STABILITY OF THE QUEUEING MODEL USING DSW MODEL WITH HEXAGONAL FUZZY NUMBER

Dr. M.S. Annie Christi
Department of Mathematics/ Providence College for Women/Coonoor
N. Priyadharshini
Department of Mathematics/ Providence College for Women/Coonoor

Abstract: This paper proposes a procedure to construct the membership functions of the performance measures in queuing systems where the inter arrival time and service time are Fuzzy numbers. We propose a Fuzzy nature in FM\FM\1 queuing system with finite capacity and calling population is infinite. Approximate method of extension namely DSW(Dong, Shah & Wong) algorithm is used to define membership functions of the performance measures for the Queuing model FM\FM\1 in which the arrival rate and service rate are Fuzzy numbers.DSW algorithm is based on the α-cut representation of fuzzy sets in a standard interval analysis. The discussion of this paper is confined to the systems with Fuzzy variables. Numerical example shows the efficiency of the algorithm.

Introduction: The classical set theory is built on the fundamental concept of “set” of which an individual is either a member or not a member. A sharp, crisp, and unambiguous distinction exists between a member and a non member for any well-defined “set” of entries. Many real-world application problems cannot be described and handled by the classical set theory including all those involving elements with only partial membership of a set. On the contrary, fuzzy set theory accepts partial memberships, and, therefore, in a sense generalizes the classical set theory to some extent.

Definitions: If X is a collection of objects denoted generically by x then a fuzzy set A in X is a set of ordered pairs:
$\boldsymbol{A}= \{(\boldsymbol{x}, \mu_{\boldsymbol{A}}(\boldsymbol{x})/\boldsymbol{x} \in \boldsymbol{X}\}$
μ_A(x) is called the membership function or grade of membership(also degree of compatibility or degree of truth)of x in A which maps X to the membership space M.(when M contains only two points 0 & 1,A is non-fuzzy & μ_A(x is identical to the characteristic function of a non-fuzzy set)
-Zimmermann.

Membership Function of Fuzzy Set: In fuzzy sets , each elements is mapped to [0,1] by membership function
μ_A:X→[0,1]
where [0,1] means real numbers between 0 and 1(including 0 and 1).

Fuzzy Sets: A fuzzy sets F is defined in a set E as follows
F={(x,μ_F(x))/x∈E}
Where μ_F: E→[0,1],x∈E→ μ_F(x) ∈[0,1], and μ_F(x) denotes the degree of membership of element x in the set E.

The (FM\FM\1):(A\FCFS)Queue Model: In this model we consider an infinite source population with first come first served discipline where both the interarrival time $\tilde{\lambda}$ and the service time $\tilde{\mu}$ follow an exponential distribution.
The expected number of customer in the system

$$L_s=\frac{\lambda}{\mu-\lambda}$$

The expected number of customers in the queue

$$L_q=\frac{\lambda^2}{\mu(\mu-\lambda)}$$

The average waiting time in the system

$$W_s=\frac{1}{\mu-\lambda}$$

The average waiting time of a customer in the queue

$$W_q=\frac{\lambda}{\mu(\mu-\lambda)}$$

Standard Internal Analysis Arithmetic: Let I_1 and I_2 be two interval numbers defined by ordered pairs of real numbers with lower upper bounds.

I_1=[a , b] , a ≤ b I_2=[c , d] , c ≤ d.

Define a general arithmetic property with the symbol *

Where * =[+ ,- ,× ,÷] symbolically the operation.

I_1 * I_2 =[a, b] * [c, d]

represents another interval.

The interval calculation depends on the magnitude and signs of the element a , b ,c ,d.

[a, b] + [c, d] = [a + c, b+ d]

[a, b] - [c, d] = [a- d , b- c]

[a, b].[c, d] =[min(ac,ad,bc,bd),max(ac,ad,bc,bd)]

[a, b]÷[c, d] =[a, b].[$\frac{1}{d}$,$\frac{1}{c}$], provided that 0 ∉ [c, d]

α[a, b] = [αa, αb] for α>0

α [a, b] = [αb ,αa] for α< 0

DSW Algorithm: DSW (Dong, Shah, Wong) is one of the approximate methods make use of intervals at various α–cut levels in defining membership functions. It was the full α–cut intervals in a standard interval analysis.

The DSW algorithm greatly simplifies manipulation of the extension principle for continuous valued fuzzy variables, such as fuzzy numbers defines on the real line. It prevent abnormality in the output membership function due to application of the discrimination reaching on the fuzzy variables domain, it can prevent the widening of the resulting functional expression by conventional interval analysis methods.

Any continuous membership function can be represented by a continuous sweep of α–cut in term from α=0 to α= 1. Suppose we have single input mapping given by $y = f(x)$ that is to be extended for fuzzy sets $\bar{A} = f(\bar{A})$and we want to decompose $\bar{A}$ into the series of α–cut intervals, say I_α.

The DSW Algorithm:

1. Select a α–cut value where 0≤α≤1.
2. Find the intervals in the input membership functions that correspond to this α.
3. Using standard binary interval operations compute the interval for the output membership function for the selected α–cut level.
4. Repeat steps 1 - 3 for different values of α to complete α–cut representation of the solution.

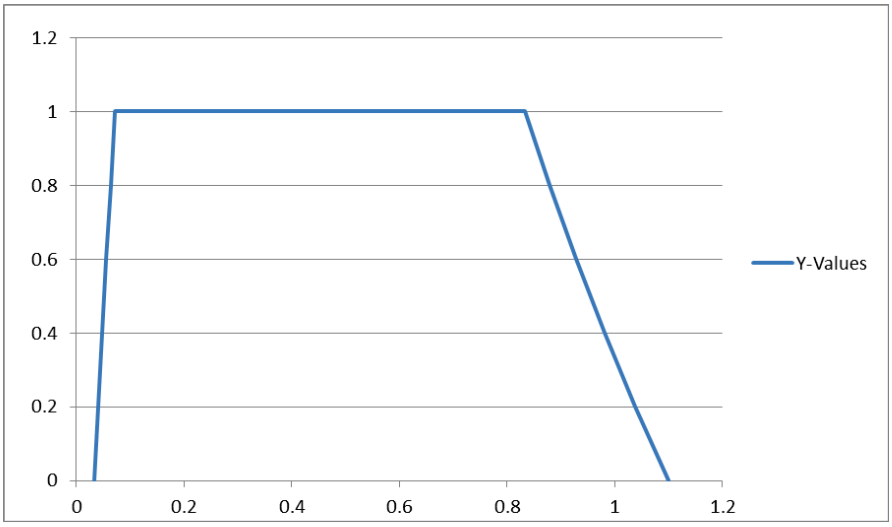
Numerical Example: ***Hexagonal Fuzzy Number***: Consider a FM/FM/1 queue, where the both the arrival rate and service rate are fuzzy numbers represented by λ̃=[1,3,5,7,9,11] and μ̃=[13,15,17,19,21,23].The interval of confidence at possibility level as [1+α,11-α] and [13+α,23-α].

$L_s = \frac{x}{y-x}$, $W_s=\frac{1}{y-x}$, $L_q = \frac{x^2}{y(y-x)}$, $W_q=\frac{x}{y(y-x)}$

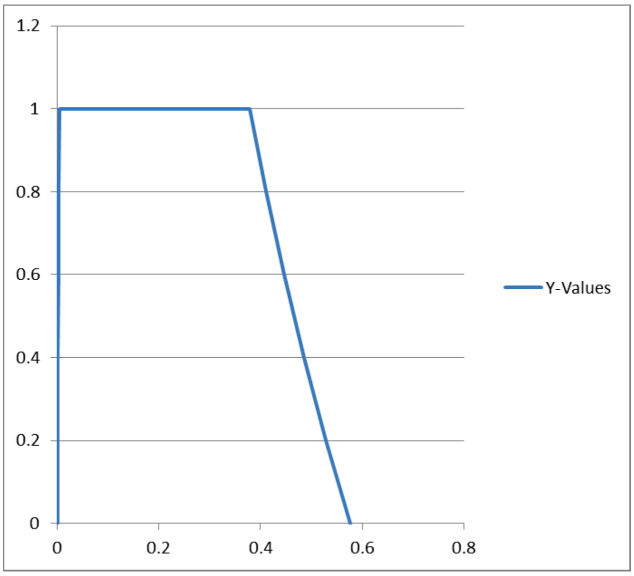
Where x=[1+α,11-α] and y=[13+α,23-α]

Table: The α- cuts of L_s,W_s,L_q,W_q at α Values

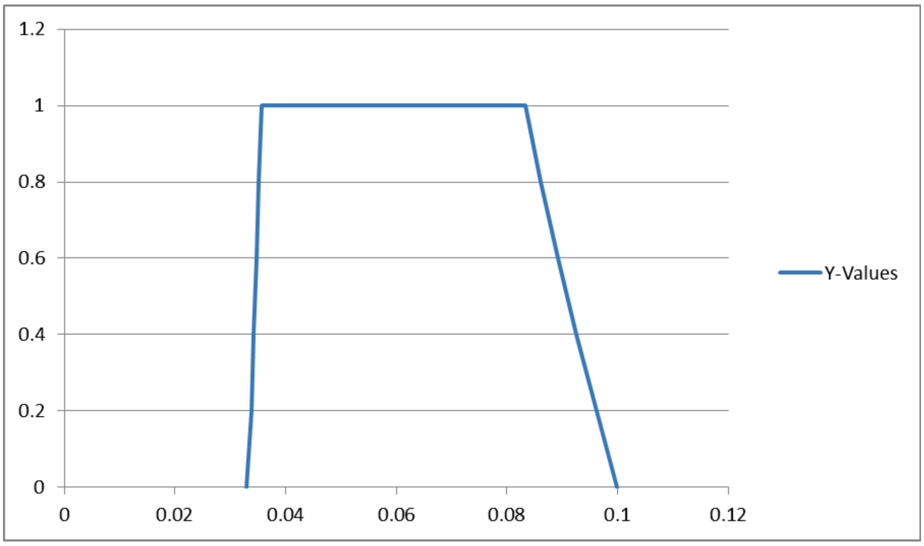
ALPHA	L_s	L_q	W_s	W_q
0	[0.0333,1.1]	[0.0011,0.5762]	[0.0333,0.1]	[0.0011,0.0523]
0.1	[0.0369,1.0686]	[0.0013,0.5521]	[0.0336,0.0980]	[0.0012,0.0506]
0.2	[0.0405,1.0385]	[0.0016,0.5290]	[0.0338,0.0962]	[0.0013,0.049]
0.3	[0.0442,1.0094]	[0.0019,0.5071]	[0.0340,0.0943]	[0.0014,0.0474]
0.4	[0.0479,0.9814]	[0.0022,0.4862]	[0.0342,0.0926]	[0.0016,0.0459]
0.5	[0.0517,0.9545]	[0.0025,0.4662]	[0.0345,0.0909]	[0.0017,0.0444]
0.6	[0.0556,0.9286]	[0.0029,0.4471]	[0.0347,0.0893]	[0.0018,0.043]
0.7	[0.0594,0.9035]	[0.0033,0.4288]	[0.035,0.0877]	[0.0019,0.0416]
0.8	[0.0634,0.8793]	[0.0037,0.4114]	[0.0352,0.0862]	[0.0021,0.0403]
0.9	[0.0673,0.8559]	[0.0041,0.3937]	[0.0355,0.0847]	[0.0022,0.0391]
1	[0.0714,0.8333]	[0.0046,0.3788]	[0.0357,0.0833]	[0.0023,0.0379]


1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
1.2
Y-Values

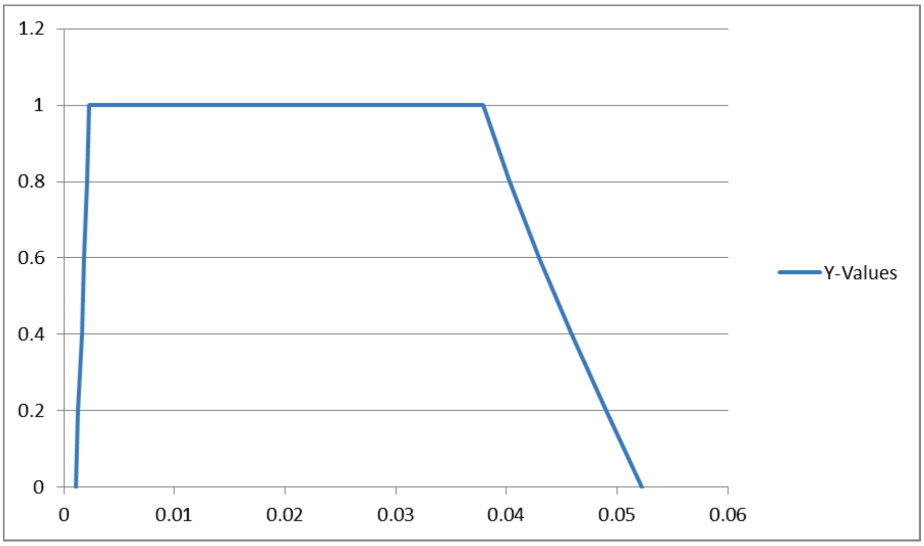
L_S


1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
Y-Values

L_q


1.2
1
0.8
0.6
0.4
0.2
0
0
0.02
0.04
0.06
0.08
0.1
0.12
Y-Values

W_s



W_q

Conclusion: In this paper,the performance measure of their queueing model is studied in a fuzzy environment. Hexagonal fuzzy numbers are used to study the efficiency of DSW algorithm. This is illustrated with an example.

References:

1. Buckely.J.J, "Elementary Queuing Theory Based on Possibility Theory", Fuzzy sets and systems37,43-52,(1990).
2. Chen .S.P, "Parametric Non Linear Programming Approach to Fuzzy Queues with Bulk, service", European Journal of Operational Research 163,434-444, (2005).
3. Elizabeth Sebastian, Priyanka Victor, A Discrete Time Sis Epidemic Model With Primary Immunodeficiency; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 99-104
4. Chen .S.P, " A Mathematics Programming Approach to the Machine Interference Problem with Fuzzy Parameters", Applied Mathematics and Computation 173,374-387,(2006).
5. Gross, D. and Haris, C.M. "Fundamentals of Queuing Theory", Wiley, Newyork, (1985). Kanufmann, " Introduction to the Theory of Fuzzy Subsets",Vol I,Academic Press, Newyork,(1975).
6. Li.R.J and Lee.E.S " Analysis of Fuzzy Queues", Computers and Mathematics with Applications 17(7),1143-1147,(1989).
7. Negi.D.S. and Lee.E.S, "Analysis and Stimulation of Fuzzy Queue, Fuzzy Sets and Systems 46;321-330,(1992).
8. Elizabeth Sebastian, Preethi Victor, A Discrete Time Model On Depletion Of Forest Resources By Human Population: Effect Of Technology On Its Conservation; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 105-108
9. Timothy J.Rose, "Fuzzy Logic and its Applications to Engineering, Wiley Eastern publishers(2005). Yovgav,R.R, " A Characterization of the Extension Principle", Fuzzy Sets and Systems 18;71-78,(1986).
10. Zadeh,L.A, "Fuzzy Sets, Information and Control., No.8pp.338-353,(1965).
11. Zimmermann,H.J., "Fuzzy Set Theory and its Applications,4th edition., Kluwer Academic, Boston,(2001).
12. Zhang,Z.J. and Xu, X.L., "Analysis for the M/M/1 Queue with Multiple Working Vacations and N-Policy", " International Journal of Information and Management sciences",Vol.19, No.3,pp.495-506,(2008).
13. Elizabeth S, Nirmal Veena S, An Analysis Of Exponential Stability For Difference Equations; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 109-112
14. Premkumar Gupta and D.S.Hira, "Operation Research" ,884-885(2007).

***