MULTI-OBJECTIVE FUZZY FULLY LINEAR PROGRAMMING TRANSPORTATION PROBLEM

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Abstract: Modeling and solving optimization problems is one of the most important issue in real world situation. In recent years, there has been a substantialnamount of research related to the fuzzy applied Linear Programming problem with transportation model(LPTP). Fuzzy methods have been developed virtually in all branches of transportation problem, including multiobjective and multistage LPTP. In this paper, the different case for solving a FLPTP using Fuzzy numbers are proposed.

Keywords: Fully Fuzzy Linear Programming Transportation Problem,LPP, Multi Objective Fuzzy Transportation Problem(MOFTP), Membership Function of A Fuzzy Set.

Introduction: Real World problems invariably involve multiple objectives. Multi objective programme is a part of mathematical programming dealing with transportation problem characterized by multiple and conflicting objective functions that are to be optimized over a feasible set of alternatives of transportation. Multi Objective linear programming transportation problem is an extension of linear programming. This section demonstrates MOLPTP with fuzzy co-efficients occurring in constraints and objective functions and fuzzy constraint cost has been considered. Here fuzzy constraint cost and co-efficients of objective and constraints of objective and constraints functions are characterized by triangular fuzzy numbers. Then the optimal solution of MOFLPTP is solved using min operator chanas (3) proposed a fuzzy programming in multi objective linear programming transportation problem and it was developed by parametric approach. Zimmermann (19) proposed a multi criteria decision making set defined as the intersection of all fuzzy goals and constraints. In this section, we have proposed MOFLPTP with mixed constraint is which right hand side and the constraint matrix are fuzzy numbers and it has been solved by min operator transportation Bablu and Tapan (8) applied transportation application of MOFLPP with a minimization objective whereas in this section the maximization objective with numerical example is proposed.

Case Analysis of Type II: The general form of MOFLPTP can be written as follows

Min Z = $[z^1, z^2, \ldots z^k]$

Subject to $\sum_{izi}^{m} xij \le \tilde{ai}$;

$\sum_{j-i}^{n} x_{ij} \le \tilde{b}_i$, | ≤ I ≤ n

$X_{ij} \ge 0$

Where | ≤ j ≤ m

$Z^k = \sum_{i-l}^{m}\sum_{j=1}^{n} C_{ij}\ x_{ij}$, k = 1, 2 .... K

where the right hand side numbers $\tilde{a}_i, \tilde{b}_j$ are triangular numbers. Let L_k and U_k lower and Upper bound for K^{th} objective.

L_k = aspired level of achievement for the K^{th} objective function.

U_k = Highest acceptable of achievement for the k^{th} objective function.

Solution Technique for MOFLPTP with Fuzzy Resources:

Step 1: The MOFLPTP (2.5) is solved as a single objective LPTP using only one objective at a time and ignoring all others.

Step 2: From the result of step 1 the corresponding value for every objective function at each solution is determined.

Step 3: The upper and lower bounds for the k[th] objective are found from the objective values derived as s step 2.

Step 4: Find $\{x_{ij}; \begin{matrix} i=1,2,....m \\ j=1,1,2...n \end{matrix}$

So as to satisfy $Z_k \le L_k$, K-1,2...k

$$\sum_{i=1}^{m} x_{ij} \le \tilde{ai}, \quad I = 1,2... m$$

$$\sum_{j=1}^{n} x_{ij} \le \tilde{b_j}, \quad j = 1,2 ....n$$

The membership functions for fuzzy transportation and fuzzy constraints of (2.6) are defined as [17]

$$\mu_{G_k}^{(x)} = \begin{cases} 0 \quad if \quad \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}\ x_{ij} \\ \dfrac{U_k - \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}\ x_{ij}}{U_k - L_k} \quad if \quad L_k \le \\ \qquad \sum_{i=1}^{m}\sum_{j=1}^{n} C^k{}_{ij}\ x_{ij} \le U_k \\ 1, \quad if \quad \sum\sum C^k{}_{rj}\ x_{ij} > U_k \end{cases}$$

$$\mu_c^{(a_i)} = \begin{cases} 0 \quad when \quad \sum_{i=1}^{m} x_{ij} > a_i + a^o{}_i \\ \dfrac{\sum_{i=1}^{m} x_{ij} - a_i}{a_i} \quad when \quad a_i \sum_{i=1}^{m} x_{ij} \le a_i + a_i \\ 1 \quad when \quad \sum_{i=1}^{m} x_{ij} < a_i \end{cases}$$

Similarly part $a_i = b_j$ in the above form.

Using Zimmermann [147] maximum operator the problem 2.5 can be rewritten as the following crisp LPTP

Min λ

Subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}\ x_{ij} + \lambda(U_k - L_k) \le U_k, K = 1,2.....k$$

$$\sum_{j=1}^{n} x_{ij} - \lambda b_j \ge a_i,\ 1 \le i \le m$$

$$\sum_{l=1}^{m} x_{ij} - \lambda\ b_j \ge b_j,\ 1 \le j \le n,\ 0 \le \lambda \le j$$

And

$\lambda, x_{ij} \ge 0$

Numerical Example of Type II: As per the case analysis problem in section (2.2.1) the problem of type 2 is analyzed and solving using MOFLPTP. In this problem two objective functions are considered where both are inarcimization type the problem can be formulated as

Min $Z^1 = C^1{}_1 x_1 + C^1{}_2 x_2 + C1^3 x_3 + C^1{}_4 x_4$

Min $Z^2 = C^2{}_1 x_1 + C^2{}_2 x_2 + C^2{}_3 x_3 + C^2{}_4 x_4$

Subject to

$$a\, x_1 + a_2\, x_2 \le \tilde{a}_1$$

$$a\, 3^{x}{}_3 + a_4\, x_4 \le \tilde{a}_2$$

$$a_1 + x_1\, a_3 x_3 \ge \tilde{b}_1$$

$$a\, x_1 + a_2\, x_2 \le \tilde{a}_1$$

$$a_2 + x_2 + a_4 x_4 \ge \tilde{b}_2$$

and $x_1, x_2, x_3 x_4 \ge 0$

The following notations are used in this study.

Z^1 - First objective function (Break even point)

Z^2 - Second objective function (Min-cost)

$C_1^1, C_2^1, C_3^1, C_4^1$ - Cost co-efficient of the first objective function

$C_1^2, C_2^2, C_3^2, C_4^2$ - Cost co-efficient of the second objective function

a_i – constraint matrix $\tilde{a}_1, \tilde{a}_2, \tilde{b}_1, \tilde{b}_2$ - triangular fuzzy numbers. The above problem can be formulated as

Min $Z_1 = 3x_1 + 4x_2 + 5x_3 + 3x_4$

Min $Z_2 = 5x_1 + 3x_2 + 7x_3 + 2x_4$

Sub $x_1 + x_2 \le 80$

$x_3 + x_4 \le 180$

$x_1 + x_3 \ge 100$

$x_2 + x_4 \ge 160$

and $x_1, x_2, x_3, x_4 \ge 0$

where $C_j^k = \begin{pmatrix} 3 & 4 & 5 & 3 \\ 5 & 3 & 7 & 2 \end{pmatrix}$

$$a_i = \begin{pmatrix} 80 \\ 180 \end{pmatrix}, \; b_j = (100 \;\; 160),$$

i, j, k = 1, 2 and

$\tilde{a}_1 = (60,80,100)$

$\tilde{a}_2 = (150,180,210)$

$\tilde{b}_1 = (80,100,120)$

$\tilde{b}_2 = (130,160,190)$

Transportation fuzzy numbers. This problem is divided into six sub problems.

(i) Min $z^{11} = 3x_1 + 4x_2 + 5x_3 + 3x_4$

Subject to

$x_1 + x_2 \leq 100$

$x_3 + x_4 \leq 210$

$x_1 + x_3 \geq 120$

$x_2 + x_4 \geq 190$

and $x_1, x_2, x_3, x_4 \geq 0$

(ii) Min $z^{12} = 3x_1 + 4x_2 + 5x_3 + 3x_4$

Subject to

$x_1 + x_2 \leq 80$

$x_3 + x_4 \leq 180$

$x_1 + x_3 \geq 100$

$x_2 + x_4 \geq 160$

and $x_1, x_2, x_3, x_4 \geq 0$

(iii) Min $z^{13} = 3x1 + 4x_2 + 5x_3 + 3x_4$

Subject to $x_1 + x_2 \leq 60$

$x_3 + x_4 \leq 150$

$x_1 + x_3 \leq 80$

$x_2 + x_4 \leq 130$

and $x_1, x_2, x_3, x_4 \geq 0$

(iv) Min $z^{21} = 5x_1 + 3x_2 + 7x_3 + 2x_4$

Subject to

$x_1 + x_2 \leq 100$

$x_3 + x_4 \leq 210$

$x_1 + x_3 \geq 180$

$x_2 + x_4 \geq 190$

and$x_1, x_2, x_3, x_4 \geq 0$

(v) Min $z^{22} = 5x_1 + 3x_2 + 7x_3 + 2x_4$

$x_1 + x_2 \leq 80$

$x_3 + x_4 \leq 180$

$x_1 + x_3 \geq 100$

$x_2 + x_4 \geq 160$

and$x_1, x_2, x_3, x_4 \geq 0$

(iv) Min $z^{23} = 5x_1 + 3x_2 + 7x_3 + 2x_4$

$x_1 + x_2 \leq 60$

$x_3 + x_4 \leq 150$

$x_1 + x_3 \geq 80$

$x_2 + x_4 \geq 130$

and$x_1, x_2, x_3, x_4 \geq 0$

Optimal solutions of the six sub problems are illustrated below respectively.

$x^{11} = (x_{11}^{1}, x_2^{11}, x_3^{11}, x_4^{11}) = (100, 0, 20, 190)$

$z^{11}(x^{11}) = 970$

$x^{12} = (x_1^{12}, x_2^{12}, x_3^{12}, x_4^{12}) = (80, 0, 20, 160)$,

$z^{12}(x^{12}) = 820$

$x^{13} = (x_1^{13}, x_2^{13}, x_3^{13}, x_4^{13}) = (60, 0, 20, 130)$

$z^{13}(x^{13}) = 670$

$x^{21} = (x_1^{21}, x_2^{21}, x_3^{21}, x_4^{21}) = (100, 0, 20, 190)$

$z^{21}(x^{21}) = 1020$

$x^{22} = (x_1^{22}, x_2^{22}, x_3^{22}, x_4^{22}) = (80, 0, 20, 160)$

$x^{22} = (x^{22}) = 860$

$x^{23} = (x_1^{23}, x_2^{23}, x_3^{23}, x_4^{23}) = (60, 0, 20, 130)$

$z^{23}(x^{23}) = 700$

Let $L_1 = \min \{z^1(x^{11}), z^1(x^{12}), z^1(x^{13})\}$

= min (970, 820, 670)
= 670
Let U_1 = max $\{z^{11}(x^{11}), z^{11}(x^{12}), z^{13}(x^{13})\}$
= max (970, 820, 670)
= 970
Let L_2 = min $\{z^{21}(x^{21}), z^{22}(x^{22}), z^{23}(x^{23})\}$
= min {1020, 860, 700}
= 700
Let U_2 = max $\{z^{21}(x^{21}), z^{22}(x^{22}), z^{23}(x^{23})\}$
= max (1020, 860, 700)
= 1020
Find $\{x_j, j=1,2\}$
So as to satisfy,

$$3x_1 + 4x_2 + 5x_3 + 3x_4 \le 670$$
$$5x_1 + 3x_2 + 7x_3 + 2x_4 \le 700$$
$$x_1 + x_2 \le 80$$
$$x_3 + x_4 \le 180$$
$$x_1 + x_3 \ge 100$$
$$x_2 + x_4 \ge 160$$

And$x_1, x_2, x_3, x_4 \ge 0$
Here membership functions for fuzzy transportation and fuzzy constraints of the problem (2.13) are defined as

$$\mu_{\tilde{(}a_i)}(3x_1 + 4x_2 + 5x_3 + 3x_4)$$

$$= \begin{cases} 0, \text{ if } \ 3x_4 + 4x_2 + 5x_3 + 3x_4 \le 670 \\ \dfrac{970 - (3x_4 + 4x_2 + 5x_3 + 3x_4)}{300} \ \text{ if} \\ 1 \ 970 \le (3x_1 + 4x_2 + 5x_3 + 3x_4) \le 670 \\ \text{if } \ 3x_1 + 4x_2 + 5x_3 = 3x_4 > 970 \end{cases}$$

$$\mu_{\tilde{a}_2}(5x_1 + 3x_2 + 7x_3 + 2x_4)$$

$$= \begin{cases} 0, \text{ if } \ 5x_4 + 3x_2 + 7x_3 + 2x_4 \le 700 \\ \dfrac{1020 - (5x_4 + 3x_2 + 7x_3 + 2x_4)}{320} \ \text{ if} \\ 700 \le (5x_1 + 3x_2 + 7x_3 + 2x_4) \le 1020 \\ 1, \text{if } (5x_1 + 3x_2 + 7x_3 + 2x_4) \ge 1020 \end{cases}$$

$$\mu_{C1}(80) = \begin{cases} 0, \text{ if } \ x_1 + x_2 > 100 \\ \dfrac{(x_1 + x_2) - 80}{20} \ \text{ if} \\ 80 \le (x_1 + x_2) \le 100 \\ 1 \text{ if } \ (x_1 + x_2) < 80 \end{cases}$$

$$\mu_{C2}(180) = \begin{cases} 0, \ if \ \ (x_3 + x_4) > 210 \\ \dfrac{(x_3 + x_4) - 180}{30}, \ \ if \\ 180 \le (x_3 + x_4) \le 210 \\ 1 \ if \ \ (x_3 + x_4) < 180 \end{cases}$$

$$\mu_{C3}(100) = \begin{cases} 0, \ if \ \ (x_1 + x_3) > 120 \\ \dfrac{(x_1 + x_3) - 100}{20} \ \ if \\ 100 \le (x_1 + x_3) \le 120 \\ 1 \ if \ \ x_1 + x_3 < 100 \end{cases}$$

$$\mu_{C4}(160) = \begin{cases} 0, \ if \ \ x_2 + x_4 > 190 \\ \dfrac{(x_2 + x_4) - 160}{30} \ \ if \\ 160 \le x_2 + x_4) \le 190 \\ 1 \ if \ \ x_2 + x_4 < 160 \end{cases}$$

Using max-min operator as per Zimmermann (147) Crisp LPTP for equation (2.5) is formulated as follows.
Min λ
Subject to

$$3x_1 + 4x_2 + 5x_3 + 3x_4 + 300\lambda \le 970$$
$$5x_1 + 3x_2 + 7x_3 + 2x_4 + 320\lambda \le 1020$$
$$x_1 + x_2 - 20\lambda \ge 80$$
$$x_3 + x_4 - 30\lambda \ge 180$$
$$x_1 + x_3 - 20\lambda \ge 100$$
$$x_2 + x_4 - 30\lambda \ge 160$$
$$0 \le \lambda \le 1 \ , \quad x_1, x_2, x_3, x_4 \ge 0$$

$$x_1 = 80 \ , x_3 = 6.667, \qquad x_4 = 173.333, z = 793.333$$

So optimal solution of MOFLPTP (2.13) are

$$x_1 = 0, x_2 = 80, x_3 = 33.33,$$
$$z_2 = 766.667$$
$$z_2 = 766.667$$
$$x_2 = 80$$
$$x_3 = 33.33$$
$$x_4 = 146.667$$

$x_2 = 80$,$x_3 = 6.667$,$x_4 = 173.333, z_2 = 793.333$
with aspiration level λ=0. The profit ascertained for the second objective function with the desired aspiration levels of maximum profitability of the organization has been forecast as minimum cost of z=Rs.766.667 with aspiration level of zero.

Conclusion: This particular industry has got lot of uncertainties due to the interferences of accidents, fire, flood and other natural irregular conditions that inhibit the usage and consumption of the particular product. Hence careful planning has to be done prior to produce the particular product without any further loss occurring due to the prevailing conditions in the economy. This will enable the company to formulate strategies for achieving the desired levels of cost and profit at different levels of production for sales as well as cost at different levels of pricing. Moreover we have taken for two products only and the optimal combination results are also given for the desired aspiration levels. This study is such a versatile are that more than two products and also with different combinations can be worked and the results can be obtained with forecast for the given condition.

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