APPLICATION OF JACOBIAN & SOR ITERATIONPROCESS IN INTUITIONISTIC FUZZY MAGDM PROBLEMS

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Abstract: The aim of this paper is to investigate the Multiple Attribute Group Decision Making (MAGDM) problems with intuitionistic fuzzy sets. The unknown decision maker weights are derived through the Jacobi iteration and SOR method by obtaining the solution of linear algebraic equations and it is utilized to solve the MAGDM problems. A numerical illustration is given to show the effectiveness and feasibility of the proposed approach.

Keywords: Intuitionistic Fuzzy Set, IFWAA and IFHA operator, Iteration Method, MAGDM.

Introduction: Multiple Attribute Group Decision Making (MAGDM) problems play a vital role in today's comprehensive world. The decision makers have difficulties in assigning crisp values as scoring to the criteria. The main characteristic of decision making problem is fuzziness and it was introduced by **Zadeh [20]**. **Atanassov [1-3]** and **Atanassov & Gargov [4]** expanded the Intuitionistic Fuzzy Set (IFS), using interval value to express membership and non-membership function of IFSs. **Chen & Tan [5]** proposed multicriteria fuzzy decision making problems based on vague sets. **Zeng & Li [21]** introduced the correlation coefficient of intuitionistic fuzzy sets. **Robinson & Amirtharaj [9-17]** and **Robinson & Jeeva [18]** defined correlation coefficient for different higher order intuitionistic fuzzy sets and utilized in MAGDM problems. **Li [7]** and **Wei [19]** investigated MAGDM models and methods using intuitionistic fuzzy sets. In this work, numerical methods are proposed for determining weights of decision makers and used for MAGDM problems. **Jain et al. [6]** and **Rice [8]** discussed several numerical methods for scientific and engineering computation. Here, Gauss-Jacobi iteration and SOR method are used to obtain the solution of linear algebraic equation, which are further utilized to derive the decision maker weights in intuitionistic fuzzy decision making problems. The feasibility and effectiveness of the proposed method are illustrated using numerical examples.

Preliminaries: In this section, some basic definitions and arithmetic aggregation operators of Intuitionistic Fuzzy Numbers (IFNs) are presented.

Definition: 1 Intuitionistic Fuzzy Set: Let a set X be fixed. An IFS Ã in X is an object of the form $\tilde{A}=\{<x,\ \mu_{\tilde{A}}(x),\gamma_{\tilde{A}}(x)>,\ x\in X\}$, Where the $\mu_{\tilde{A}}(x):X\rightarrow[0,\ 1]$ and $\gamma_{\tilde{A}}(x):E\rightarrow[0,\ 1]$ define the degree of membership and degree of non-membership respectively, of the element x∈X to the set Ã, which is a subset of X, for every element x ∈ X, $0\ \leq\mu_{\tilde{A}}(x)\ +\gamma_{\tilde{A}}(x)\leq\ 1$.

Definition: 2 Let $\tilde{a}_j=(\mu_j,\gamma_j)$, for all $j=1,2,...,n$ be a collection of Intuitionistic fuzzy values. The Intuitionistic Fuzzy Weighted Arithmetic Averaging Operator (IFWAA), $IFWAA:Q^n\rightarrow Q$ is defined as $IFWAA(\tilde{a}_1,\tilde{a}_2,....,\tilde{a}_n)=\sum_{j=1}^{n}\tilde{a}_{\sigma(j)}w_j=\left(1-\prod_{j=1}^{n}\left(1-\dot{\mu}_{a_{\sigma(j)}}\right)^{w_j},\prod_{j=1}^{n}\left(\dot{\gamma}_{a_{\sigma(j)}}\right)^{w_j}\right)$, where $(\sigma(1),\sigma(2),...,\sigma(n))$ is a permutation of (1,2,....n) such that $\tilde{a}_{\sigma(j-1)}\geq\tilde{a}_{\sigma(j)}$ for all j=2,....n. Here $W=(w_1,w_2,...,w_n)^T$ be the weighting vector of $\tilde{a}_j$ for all ∋ $w_j>0$ and $\sum_{j=1}^{n}w_j=1.$

Definition: 3 An Intuitionistic Fuzzy Hybrid Aggregation (IFHA) operator of dimension n is a mapping $IFHA : Q^n \to Q$ that has an associated vector $w = (w_1, w_2, \ldots, w_n)^T$ such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$.

$$IFHA(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \sum_{j=1}^{n} \dot{\tilde{a}}_{\sigma(j)} w_j$$

$$= \left(1 - \prod_{j=1}^{n} \left(1 - \dot{\mu}_{\tilde{a}_{\sigma(j)}} \right)^{w_j}, \prod_{j=1}^{n} \left(\dot{\gamma}_{\tilde{a}_{\sigma(j)}} \right)^{w_j} \right),$$

where $\dot{\tilde{a}}_{\sigma(j)}$ is the j[th] largest of the weighted IFN $\dot{\tilde{a}}_j \left(\dot{\tilde{a}}_j^{n\,\omega j}, \text{ j=1,2,...,n} \right)$. $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ be the weight vector of $\tilde{a}_j$ (j=1,2,...,n), and $\omega_j > 0$, $\sum_{j=1}^{n} \omega_j = 1$ and n is the balancing co-efficient.

Determining Experts Weights For Magdm Problems Using Jacobi-Iteration And SOR Method:

Iteration Method: Iteration methods in numerical methods are attractive for sparse matrices, because they use much less memory than direct methods, and so they might be used even though they require more execution time. Many of the problems of numerical analysis can be deduced to the problem of solving linear systems of equations. The use of matrix notation is not only convenient, but extremely powerful, in bringing out the relationship between variables. Now let us see about Gauss-Jacobi iteration and SOR method.

Gauss- Jacobi Iteration Method: We assume that the quantities a_{ii} are pivot elements. The exact solution may be written in the form

$$a_{11}x_1 = -\left(a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n \right) + b_1$$

$$a_{22}x_2 = -\left(a_{21}x_1 + a_{23}x_3 + \ldots + a_{2n}x_n \right) + b_2$$

$$\vdots$$

$$a_{nn}x_n = -\left(a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{n,n-1}x_{n-1} \right) + b_n.$$

The Jacobi iteration method may now be defined as

$$x_1^{(k+1)} = -\frac{1}{a_{11}} \left(a_{12}x_2^{(k)} + a_{13}x_3^{(k)} + \ldots + a_{1n}x_n^{(k)} - b_1 \right)$$

$$x_2^{(k+1)} = -\frac{1}{a_{22}} \left(a_{21}x_1^{(k)} + a_{23}x_3^{(k)} + \ldots + a_{2n}x_n^{(k)} - b_2 \right) \quad x_n^{(k+1)} = -\frac{1}{a_{nn}} \left(a_{n1}x_1^{(k)} + a_{n2}x_2^{(k)} + \ldots + a_{n,n-1}x_{n-1}^{(k)} - b_n \right),$$

$$\vdots \qquad k = 0,1,2\ldots$$

This method is called the method of simultaneous displacement. In the matrix form, the method can be written as: $x^{(k+1)} = -D^{-1}(L+U)x^{(k)} + D^{-1}b$

$$= H\, x^{(k)} + c, \quad k = 0,1,2\ldots \qquad (1)$$

Where $H = -D^{-1}(L+U)$ & $c = D^{-1}b$. L & U are lower and upper triangular matrices with zero diagonal entries. D is the diagonal matrix such that A=L+D+U.

Equation (1), can alternatively be written as $x^{(k+1)} = x^{(k)} + D^{-1}\left[b - Ax^{(k)} \right]$ $v^{(k)} = -D^1 r^{(k)}$. Where $v^{(k)} = x^{(k+1)} - x^{(k)}$ is the error in the approximation and $r^{(k)} = b - Ax^{(k)}$ or $Dv^{(k)} = r^{(k)}$ is the residual vector. The Jacobi iteration method in an error format is $x^{(k+1)} = x^{(k)} + v^{(k)}$.

Consider, the system of equations

$4x_1 + x_2 + x_3 = 2;\ x_1 + 5x_2 + 2x_3 = 6;\ x_1 + 2x_2 + 3x_3 = 4.$ By using Gauss Jacobi iteration method, we have

$$A = \begin{bmatrix} 0.375 & -0.0034 & 0.1833 \\ 1.02 & 0.7384 & 0.9892 \\ 0.9666 & 0.5284 & 0.8399 \end{bmatrix}$$

By using Gauss-Jacobi method we get the approximate solution and the weight vector is obtained by decomposing the approximate solution which is given by:

$\gamma = (0.118510818, 0.468475346, 0.413013859)$.

Successive Over Relaxation (SOR) Method: This method is a generalization of the Gauss-Seidal method. This method is often used when the co-efficient matrix of the system of equations is symmetric.

We define an auxiliary vector $\hat{x}$ as $\hat{x}^{(k+1)} = -D^{-1}L\,x^{(k+1)} - D^{-1}U\,x^{(k)} + D^{-1}b.$ (2)

The final solution is now written a $x^{(k+1)} = x^{(k)} + w\left(\hat{x}^{(k+1)} - x^{(k)}\right)$ or $x^{(k+1)} = (1-w)\,x^{(k)} + w\,\hat{x}^{(k+1)}$. (3)

Substituting (2) in (3), we get $x^{(k+1)} = Hx^{(k)} + c, \quad k = 0,1,2,....$ Where $H_{SOR} = (D+wL)^{-1}\left[(1-w)D - wU\right]$ and $c = w(D+wL)^{-1}b.$ Equation (3), can alternatively be written as $x^{(k+1)} = x^{(k)} + w(D+wL)^{-1}r^{(k)}.$ where $r^{(k)} = b - A\,x^{(k)}$ is the residual. $v^{(k)} = w(D+wL)^{-1}r^{(k)}$ or $(D+wL)v^{(k)} = wr^{(k)}$. This equation describes the SOR method in its error format. When $w=1$, The above equation reduces to the Gauss-Seidal method. The quantity w is called the relaxation parameter and $x^{(k+1)}$ is a weight mean of $\hat{x}^{(k+1)}$ & $x^{(k)}$.

Consider the system of equations

$2x_1 - 1x_1 + 0x_3 = 7;\ -1x_1 + x_2 - 1x_3 = 1;\ 0x_1 - 1x_2 + 2x_3 = 1.$ By using SOR method, we have

$$B = \begin{bmatrix} 4.1006 & 5.1472 & 5.8281 \\ 2.9879 & 4.4569 & 4.8729 \\ 2.3361 & 2.7958 & 2.9606 \end{bmatrix}$$

By using SOR method we get the approximate solution and the weight vector by decomposing the approximate solution which is given by:

$\omega = (0.4242505474, 0.3493854095, 0.2263640428).$

An Approach to Group Decision Making with Intuitionistic Fuzzy Information:

Step: 1 Utilize the IFWAA operator, to derive the individual overall preference IFS values.

Step: 2 Utilize the IFHA operator to derive the collective overall IFS values of the alternatives A_i.

Step: 3 Calculate the correlation [21] between the collective overall preference values r_i and the positive ideal value $\tilde{r}_i$, where $\tilde{r}_i = (0,1)$. The correlation of $A, B \in IFSs(x)$ is given by a formula

$$C_{ZL}(A,B) = \frac{1}{n}\sum_{i=1}^{n}\begin{bmatrix} u_A(x_i)u_B(x_i) + \gamma_A(x_i)\gamma_B(x_i) \\ +\pi_A(x_i)\pi_B(x_i) \end{bmatrix}$$

Step: 4 Calculate the correlation coefficient [21] from the following equation $\rho_{ZL}(A,B) = C_{ZL}(A,B) \big/ \sqrt{C_{ZL}(A,A)\cdot C_{ZL}(B,B)}$. **Step: 5** Rank all the alternatives $A_i\ (i = 1,2,\dots,m)$ and select the best one in accordance with the correlation coefficient obtained in step 4.

Numerical Illustration: Let us suppose there is a risk investment company, which wants to invest a sum of money in the best option. There is a panel with five possible alternatives to invest the money with the following four attributes: G1 is the risk analysis. G2 is the growth analysis. G3 is the social-political impact analysis. G4 is the environmental impact analysis. The weighting vector is obtained by normalizing the solution of Gauss-Jacobi and SOR methods are $\gamma = (0.118510818, 0.468475346, 0.413013859)^T$, $\omega = (0.4242505474, 0.3493854095, 0.2263640428)^T$ under the above four attributes weights $w = (0.2, 0.1, 0.3, 0.4)^T$ and construct, respectively, the decision matrices as listed in the following matrices $R = \left(r_{2ij}^{(k)}\right)5*4\,(k=1,2,3)$ As follows:

$$R_1 = \begin{pmatrix} (0.5,\ 0.4) & (0.6,\ 0.3) & (0.3,\ 0.6) & (0.2,\ 0.7) \\ (0.7,\ 0.3) & (0.7,\ 0.2) & (0.7,\ 0.2) & (0.4,\ 0.5) \\ (0.6,\ 0.4) & (0.5,\ 0.4) & (0.5,\ 0.3) & (0.2,\ 0.3) \\ (0.8,\ 0.1) & (0.6,\ 0.3) & (0.3,\ 0.4) & (0.2,\ 0.6) \\ (0.6,\ 0.2) & (0.4,\ 0.3) & (0.7,\ 0.1) & (0.1,\ 0.3) \end{pmatrix} \quad R_2 = \begin{pmatrix} (0.4,\ 0.3) & (0.5,\ 0.2) & (0.2,\ 0.5) & (0.1,\ 0.6) \\ (0.6,\ 0.2) & (0.6,\ 0.1) & (0.6,\ 0.1) & (0.3,\ 0.4) \\ (0.5,\ 0.3) & (0.4,\ 0.3) & (0.4,\ 0.2) & (0.5,\ 0.2) \\ (0.7,\ 0.1) & (0.5,\ 0.2) & (0.2,\ 0.3) & (0.1,\ 0.5) \\ (0.5,\ 0.1) & (0.3,\ 0.2) & (0.6,\ 0.2) & (0.4,\ 0.2) \end{pmatrix}$$

$$R_3 = \begin{pmatrix} (0.4,\ 0.5) & (0.5,\ 0.4) & (0.2,\ 0.7) & (0.1,\ 0.8) \\ (0.6,\ 0.4) & (0.6,\ 0.3) & (0.6,\ 0.3) & (0.3,\ 0.6) \\ (0.5,\ 0.5) & (0.4,\ 0.5) & (0.4,\ 0.4) & (0.5,\ 0.4) \\ (0.7,\ 0.2) & (0.5,\ 0.4) & (0.2,\ 0.5) & (0.1,\ 0.7) \\ (0.5,\ 0.3) & (0.3,\ 0.4) & (0.6,\ 0.2) & (0.4,\ 0.4) \end{pmatrix}$$

By using algorithm, we get the correlation coefficients as:

$\rho_{ZL}(r_1,\tilde{r}_1) = 0.471650703;\ \rho_{ZL}(r_1,\tilde{r}_1) = 0.138684852;$

$\rho_{ZL}(r_1,\tilde{r}_1) = 0.0997173;\ \rho_{ZL}(r_1,\tilde{r}_1) = 0.084134375;$

$\rho_{ZL}(r_1,\tilde{r}_1) = 0.053130994.$

Rank all the alternatives $A_i\ (i = 1,2,3,4,5)$.

$A_1 > A_2 > A_3 > A_4 > A_5$. Hence, the best alternative is A_1.

Conclusion: In this paper, Gauss-Jacobi and SOR method are used to obtain the solution of linear algebraic equations and it is utilized to derive the decision maker weights in MAGDM problems under intuitionistic fuzzy sets. In the process of determining weights, multi criteria are explicitly considered, the numerical solutions are decomposed, and the decision maker's weights for attributes and corresponding decision making methods have also been proposed. The feasibility and effectiveness of the proposed method are illustrated using numerical examples.

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