TOTAL WEIGHT DOMINATING VERTEX SET ON S - VALUED GRAPHS

S.Jeyalakshmi

Assistant Professor/ Devanga Arts College/Aruppukottai /Tamilnadu

M.Chandramouleeswaran

Associate Professor/ Head /Saiva Bhanu Kshatriya College/Aruppukottai /Tamilnadu

Abstract: In [1] and [3] the authors studied the notion of domination in Graphs. In [6] we have studied the notion of S– valued graphs, where S is a semiring. In [5], we have studied the notion of vertex domination in S-valued Graphs. In this paper, we introduce the notion of Total weight dominating vertex set on S– valued graphs and prove simple properties.

Keywords: Semirings, Graphs, S–Valued Graphs.

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Introduction: In [1], Berge introduced the notion of domination in graphs. In [3], Haynes and others continued the study of the notion of domination in Graphs. In [2], the authors studied the notion of total domination in Graphs. Motivated by the concept of S-valued graphs in [4] by Golan, in [6] we have studied the notion of S– valued graphs in detail, where S is a semiring. In [5], we have studied the notion of vertex domination in S-valued Graphs. In this paper, we introduce the notion of Total weight dominating vertex set on S–valued graphs and prove simple properties.

Preliminaries: In this section, we recall some basic definitions that are needed for our work.

Definition 2.1: [4] A semiring $(S,+,\cdot)$ is an algebraic system with a non-empty set S together with two binary operations + and $\cdot$ such that

1. $(S, +, 0)$ is a monoid.
2. $(S, \cdot)$ is a semigroup.
3. 3. for all $a,b,c \in S$, $a \cdot (b+c) = a \cdot b + a \cdot c$ and $(a + b) \cdot c = a \cdot c + b \cdot c$.
4. $0 \cdot x = x \cdot 0 = 0, \forall x \in S$.

Definition 2.2: Let $(S, +, \cdot)$ be a semiring. A canonical Pre-order $\precsim$ in S is defined as follows: for $a, b \in S$, $a \precsim b$ if and only if, there exist a $c \in S$ such that $a + c = b$.

Definition 2.3: [6] Let $G = (V,E)$ be a given graph with $V,E \neq \emptyset$. For any semiring $(S, +, \cdot)$ a semiring-valued graph (or a S – valued graph), G^s, is defined to be the graph $G^S = (V, E, \sigma, \psi)$ where $\sigma : V \to S$ and $\psi : E \to S$ is defined to be $\psi(x,y) = \begin{cases} \min\{\sigma(x),\sigma(y)\} & \text{if } \sigma(x) \precsim \sigma(y) \text{ or } \sigma(y) \precsim \sigma(x) \\ 0 & \text{otherwise} \end{cases}$

for every unordered pair (x,y) of $E \subset V \times V$. We call σ, a S– vertex set and ψ, a S edge set of G^S.

Definition 2.6: [5] A vertex υ in G^S is said to be a weight dominating vertex if

$\sigma(u) \precsim \sigma(v), \forall u \in V$.

Definition 2.6: [5] A subset $D \subseteq V$ is said to be a weight dominating vertex set of G^S if for each $v \in D$, $\sigma(u) \precsim \sigma(v), \forall u \in N_S(v)$.

Definition 2.7: [5] If D is a weight dominating vertex set of G^S, then the scalar cardinality of D, denoted by $|D|_S$, is defined by

$|D|_S = \sum_{v \in D} \sigma(v)$.

Definition 2.8: [5] The cardinality of the minimal weight dominating vertex set D ⊆ V is called the weight dominating vertex number of G^S . It is denoted by $\gamma_V^S(G^S)$. That is ,

$\gamma_V^S(G^S)$ = ($|D|_S$, |D|).

Total Weight Dominating Vertex Set on S-Valued Graphs: In this section, we introduce the notion of total weight dominating vertex set on S-valued Graphs and obtain some simple results. First we fix a notation.

Notation: Consider the S-valued graph G^S = (V, E, σ, ψ). Denote $V_S = \{(\sigma(v_i), v_i) / \ i = 1,2,3, \dots n$ and $\sigma(v_i) \in$ S}.

Definition 3.1: Consider the S-valued graph G^S = (V, E, σ, ψ). Let $T_D^S \subseteq$ V. The open neighbourhood of T_D^S, denoted by $N_S(T_D^S)$, is defined by $N_S(T_D^S) = \bigcup_{v \in T_D^S} N_S(v)$.

Definition 3.2: Consider the S-valued graph G^S = (V, E, σ, ψ). Let $T_D^S \subseteq$ V. The closed neighbourhood of T_D^S, denoted by $N_S[T_D^S]$, is defined by $N_S[T_D^S] = \bigcup_{v \in T_D^S} N_S[v]$.

Definition 3.3: Consider the S-valued graph G^S = (V, E, σ, ψ). Let $T_D^S \subseteq$ V. For any v∈ T_D^S The private neighbour set of v with respect to T_D^S, denoted by $PN_S[v, T_D^S]$, is defined by $PN_S[v, T_D^S]$= $N_S[v]$ - $N_S[T_D^S - \{v\}]$.

Definition 3.4: If $N_S(T_D^S) = V_S$, then T_D^S is called a total weight dominating vertex set of G^S.

Example 3.5: Let (S = {o,a,b,c}, +, .) be a semiring with the following cayley tables :

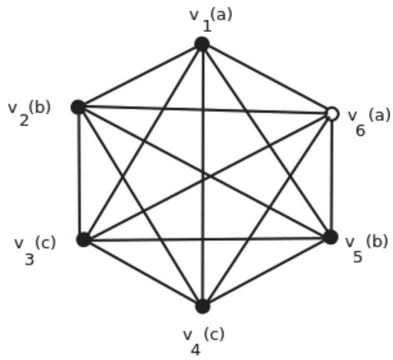
+	o	a	b	c
o	o	a	b	c
a	a	a	a	a
b	b	a	b	b
c	c	a	b	c

.	o	a	b	c
o	o	o	o	o
a	o	a	a	a
b	o	b	b	b
c	o	b	b	b

Let ≾ be a canonical pre-order in S, given by

o ≾ o, o ≾ a,o ≾ b, o ≾ c, a ≾ a, b ≾ b,b ≾ a, c ≾ c, c ≾ a, c ≾ b.

Consider the S-valued graph G^S:



Consider $T_D^S = \{v_1(a), v_2(b), v_3(c), v_4(c), v_5(b)\}$

Hence $N_S[T_D^S] = \{v_1(a), v_2(b), v_3(c), v_4(c), v_5(b), v_6(a)\}$

= V_S

∴ $T_D^S = \{v_1(a), v_2(b), v_3(c), v_4(c), v_5(b)\}$ is a total weight dominating vertex set of the given G^S.

Definition 3.6: Consider the S-valued graph G^S = (V, E, σ, ψ). Let $T_D^S \subseteq$ V is said to be a minimal total weight dominating vertex set if T_D^S is a total weight dominating vertex set and no proper subset of T_D^S is a total weight dominating vertex set.

Example 3.7: From the above example $T_D^S = \{v_1(a), v_2(b), v_3(c), v_4(c), v_5(b)\}$ and $T_D^S = \{v_2(b), v_3(c), v_4(c), v_5(b), v_6(a)\}$ and $T_D^S = \{v_1(a), v_2(b), v_3(c), v_5(b), v_6(a)\}$ are total weight dominating vertex sets of given G^S.

Definition 3.8: Consider the S-valued graph G^S = (V, E, σ, ψ). The cardinality of the minimal weight dominating vertex set is called the total weight dominating vertex number of G^S. That is, $\gamma_{T_D^S}^S$(GS) = $(|T_D^S|_S, |T_D^S|)$ where T_D^S is a minimal total weight dominating vertex set of G^S.
Remark: Minimal weight dominating vertex set in a S-valued graph need not be, in general, unique.

Definition 3.9: If G^S = (V, E, σ, ψ) is a S-valued graph. Let $T_D^S \subseteq V$ is said to be a independent total weight dominating vertex set if
i) T_D^S is a total weight dominating vertex set.
ii) No two vertices of T_D^S are adjacent in a total weight dominating vertex set.
And is denoted $\alpha_V^S(G^S)$, is defined as
$\alpha_{T_D^S}^S(G^S) = (|T_D^S|_S, |T_D^S|)$, where T_D^S is a minimal independent total weight dominating vertex set.
Remark: By the definition of the total weight dominating vertex set and the above remark we observe that an minimal independent weight dominating vertex set is also a total weight dominating vertex set of G^S.

Definition 3.10: An upper total weight dominating vertex number of G^S = (V, E, σ, ψ), denoted by $\Gamma_{T_D^S}^S(G^S)$ = max{$(|T_D^S|_S, |T_D^S|)$, where T_D^S is a minimal total weight dominating vertex set of G^S.

Theorem 3.11: Consider the S-valued graph G^S = (V, E, σ, ψ). A star on n vertices, where n≥1 then $\gamma_{T_D^S}^S(G^S)$ = (σ(v),1), where v is the pole of the star graph G^S.
Proof: Let G^S be a star. Then the pole $\{v_1, σ(v_1)\}$ with maximum σ value in S dominates all other vertices showing that $T_D^S = \{v_1, σ(v_1)\}$.
Hence $N_S(T_D^S) \subseteq (V\text{-}D)_S$ and $N_S[T_D^S] = V_S$.
Therefore $\{v_1, σ(v_1)\}$ is a total weight dominating vertex set in G^S.
$\gamma_{T_D^S}^S(G^S)$ = (σ(v), 1), where v is the pole of the star graph G^S.

Theorem 3.12: Consider the S-valued graph G^S = (V, E, σ, ψ). For any cycle on n vertices, where n≥1 the total weight dominating vertex number $\gamma_{T_D^S}^S(G^S) = \begin{cases} (\sum \sigma(v_i),\ \frac{n}{2})\ \textit{where n is even.} \\ (\sum \sigma(v_i),\ \frac{n+1}{2})\ \textit{where n is odd.} \end{cases}$
Proof: Let G^S= (V, E, σ, ψ) be a given S-valued graph. Consider the cycle on n vertices, C_n^S.
Let $\{v_1, σ(v_1)\} \in V_S$ be any vertex in the cycle C_n^S with $N_S(v_1) = \{(v_2, σ(v_2)), (v_n, σ(v_n))\}$.
This implies that $\{(v_1, σ(v_1))\}$ dominates both $(v_2, σ(v_2))$, *and* $(v_n, σ(v_n))$.
Take $\{v_3, σ(v_3)\} \in V_S$ that lies on the cycle C_n^S then $\{v_3, σ(v_3)\}$ dominates $(v_2, σ(v_2))$ and $(v_4, σ(v_4))$.
Continuing in this way, we choose vertices not in $N_S(v_i)$ for any v_i chosen previously. Then
$N_S(v_{n-1}) = \{(v_{n-2}, σ(v_{n-2})), (v_n, σ(v_n))\}$.
When **n is even,** there will be no vertices remaining and hence we conclude that
$T_D^S = \{(v_1, σ(v_1)), (v_3, σ(v_3)), \ldots (v_{n-1}, σ(v_{n-1}))\}$ is a total weight dominating vertex set showing that $N_S[T_D^S] = \{(v_1, σ(v_1)), (v_2, σ(v_2)), \ldots, (v_n, σ(v_n))\}$. Therefore
$\gamma_{T_D^S}^S(G^S) = \sum \sigma(v_i),\ \frac{n}{2})$ *where n is even.*
When **n is odd, t**he edge ending with $(v_{n-1}, σ(v_{n-1}))$ in C_n^S, will not be covered. Hence we need to add one more vertex which will dominate both v_{n-1} and v_n. Then the cycle will have (n+1) vertices. Hence
$\gamma_{T_D^S}^S(G^S) = (\sum \sigma(v_i),\ \frac{n+1}{2})$ *where n is odd.*

References:

1. C. Berge, " Theory of Graphs and its Applications", Methuen, London, (1962).
2. V. Kokilavani, M. Vivek Prabu, A Study On Semi #Generalized -Closed Sets In Bi-ˇECH Closure Spaces; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 143-145

3. E. J. Cockayne, R. M. Dawes, S. T. Hedetniemi, Total Domination in Graphs. Networks. John Wiley and Sons, Vol. 10 (1980) 211-219.
4. W.Haynes, S.T. Heditniemi and Peter J. Slater " Fundamentals of Domination in Graphs", Newyork 1998.
5. V.Mathan Kumar, S.Shanmugasundaram, A Linear Programming Approach For Optimum Duration And Associated Cost Using Lindo; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 149-153
6. Jonathan Golan, "Semirings and Their Applications", Kluwer Academic Publishers,London.
7. S.Jeyalakshmi and M.Chandramouleeswaran, "Vertex domination on S-valued graphs", IOSR journal of Mathematics, vol. 12(III), 08 – 12, 2016.
8. M.Rajkumar, S.Jeyalakshmi and M.Chandramouleeswaran,"Semiring-valued graphs", International Journal of Math.Sci.and Engg.Appls.,vol.9(III),141-152, 2015.
9. M. Jerlin Seles, Dr. U. Mary, R. Sahana, A Study On Mcpherson Number Of Jn (1) ; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 146-148
10. G. K. Jagatheswari, Murugesan R, Gajanan V Honnavar, Compression Of Two Dimensional Nonlinear Functions Using Two Dimensional Wavelet Transforms; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 161-164
11. M.Bharathi, Dr. V. Sangeetha, Performance Study Of Combinatorial Testing To Improve Software System Quality; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 154-160
12. Dr. D.Vijayalakshmi, N.Nithya Devi, A Study On Achromatic Number Of The Transformation Graph G++-; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 165-166

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