a,0), (b,1), (c,1), (d,1)} and γ*={(a,1), (b,0), (c,1), (d,1)} but β*∧ γ*={(a,0), (b,0), (c,1), (d,1)} which is not a fuzzy srw-closed set in X.

Theorem 3.18: If A be a fuzzy subset of X is both fuzzy regular semi-open and fuzzy rw-closed set and then it is a fuzzy srw-closed set in X.

Proof: Let A be a fuzzy subset of X, which is both fuzzy regular semi-open and fuzzy rw-closed. Let A≤G where G is fuzzy rw-open set in X. Now A≤A, by the Definition 2.1(5), cl(A)≤A but scl(A)≤cl(A) implies that scl(A)≤G. Hence A is fuzzy srw-closed set in X.

Remark 3.19: If A is a fuzzy subset of X is both fuzzy regular semi open and fuzzy srw-closed, then A need not be fuzzy rw-closed set in general, as seen from the following example.

Example 3.20: Let X={a, b, c, d} and fuzzy subsets are 0_X={(a,0), (b,0), (c,0), (d,0)}=0, α_1={(a,1), (b,0), (c,0), (d,0)}, α_2={(a,0), (b,1), (c,1), (d,0)}, α_3={(a,1), (b,1), (c,1), (d,0)} and α_X={(a,1), (b,1), (c,1), (d,1)}=1. Then fuzzy topology of X is T={0_X, α_1, α_2, α_3, α_X}. Let β={(a,0), (b,1), (c,1), (d,0)} is both fuzzy regular semi-open and fuzzy srw-closed set but not fuzzy rw-closed set in X.

Theorem 3.21: If A be a fuzzy subset of X is both fuzzy regular open and fuzzy srw-closed set then A is semi-closed in X.

Proof: Suppose a fuzzy subset A of X is fuzzy regular open and fuzzy srw-closed set. As every regular open set is fuzzy rw-open set in X. Now A≤A then by definition of fuzzy srw-closed set, scl(A) ≤A and also A≤scl(A) then we have scl(A)=A. Hence A is fuzzy semi-closed set in X.

Theorem 3.22: If A is a fuzzy srw-closed set of X and scl(A)∧$(scl(A))^c$=0, then scl(A)-A does not contain non-zero fuzzy rw-closed set in X.

Proof: Suppose A is fuzzy srw-closed subset of X and scl(A)∧$(scl(A))^c$=0. By contrary, let B be a fuzzy rw-closed set such that scl(A)-A≥B and B≠0. Now B≤scl(A)-A i.e. B≤A^c implies that A≤B^c. Since B is a fuzzy rw-closed set, by the Definition 3.28[4], B^c is fuzzy rw-open set in X. Since A is a fuzzy srw-closed set in X, by definition scl(A)≤B^c. So B≤$(scl(A))^c$, therefore B≤(scl(A))∧$(scl(A))^c$=0, by the hypothesis B=0 which is contradiction. Hence scl(A)-A does not contain any non-zero fuzzy rw-closed set in X.

Corollary 3.23: If A is a fuzzy srw-closed set in X and scl(A)∧$(scl(A))^c$=0, then scl(A)-A does not contain non-zero fuzzy rw-open set in X.

Proof: This follows from Theorem 3.22 and the fact that every fuzzy open set is fuzzy rw-open set in X.

Theorem 3.24: Let A be a fuzzy srw-closed set of X and suppose A≤B≤scl(A), then B is also a fuzzy srw-closed set in X.

Proof: Let A be any fuzzy srw-closed set in X and A≤B≤scl(A). Let G be any fuzzy rw-open set in X such that B≤A. Then A≤A and A is fuzzy srw-closed, scl(A)≤A but scl(B)≤scl(A) and hence scl(B) ≤G.Therefore B is a fuzzy srw-closed set in X.

The converse of the above Theorem need not be true in general as seen from following example.

Example 3.25: Let X={a, b, c, d} and fuzzy subsets are 0_X={(a,0), (b,0), (c,0), (d,0)}=0, α_1={(a,1), (b,0), (c,0), (d,0)}, α_2={(a,0), (b,1), (c,1), (d,0)}, α_3={(a,1), (b,1), (c,1), (d,0)} and α_X={(a,1), (b,1), (c,1), (d,1)}=1. Then fuzzy topology of X is T={0_X, α_1, α_2, α_3, α_X}. Let β={(a,0), (b,0), (c,0), (d,1)} and γ={(a,1), (b,0), (c,0), (d,1)} are fuzzy srw-closed sets in X, but β ≤ γ is not fuzzy subset in fscl(β), since fscl(β)={(a,0), (b,0), (c,0), (d,1)}.

Theorem 3.26: In a fuzzy topological space X, if FRWO(X)={1,0}, where FRWO(X) is the family of all fuzzy rw-open sets, then every fuzzy subset of X is fuzzy srw-closed set in X.

Proof: Let X be any fuzzy topological space and FRWO(X)={1,0}. Let A be any fuzzy subset of X. Suppose A=0, then 0 is a fuzzy srw-closed set in X. Suppose A≠0, then 1 is the only fuzzy rw-open set containing A and so scl(A)≤1. Hence A is a fuzzy srw-closed set in X.

Theorem 3.27: Let A be a fuzzy srw-closed set in X and scl(A))∧(scl(A))^c=0, then A is a fuzzy semi-closed set if and only if scl(A)-A is a fuzzy rw-closed set in X.

Proof: Suppose A is a fuzzy semi-closed set in X. Then scl(A)=A which implies scl(A)-A=0, which is the fuzzy rw-open set in X.

Conversely, suppose scl(A)-A is fuzzy srw-closed, then by Theorem 3.22, scl(A)-A=0, that is scl(A)=A and hence A is fuzzy semi-closed set in X.

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