# BOUNDS OF WEIGHT DOMINATING EDGE NUMBER IN S VALUED GRAPHS

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**Abstract:** In this paper we define and study some values of edge domination number for certain special types of graphs and obtain some results on the bounds.

**Keywords:** S-Valued Graphs, Domination in S-Valued Graphs, Edge Domination on S-Valued Graphs , Edge Domination Number of S-Valued Graphs.

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**Introduction:** The concept of edge domination was introduced by Mitchell and Hedetniemi in [7]. Further it was studied by Arumugam and Velammal [1] and B.Basavanagoud and Sunil kumar M. Hosamani [2]. Motivated by the concept of S-valued graph by Golan in [3], Chandramouleeswaran and others introduced and studied the notion of Semiring valued graphs called as S-valued graphs in [8]. In [4] the authors, studied the vertex domination on S-valued graphs. Motivated by this we discuss the notion of edge domination on S-valued graphs in [5]. In this paper, we define and study some values of edge domination number for certain special types of graphs and obtain some results on the bounds.

**Preliminaries:** In this section, we recall some basic definitions that are needed for our work.

**Definition 2.1:** [3] A semiring (S, +, .) is an algebraic system with a non-empty set S together with two binary operations + and . such that

1. (S, +, 0) is a monoid.
2. (S, . ) is a semigroup.
3. For all a, b, c $\in$ S , a . (b+c) = a . b + a . c and (a+b) . c = a . c + b . c
4. 0. x = x . 0 = 0 $\forall$ x $\in$ S.

**Definition 2.2:** [3] Let (S, +, .) be a semiring. $\preceq$ is said to be a Canonical Pre-order if for a, b $\in$ S, a $\preceq$ b if and only if there exists an element c $\in$ S such that a + c = b.

**Definition 2.3:** [8] Let G = (V, E $\subset$ V $\times$V ) be a given graph with V, E$\neq \phi$. For any semiring (S, +, .), a semiring-valued graph (or a S - valued graph), $G^S$ is defined to be the graph $G^S$ = (V, E,$\sigma,\psi$) where $\sigma : V \to S$ and $\psi : E \to S$ is defined to be $\psi(x,y) = \begin{cases} \min\{\sigma(x),\sigma(y)\}, if \sigma(x) \preceq \sigma(y) \ or \ \sigma(y) \preceq \sigma(x) \\ 0, otherwise \end{cases}$ for every unordered pair (x, y) of E $\subset$ V $\times$V. We call $\sigma$, a S - vertex set and $\psi$, a S - edge set of S - valued graph $G^S$. Henceforth, we call a S – valued graph simply as a S - graph.

**Definition 2.4:** [6] Let $G^S = (V, E, \sigma, \psi)$ be a S-valued graph. Let $e \in E$. The open neighbourhood of 'e' denoted by $N_S(e)$ is defined to be the set

$$N_S(e) = \{(e_i, \psi(e_i)) / e \ and \ e_i \in E \ are \, adjacent\}$$

The closed neighbourhood of 'e', denoted by $N_S[e]$ is defined to be the set

$N_S[e] = N_S(e) \cup (e, \psi(e))$

**Definition 2.5:** [4] Consider the S- valued graph $G^S = (V, E, \sigma, \psi)$. Let $D \subseteq E$. An edge $e \in D$ of $G^S$ is said to be an S- isolate edge if $N_S(e) \subseteq D'$ where $D' = E - D$.

**Definition 2.6:** [6] Let $G^S = (V, E, \sigma, \psi)$ be a S- valued graph. The degree of the edge 'e' is defined as $\deg_S(e) = \left( \sum_{e_i \in N_S(e)} \psi(e_i), m \right)$, where m is the number of edges adjacent to e.

**Definition 2.7:** [5] Consider the S- valued graph $G^S = (V, E, \sigma, \psi)$. An edge 'e' in $G^S$ is said to be a weight dominating edge if $\psi(e_i) \preceq \psi(e), \forall e_i \in N_S[e]$.

**Definition 2.8:** [5] Consider the S- valued graph $G^S = (V, E, \sigma, \psi)$. A subset $D \subseteq E$ is said to be a weight dominating edge set if for each $e \in D$, $\psi(e_i) \preceq \psi(e), \forall e_i \in N_S[e]$.

**Definition 2.9:** [5] Consider the S- valued graph $G^S = (V, E, \sigma, \psi)$. If D is a weight dominating edge set of $G^S$, then the scalar cardinality of D is defined by $|D|_S = \sum_{e \in D} \psi(e)$.

**Definition 2.10:** [5] The edge domination number of $G^S$ denoted by $\gamma_E^S(G^S)$, is defined by $\gamma_E^S(G^S) = (|D|_S, |D|)$, where D is the minimal weight dominating edge set.

**Theorem 2.11:** [5] A weight dominating edge set D of a graph $G^S$ is a minimal weight dominating edge set of G iff every edge $e \in D$ satisfies atleast one of the following properties:

(i) there exist an edge $f \in E - D$, such that $N_S(f) \cap (D \times S) = \{(e, \psi(e))\}$

(ii) e is adjacent to no edge of D.

**Definition 2.12:** [8] Let $G^S = (V, E, \sigma, \psi)$ be the S-graph corresponding to a given underlying graph G = ( V , E ). An S-graph $H^S = (P, L, \tau, \gamma)$ is called a S-subgraph of $G^S$ if H = (P, L) is a subgraph of G with $P \subset V$, $L \subset E$, $\tau \subset \sigma$ and $\gamma \subset \psi$.

Here, $\tau \subset \sigma \Rightarrow \tau(x) \preceq \sigma(x), x \in P$ and $\gamma \subset \psi \Rightarrow \gamma(x, y) \preceq \psi(x, y), (x, y) \in L \subset P \times P$.

**Bounds of Edge Domination Number:** In this section, we obtain some results on weight dominating edge number in $G^S$.

**Theorem 3.1:** Let $G^S = (V, E, \sigma, \psi)$ be a given S- valued graph with n vertices.

Then $(0,1) \preceq \gamma_E^S(G^S) \preceq \left( \sum_{e \in E} \psi(e), \frac{n(n-1)}{2} \right)$

**Proof:** Since S is a semiring, it has the zero element and $0 \preceq a$ for every $a \in S$.

Let $D \subseteq E$ be a minimal weight dominating edge set of $G^S$, then D should have atleast one edge which will be assigned atleast the weight zero. Hence $(0,1) \preceq \gamma_E^S(G^S)$.

The scalar cardinality of D is $|D|_S = \sum_{e \in D} \psi(e)$.

Since $D \subseteq E$, $|D|_S = \sum_{e \in D} \psi(e) \preceq \sum_{e \in E} \psi(e) = |E|_S$.

Since $G^S$ is a graph with n vertices the number of edges in $G^S$ is atmost $\frac{n(n-1)}{2}$.

Hence $|D| \preceq \frac{n(n-1)}{2}$.

Thus, $\gamma_E{}^S(G^S) = (|D|_S, |D|) \preceq \left(\sum_{e\in E}\psi(e), \frac{n(n-1)}{2}\right)$.

Hence $(0,1) \preceq \gamma_E{}^S(G^S) \preceq \left(\sum_{e\in E}\psi(e), \frac{n(n-1)}{2}\right)$.

In the following, we prove the analogous of Ore's theorem for S-valued graphs.
**Theorem 3.2:** Let $G^S$ be a given edge regular S- valued graph with |V| = m and |E| = n.

$$\text{Then } \gamma_E{}^S(G^S) \preceq \begin{cases} \left(\sum_{e\in D}\psi(e), \frac{n}{2}\right) & , \text{ if } |D| \le |E-D| \\ \left(\sum_{e\in E-D}\psi(e), \frac{n}{2}\right) & , \text{ if } |E-D| \le |D| \end{cases}$$

**Proof:** The second condition of theorem 2.11 gives the analogous of Ore's theorem for the graph $G^S$.
Let $G^S$ be a given edge regular S- valued graph with |V| = m and |E| = n. If D is a minimal weight dominating edge set, then E-D is also a weight dominating edge set. Thus |D| or |E-D| is at most $\frac{n}{2}$.

**When** $|D| \le \frac{n}{2}$, $\sum_{e\in D}\psi(e) \preceq \sum_{e\in E}\psi(e) \Rightarrow \gamma_E{}^S(G^S) \preceq \left(\sum_{e\in D}\psi(e), |E-D|\right)$.

When $|E-D| \le \frac{n}{2}$, $\sum_{e\in E-D}\psi(e) \preceq \sum_{e\in E}\psi(e) \Rightarrow \gamma_E{}^S(G^S) \preceq \left(\sum_{e\in E-D}\psi(e), |D|\right)$.

Thus we have proved the analogous of Ore's theorem for the S-valued graph $G^S$.
**Theorem3.3:** Let $G^S$ be a given S- valued graph and D be a weight dominating edge set.
Let |E| = n. Then the following are true.

1. If $G^S$ has no S-isolated edges $\gamma_E{}^S(G^S) + \gamma_E{}^S(\overline{G^S}) \preceq \left(\sum_{e\in E}\psi(e), n\right)$
2. If $G^S$ has an S-isolated edge $\gamma_E{}^S(G^S) + \gamma_E{}^S(\overline{G^S}) \preceq \left(\sum_{e\in E}\psi(e), n+1\right)$

**Proof:** If $G^S$ has no S-isolated edges then $\gamma_E{}^S(G^S) \preceq \begin{cases} \left(\sum_{e\in D}\psi(e), \frac{n}{2}\right) & , \text{ if } |D| \le |E-D| \\ \left(\sum_{e\in E-D}\psi(e), \frac{n}{2}\right) & , \text{ if } |E-D| \le |D| \end{cases}$

If $\overline{G^S}$ has no S-isolated edges then $\gamma_E{}^S(\overline{G^S}) \preceq \begin{cases} \left(\sum_{e\in D}\psi(e), \frac{n}{2}\right) & , \text{ if } |D| \le |E-D| \\ \left(\sum_{e\in E-D}\psi(e), \frac{n}{2}\right) & , \text{ if } |E-D| \le |D| \end{cases}$

$\therefore \gamma_E{}^S(G^S) + \gamma_E{}^S(\overline{G^S}) \preceq \left(\sum_{e\in E}\psi(e), n\right)$ thus proving 1.

If $G^S$ has an S-isolated edge and $\overline{G^S}$ has no S-isolated edges then

$$\gamma_E{}^S\left(G^S\right) \preceq \left(\sum_{e\in E}\psi(e), n\right) \text{ and } \gamma_E{}^S\left(\overline{G^S}\right) = (0,1).$$

Then we have $\gamma_E{}^S\left(G^S\right)+\gamma_E{}^S\left(\overline{G^S}\right) \preceq \left(\sum_{e\in E}\psi(e), n+1\right)$.

If $\overline{G^S}$ has an S-isolated edge and $G^S$ has no S-isolated edges then

$$\gamma_E{}^S\left(\overline{G^S}\right) \preceq \left(\sum_{e\in E}\psi(e), n\right) \text{ and } \gamma_E{}^S\left(G^S\right) = (0,1).$$

$\Rightarrow \gamma_E{}^S\left(G^S\right)+\gamma_E{}^S\left(\overline{G^S}\right) \preceq \left(\sum_{e\in E}\psi(e), n+1\right)$, thus proving 2.

**Theorem3.4:** For the S-valued graph $K_p{}^S$, $\gamma_E{}^S\left(K_p{}^S\right) \preceq (\psi(e),1)$.

**Proof:** Consider the S-valued graph $K_p{}^S = (V, E, \sigma, \psi)$

Let $F \subseteq E$ be a weight dominating edge set.

Since $G^S = K_p{}^S$, is a complete graph on 'p' vertices and degree of 'e' are all equal for all $e \in E$.

Let $e \in F$ be such that $\psi(f) \preceq \psi(e)$ for all $f \in E - \{e\}$.

Then this $e \in F$ will dominate every other edges in $K_p{}^S$.

Therefore $F = \{e\}. \Rightarrow |F|_S = (\psi(e))$. Thus $\gamma_E{}^S\left(K_p{}^S\right) = \left(|F|_S, |F|\right) \preceq (\psi(e),1)$.

**Theorem3.5:** For the S-valued graph $C_p{}^S, \gamma_E{}^S\left(C_p{}^S\right) \preceq \left(\sum_{e\in F}\psi(e), p-2\right)$

**Proof:** Let $G^S = C_p{}^S$ be a S- valued cycle on p vertices. Therefore for any edge e in $C_p{}^S$, deg(e) = 2.

Let $F \subseteq E$ be a weight dominating edge set in $G^S$. For each $e \in F$, $\psi(f) \preceq \psi(e)$ for all $f \in N_S[e]$.

This shows that the set F will dominate every other edges in E-F.

Therefore $|F|_S = \sum_{e\in F}\psi(e)$ and $|F| = p - \Delta(G) = p-2$.

Hence $\gamma_E{}^S\left(C_p{}^S\right) = \left(|F|_S, |F|\right) \preceq \left(\sum_{e\in F}\psi(e), p-2\right)$.

**Theorem3.6:** Let $G^S$ be any S-valued graph and $H^S$ be a spanning subgraph of $G^S$ then $\gamma_E{}^S\left(G^S\right) \preceq \gamma_E{}^S\left(H^S\right)$.

**Proof:** Let $G^S = (V, E, \sigma, \psi)$ be an S- valued graph and $H^S = (P, L, \tau, \gamma)$ be its spanning S- subgraph. That is, $H^S$ is called a spanning S- subgraph of $G^S$ if P = V, $L \subset E$, $\tau(v) = \sigma(v)$, for every $v \in P$ and $\gamma(e) = \psi(e)$ for every $e \in L$.

Let F be a minimal weight dominating edge set of $G^S$, such that $\gamma_E{}^S\left(G^S\right) = \left(|F|_S, |F|\right)$.

Since $H^S$ is a spanning S- subgraph of $G^S$, $F \subseteq L = E$ and hence, $|F|_S = \sum_{e\in F}\psi(e) \preceq \sum_{e\in E}\psi(e) = |L|_S$.

Also, |F| < |L| = |E|. Hence $\gamma_E{}^S\left(G^S\right) = \left(|F|_S, |F|\right) \preceq \left(|L|_S, |L|\right) = \gamma_E{}^S\left(H^S\right)$.

**References:**


1. S. Arumugam and S. Velammal: Edge Domination in Graphs, Taiwanese Journal of Mathematics,Vol 2, No 2, pp 173-179, June 1998.

2. B.Basavanagoud and Sunilkumar M.Hosamani: Edge Dominating Graph of a Graph, Tamkang Journal of Mathematics, Volume 43, Number 4, pp 603-608.
3. B. Mayilvaganan, S.V.Gomathy, A Direct Cole-Hopf Transformation Of A Generalized Burgers Equation With Variable Viscosity To Second Order Linear Ode; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 167-170
4. T.Madhumathi, Dr. F. Nirmala Irudayam, On Ωgb+ And Ʊgb+ Sets In Simple Extension Ideal Topological Spaces; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 187-190
5. Jonathan Golan: Semirings and Their Applications, Kluwer Academic Publishers, London.
6. Dr. A. Sahaya Sudha, M. Revathy , Analysing An Intuitionistic Fuzzy Linear Programming Problem Using (A, B)-Cuts; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 176-178
7. Kiruthiga Deepa.S and Chandramouleeswaran.M: Edge Vertex Mixed Domination on S- valued graphs, Asian Eureopean journal of Mathematics(Submitted)
8. Kiruthiga Deepa.S, Mangala Lavanya.S and Chandramouleeswaran.M: Edge Domination on S- valued graph, Journal of mathematical and computational Science.Journal of Mathematics and Computational Science,Volume 7, No 1(2017)59-67.
9. Vaiyomathi.K, Dr.F.Nirmala Irudayam, A New Form Of B-Open Sets In Infra Topological Space; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 191-194
10. R. Jayasree, A. Kulandai Therese, U. Mary, Johan Kok, Harmonic Polynomial And Chromatic Harmonic Polynomial And Indices For Linear Jaco Graph; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 171-175
11. Mangala Lavanya.S and Chandramouleeswaran.M: Degree Regularity on Edges of S-Valued Graph,IOSR Journal of Mathematics (IOSR-JM) Volume 12, Issue 5 Ver. VII (Sep. -Oct.2016), PP 22-27.
12. R.Rajalakshmi, Dr.K.Julia Rose Mary, Operating Characteristics Of M/M (Alb)/1/Mwv/Bd Queueing Systems; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 179-182
13. S. Mitchell and S. T. Hedetniemi: Edge domination in trees, Congr. Numer.19 (1977), pp 489-509.
14. Rajkumar.M., Jeyalakshmi.S and Chandramouleeswaran.M: Semiring-valued Graphs , International Journal of Math. Sci. and Engg. Appls. , Vol. 9 (III), 2015, 141 - 152.
15. S.Pavulin Rani, Dr.M. Trinita Pricilla, A Note On Various Types Of Continuity Via Jμþ – Closed Sets And Jμ*Þ -Closed Sets; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 183-186


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