NUMERICAL SOLUTION OF INTUITIONISTIC FUZZY DIFFERENTIAL EQUATION BY ADAMS'S PREDICTOR- CORRECTOR METHOD UNDER GENERALISED DIFFERENTIABLITY

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Abstract: Now days, many real life problems are identified with Fuzzy set theory. The Fuzzy set theory is a useful tool to describe the situation in which data are imprecise or vague or uncertain. This set theory is completely described by its membership function. A membership function of a classical Fuzzy set assigns to each element of the universe of discourse a number from the interval [0,1] to indicate the degree of belongingness to the set under consideration. The degree of non belongingness is just automatically the compliment to "1" of the membership degree. But, a human being who expresses the degree of membership of given element in a Fuzzy set very often does not express corresponding degree of non-membership as the complement to "1". This reflects a well known psychological fact that the linguistic negation not always identifies with logical negation. There may be some hesitation about the belongingness and non belongingness. This missing data or hesitation is accomplished by a set known As Intuitionistic Fuzzy set. In this paper, Adams's Predictor - Corrector method is used for finding numerical solution of an Intuitionistic Fuzzy Differential Equation (IFDE). The proposed method is based on generalized characterization theorem. Using the generalized characterization theorem, IFDE is transformed into four ordinary differential systems. Also, the convergence and stability of the proposed method is given and its applicability is illustrated by solving a first order IFDE.

Keywords: Intuitionistic Fuzzy Differential Equation, Numerical methods, Adams's Predictor – Corrector, Generalized Differentiability.

Introduction: In 1965 , Zadeh introduced the fuzzy set theory [1] and Atanassov extended the concept of fuzzy set theory to Intuitionistic fuzzy set (IFS) theory [2] . fuzzy differential equation (FDE) methods have wide range of application in many branches of engineering and in the field of medicine. Many research paper are focused on numerical solution of fuzzy initial value problem (FIVP) . Ming Ma et al introduced Euler method for solving FDE's numerically under H-derivative [3] . Numerical solutions for FIVP's using H-derivatives have been studied and can be found out in [4] to [9]. But they have the some disadvantages that the diameter of the solution becomes infinite as the independent variable increases. To overcome this disadvantage, Bede and Gal introduced the strongly generalized differentiability to FDEs[10] and first order fuzzy differential equation has been studied under generalised differentiability in [11]. Following Bede and Gal, Chalco-Cano and Roman-Flores studied numerical solution of fuzzy differential equations by lateral H-derivative [12]. This opened a way to study numerical solutions of FDEs under generalised differentiability concept. Bede[13] has also proved a characterization theorem which states that under certain conditions a FDE under H-derivative is equivalent to a system of ordinary differential equations(ODEs)[13]. So any numerical method which is used to solve ODEs, can be extended to solve FDEs. Using this characterization theorem, numerical solutions of FDEs have been studied in [14, 15, 16] and the numerical solutions studied through generalised differentiability can be found out in [17, 18, 19, 20, 21, 22, 23].
Melliani and Chadli have discussed differential and partial differential equations under intuitionistic fuzzy environment [24, 25]. Allah Viranloo discussed numerical solution of FDE by Runge Kutta method with

intuitionistic treatment [26]. Sneh lata and Amit Kumar have introduced time dependent Intuitionistic fuzzy linear differential equationand have proposed a method to solve it[27]. Sankar Prasad Mondal and Tapan Kumar Roy have discussed strong and weak solution of intuitionistic fuzzy ordinary differential equation [28]. Numerical solutions of IFDE under generalised differentiability by Euler method, Modified Euler method, fourth order Runge-Kutta method,Adams,s predictor- Corrector have been discussed respectively in [29, 30, 31]. In this paper, intuitionistic fuzzy Cauchy problem is solved numerically by Adms's Predictor-Corrector method under generalised differentiability concept.

This paper is arranged as follows: Section 2 consists of basic definitions related to Intuitionistic fuzzy set theory. Intuitionistic fuzzy Cauchy problem is given in section 3. Milne's Predictor-Corrector method for IFDE is presented in section 4. The convergence and stability of the proposed method is presented in section 5. Section 6 consists of a numerical example and conclusion of the paper is in section 7.

Preliminaries:

Definition 2.1[2]: Let X be a universe of discourse .An IFS "A" in X is an object having the form: $A=\{(x,\mu_A(x),\nu_A(x))\setminus x\in X\}$ which is characterized by a membership function $\mu_A : X\rightarrow[0,1]$ and a non membership function $\nu_A : X\rightarrow[0,1]$with the condition $0\le\mu_A(x)+\nu_A(x)\le 1,$ for all $x\in X.$

For each x the number $\mu_A(x)$ and $\nu_A(x)$ represents the degree of membership and degree of non - membership of the element $x\in X$ to $A\subset X$, respectively.

For each IFS A in X. if $\pi_A(x)=1-\mu_A(x)-\nu_A(x).$ $\pi_A(x)$is called the degree of indeterminacy or hesitancy of x to A .

Especially, if $\pi_A(x)=1-\mu_A(x)-\nu_A(x)=0$. for each $x\in X$ then the IFS is reduced to fuzzy set.

Definition2.2:[32] An Intuitionistic fuzzy set $A=\{(x,\mu_A(x),\nu_A(x))\setminus x\in R\}$such that $\mu_A(x)$ and $(1-\nu_a(x))=1-\nu_A(x).\forall x\in R$ are fuzzy numbers is called an intuitionistic fuzzy number.

Therefore IFS $A=\{(x,\mu_A(x),\nu_A(x))\setminus x\in R\}$ is a conjunction of two fuzzy numbers A^+ with a membership function $\mu_{A^+}=\mu_A(x)$ and A^- with a membership function $\mu_{A^-}=1-\nu_A(x)$.

Definition2.3:[32] The α-cut of an IFN $A=\{(x,\mu_A(x),\nu_A(x))\setminus x\in R\}$ defined as follows $A_\alpha=\{(x,\mu_A(x),\nu_A(x))\setminus x\in R,\mu_A(x)\ge\alpha\, and\, \nu_A(x)\le 1-\alpha\},\forall\alpha\in[0,1].$

The α-cut representation of IFN A generates the following pair of intervals and is denoted by $[A]_\alpha=\{[A_L^+(\alpha),A_U^+(\alpha)],[A_L^-(\alpha),A_U^-(\alpha)]\}$ where $A_L^+(\alpha)=\inf\{x\in R/\mu_A(x)\ge\alpha\}$, $A_U^+(\alpha)=\sup\{x\in R/\mu_A(x)\ge\alpha\}$, $A_L^-(\alpha)=\inf\{x\in R/\nu_A(x)\le 1-\alpha\}$, $A_U^-(\alpha)=\sup\{x\in R/\nu_A(x)\le 1-\alpha\}$ where $[A^+]_\alpha=[A_L^-(\alpha),A_U^+(\alpha)]$ and $[A^-]_\alpha=[A_L^-(\alpha),A_U^-(\alpha)]$ are fuzzy number with membership function μ_A and ν_A respectively.

Definition2.4:[32] Triangular Intuitionistic fuzzy number (TIFN) A is an intuitionistic fuzzy set in R with the following membership function $\mu_A(x)$ and non membership function $\nu_A(x)$ given as follows

$$\mu_A(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}\; for\; a_1\le x\le a_2\\ \dfrac{a_3-x}{a_3-a_2}\; for a_2\le x\le a_3\\ 0\; otherwise\end{cases} \quad \text{and} \quad \nu_A(x)=\begin{cases}\dfrac{a_2-x}{a_2-a_1'}\; for\; a_1'\le x\le a_2\\ \dfrac{x-a_2}{a_3'-a_2}\; for a_2\le x\le a_3'\\ 1\; otherwise\end{cases}$$

where $a_1'\le a_1\le a_2\le a_3\le a_3'$ and TIFN is denoted by $A=(a_1,a_2,a_3;a_1',a_2',a_3')$

Arithmetic operation over TIFNs can be found in [31].

Let $E_n=\{Fuzzy\ Number\ over\ R^n\}$.

Definition2.5: [34] For arbitrary $u, v \in E_n$ the quantity $D(u,v) = \sup_{0 \le \alpha \le 1} d([u]^\alpha, [v]^\alpha)$ is the distance between u and v, where d is the Hausdroff metric in E_n

Let $W^n = \{Intuitionistic\ Fuzzy\ Number\ over\ R^n\}$ and then by definition of α-cut of an Intuitionistic fuzzy function is defined as follows:

Definition2.6:[29] Let mapping $f : I \rightarrow W^n$ for some interval I be an intuitionistic fuzzy function.

The α-cut of f is given by $[f(t)]_\alpha = \{[\underline{f^+(t,\alpha)}, \overline{f^+}(t,\alpha)], [\underline{f^-}(t,\alpha), \overline{f^-}(t,\alpha)]\}$

Where $\underline{f^+}(t,\alpha) = \min\{f^+(t,\alpha)/ t \in I, 0 \le \alpha \le 1\}$, $\overline{f^+}(t,\alpha) = \max\{f^+(t,\alpha)/ t \in I, 0 \le \alpha \le 1\}$

$\underline{f^-}(t,\alpha) = \min\{f^-(t,\alpha)/ t \in I, 0 \le \alpha \le 1\}$ and $\overline{f^-}(t,\alpha) = \max\{f^-(t,\alpha)/ t \in I, 0 \le \alpha \le 1\}$

With respect to Bede & Gal [6] , the generalized differentiability concept for intuitionistic fuzzy function is defined as follow :

Definition2.7:[29] let $F : (a,b) \rightarrow W^1$ and $x_0 \in (a,b)$. It is said that F is strongly generalized differentiable on x_0. If there exist elements $F^{+'}(x_0), F^{-'}(x_0) \in E^1$, such that

(i)for all $h > 0$ sufficiently small , $\exists F^+(x_0 + h) - F^+(x_0), F^+(x_0) - F^+(x_0 - h)$ and the limits (in the metric D)

$$\lim_{h \to 0} \frac{F^+(x_0 + h) - F^+(x_0)}{h} = \lim_{h \to 0} \frac{F^+(x_0) - F^+(x_0 - h)}{h} = F^{+'}(x_0)$$

(ii) for all $h > 0$ sufficiently small , $\exists F^+(x_0) - F^+(x_0 + h), F^+(x_0 - h) - F^+(x_0)$ and the limits

(iii) for all $h > 0$ sufficiently small , $\exists F^+(x_0 + h) - F^+(x_0), F^+(x_0 - h) - F^+(x_0)$ and the limits

$$\lim_{h \to 0} \frac{F^+(x_0 + h) - F^+(x_0)}{h} = \lim_{h \to 0} \frac{F^+(x_0 - h) - F^+(x_0)}{-h} = F^{+'}(x_0)$$

(or)

(iv) for all $h > 0$ sufficiently small , $\exists F^+(x_0) - F^+(x_0 + h), F^+(x_0) - F^+(x_0 - h)$ and the limits

$$\lim_{h \to 0} \frac{F^+(x_0) - F^+(x_0 + h)}{-h} = \lim_{h \to 0} \frac{F^+(x_0) - F^+(x_0 - h)}{h} = F^{+'}(x_0)$$

Result similar to (i) to (iv) can be defined to $F^{-'}(x_0) \in E^1$

Remark 2.1 A function that is strongly differentiable as in cases (i)and (ii) of definition 2.11 ,will be referred as (i)-differentiable or as (ii)-differentiable respectively .following

The ideas of chalco-cano and roman-Flores (10) the fuzzy lateral H-derivative for a fuzzy mapping $F : (a,b) \rightarrow E^n$ where E^n denotes set of fuzzy number over R^n,is extended to intuitionstic fuzzy H-derivative for a Intuitionistic fuzzy mapping $F : (a,b) \rightarrow W^n$ as follows:

Definition2.8:[29] Let be $F : (a,b) \rightarrow W^n$ and $x_0 \in (a,b)$ we say that

(i)if there exist an element $F^{+'}(x_0), F^{-'}(x_0) \in E^n$ such that, for all $h > 0$ sufficiently near to zero, $\exists F^+(x_0 + h) - F^+(x_0)$, $\exists F^+(x_0) - F^+(x_0 - h)$, $\exists F^-(x_0 + h) - F^-(x_0)$ $\exists F^-(x_0) - F^-(x_0 - h)$ and the limits (in metric D)

$$\lim_{h \to 0^+} \frac{F^+(x_0 + h) - F^+(x_0)}{h} = \lim_{h \to 0^+} \frac{F^+(x_0) - F^+(x_0 - h)}{h} = F^{+'}(x_0).$$

$$\lim_{h \to 0^+} \frac{F^-(x_0 + h) - F^-(x_0)}{h} = \lim_{h \to 0^+} \frac{F^-(x_0) - F^-(x_0 - h)}{h} = F^{-'}(x_0).$$

(or)

(ii)if there exist element $F^{+'}(x_0), F^{-'}(x_0) \in E^n$ such that , for all $h < 0$ sufficiently near to zero, $\exists F^+(x_0+h) - F^+(x_0)$, $\exists F^+(x_0) - F^+(x_0 - h)$, $\exists F^-(x_0+h) - F^-(x_0)$, $\exists F^-(x_0) - F^-(x_0 - h)$ and the limits

$$\lim_{h\to 0-} \frac{F^+(x_0+h) - F^+(x_0)}{h} = \lim_{h\to 0-} \frac{F^+(x_0) - F^+(x_0-h)}{h} = F^{+'}(x_0)$$

$$\lim_{h\to 0-} \frac{F^-(x_0+h) - F^-(x_0)}{h} = \lim_{h\to 0-} \frac{F(x) - F(x-h)}{h} = F^{-'}(x_0)$$

Intuitionistic Fuzzy Cauchy Problems: A first order Intuitionistic fuzzy differential equation is a differential equation of the form

$y' = f(t, y), t \in I = [a, b]$ (*)

where (i) y is an intuitionistic fuzzy function of the crisp variable t (ii) $f(t, y)$ is an intuitionistic fuzzy function of the crisp variable t and the intuitionistic fuzzy variable y and (iii) y' is the intuitionistic fuzzy derivative.

if an initial value $y(x_0) = y_0 \{\text{intuitionistic fuzzy number}\}$, we get an Intuitionistic fuzzy Cauchy problem of first order $y' = f(t, y), y(t_0) = y_0$.

As each intuitionistic fuzzy number is a conjunction two fuzzy number , equation (*) can be replaced by an equivalent system as follows:

$y'(t) = \left\{ \left[\underline{y}'^{+}(t), \overline{y}'^{+}(t) \right], \left[\underline{y}'^{-}(t), \overline{y}'^{-}(t) \right] \right\}$, where

$$\underline{y}'^{+}(t) = \min\left\{ f(t,u) \setminus u \in \left[\underline{y}^+, \overline{y}^+\right] \right\} = F(t, \underline{y}'^{+}(t), \overline{y}'^{+}(t)), \underline{y}^+(t_0) = \underline{y}_0^+ \cdots (1)$$

$$\overline{y}'^{+}(t) = \max\left\{ f(t,u) \setminus u \in \left[\underline{y}^+, \overline{y}^+\right] \right\} = G(t, \underline{y}'^{+}(t), \overline{y}'^{+}(t)), \overline{y}^+(t_0) = \overline{y}_0^+ \cdots (2)$$

$$\underline{y}'^{-}(t) = \min\left\{ f(t,u) \setminus u \in \left[\underline{y}^-, \overline{y}^-\right] \right\} = F(t, \underline{y}'^{-}(t), \overline{y}'^{-}(t)), \underline{y}^-(t_0) = \underline{y}_0^- \cdots (3)$$

$$\overline{y}'^{-}(t) = \max\left\{ f(t,u) \setminus u \in \left[\underline{y}^-, \overline{y}^-\right] \right\} = G(t, \underline{y}'^{-}(t), \overline{y}'^{-}(t)), \overline{y}^-(t_0) = \overline{y}_0^- \cdots (4)$$

The system of equation given in (1) and (2) will have unique solution $\left[\underline{y}^+, \overline{y}^+\right] \in B = \overline{C}[0,1] X \overline{C}[0,1]$ and the system of equation in (3) and (4) will have unique solution $\left[\underline{y}^-, \overline{y}^-\right] \in B = \overline{C}[0,1] X \overline{C}[0,1]$. there for the system given from (1) to (4) possesses unique solution $y'(t) = \left\{ \left[\underline{y}'^{+}(t), \overline{y}'^{+}(t) \right], \left[\underline{y}'^{-}(t), \overline{y}'^{-}(t) \right] \right\} \in BXB$ which is Intuitionistic fuzzy function (i.e) for each t. $y'(t;r) = \left\{ \left[\underline{y}'^{+}(t;r), \overline{y}'^{+}(t;r) \right], \left[\underline{y}'^{-}(t;r), \overline{y}'^{-}(t;r) \right] \right\}, r \in [0,1]$ is an Intuitionistic fuzzy number . the parametric form of the system of equation (1) to (4) is given by

$$\underline{y}'^{+}(t;r) = F(t, \underline{y}^+(t;r), \overline{y}^+(t;r)), \underline{y}^+(t_0;r) = \underline{y}_0^+(r) \cdots (5)$$

$$\overline{y}'^{+}(t;r) = G(t, \underline{y}^+(t;r), \overline{y}^+(t;r)), \overline{y}^+(t_0;r) = \overline{y}_0^+(r) \cdots (6)$$

$$\underline{y}'^{-}(t;r) = H(t, \underline{y}^-(t;r), \overline{y}^-(t;r)), \underline{y}^-(t_0;r) = \underline{y}_0^-(r) \cdots (7)$$

$$\overline{y}'^{-}(t;r) = I(t, \underline{y}^-(t;r), \overline{y}^-(t;r)), \overline{y}^-(t_0;r) = \overline{y}_0^-(r) \cdots (8) \text{ for } r \in [0,1].$$

A solution to a system of equation (5) to (8) must solve (1) to (4) . in some case, the system given in equation (5) to (8) can be solved analytically . But in most case, however , analytical solution to equation (5) to (8) may not be found and a numerical approach must be considered. For every prefixed r ,each equation from (5) to (8) represents an ordinary Cauchy problem for which any converging classical numerical procedure can be applied.

The Adams's Predictor-Corrector Method: In this section, we will present the Adam's predictor-corrector method for intuitionistic fuzzy differential equation. to solve the intuitionistic fuzzy system of ordinary differential system in $\lfloor t_0, t_1 \rfloor, \lfloor t_1, t_2 \rfloor, \cdots \lfloor t_K, t_{k+1} \rfloor \cdots$, for $r \in [0,1]$ .we will replace each interval $\lfloor t_k, t_{k+1} \rfloor$ by a set $N_k + 1$ regularly spaced points. The grid points on $\lfloor t_k, t_{k+1} \rfloor$ will be $t_{k,n} = t_k + nh_k$ where $h_k = \frac{t_{k+1} - t_k}{N_k}$ and $0 \le n \le N_k$. Now we will give algorithm to numerically solve the system in $\lfloor t_0, t_1 \rfloor, \lfloor t_1, t_2 \rfloor, \cdots \lfloor t_K, t_{k+1} \rfloor \cdots$

Algorithm: to approximate the solution of the fuzzy initial value problem given by the system of equation in equation (5) and (6)

Case 1: (i) differentiability

Step 1: Let $h_k = \frac{t_{k+1} - t_k}{N_k}$;

$$\underline{w_r^+}(t_{k,0}) = \alpha_0 \;;\; \underline{w_r^+}(t_{k,1}) = \alpha_1 \;;\; \underline{w_r^+}(t_{k,2}) = \alpha_2 \;;\; \underline{w_r^+}(t_{k,3}) = \alpha_3 \;;$$

$$\overline{w_r^+}(t_{k,0}) = \beta_0 \;;\; \overline{w_r^+}(t_{k,1}) = \beta_1 \;;\; \overline{w_r^+}(t_{k,2}) = \beta_2 \;;\; \overline{w_r^+}(t_{k,3}) = \beta_3 \;;$$

Step 2: $i = 0$

Step 3: let $\underline{w_{r,p}^+}(t_{k,i+4}) = \underline{w_r^+}(t_{k,i}) + \frac{h}{24}\left[55\underline{f_r^+}(t_{k,i+1}) - 59\overline{f_r^+}(t_{k,i+2}) + 37\underline{f_r^+}(t_{k,i+3}) - 9\underline{f_r^+}(t_{k,i+4})\right]$;

$$\overline{w_{r,p}^+}(t_{k,i+4}) = \overline{w_r^+}(t_{k,r}) + \frac{h}{24}\left[55\overline{f_r^+}(t_{k,i+1}) - \underline{59 f_r^+}(t_{k,i+2}) + 37\overline{f_r^+}(t_{k,i+3}) - 9\overline{f_r^+}(t_{k,i+4})\right];$$

$$\underline{w_{r,c}^+}(t_{k,i+4}) = \underline{w_r^+}(t_{k,i+2}) + \frac{h}{24}\left[\underline{9 f_r^+}(t_{k,i+2}) + 19\underline{f_r^+}(t_{k,i+3}) - 5\underline{f_r^+}(t_{k,i+4}) + \underline{f_r^+}(t_{k,i+4})\right];$$

$$\overline{w_{r,c}^+}(t_{k,i+4}) = \overline{w_r^+}(t_{k,i+2}) + \frac{h}{24}\left[9\overline{f_r^+}(t_{k,i+2}) + 19\overline{f_r^+}(t_{k,i+3}) - 5\overline{f_r^+}(t_{k,i+4}) + \overline{f_r^+}(t_{k,i+4})\right];$$

Step 4: $t_{k,i+1} = t_{k,0} + (i+1)h_k$;

Step 5: let $i = i + 1$;

Step 6: if $i \le N$, go to step 3.

Step 7 : The algorithm ends and $\left[\underline{w_r^+}(t_{k+1}), \overline{w_r^+}(t_{k+1})\right]$ approximate the value of the exact solution $\left[\underline{Y_r^+}(t_{k+1}), \overline{Y_r^+}(t_{k+1})\right]$.

Case 2: (ii) differentiability

Step 1: Let $h_k = \frac{t_{k+1} - t_k}{N_k}$;

$$\underline{w_r^+}(t_{k,0}) = \alpha_0 \;;\; \underline{w_r^+}(t_{k,1}) = \alpha_1 \;;\; \underline{w_r^+}(t_{k,2}) = \alpha_2 \;;\; \underline{w_r^+}(t_{k,3}) = \alpha_3 \;;$$

$$\overline{w_r^+}(t_{k,0}) = \beta_0 \;;\; \overline{w_r^+}(t_{k,1}) = \beta_1 \;;\; \overline{w_r^+}(t_{k,2}) = \beta_2 \;;\; \overline{w_r^+}(t_{k,3}) = \beta_3 \;;$$

Step 2: $i = 0$

Step 3: let

$$\underline{w_{r.p}^+}(t_{k,i+4}) = \underline{w_r^+}(t_{k,r}) + \frac{h}{24}\left[55\overline{f_r^+}(t_{k,i+1}) - 59\underline{f_r^+}(t_{k,i+2}) + 37\overline{f_r^+}(t_{k,i+3}) - 9\overline{f_r^+}(t_{k,i+4})\right] \;;$$

$$\overline{w_{r,p}^{+}}\left(t_{k,i+4}\right)=\overline{w_{r}^{+}}\left(t_{k,i}\right)+\frac{h}{24}\left[55\underline{f_r^{+}}\left(t_{k,i+1}\right)-59\overline{f_r^{+}}\left(t_{k,i+2}\right)+37\underline{f_r^{+}}\left(t_{k,i+3}\right)-9\underline{f_r^{+}}\left(t_{k,i+4}\right)\right];$$

$$\underline{w_{r,c}^{+}}\left(t_{k,i+4}\right)=\underline{w_{r}^{+}}\left(t_{k,i+2}\right)+\frac{h}{24}\left[9\overline{f_r^{+}}\left(t_{k,i+2}\right)+19\overline{f_r^{+}}\left(t_{k,i+3}\right)-5\overline{f_r^{+}}\left(t_{k,i+4}\right)+\overline{f_r^{+}}\left(t_{k,i+5}\right)\right];$$

$$\overline{w_{r,c}^{+}}\left(t_{k,i+4}\right)=\overline{w_{r}^{+}}\left(t_{k,i+2}\right)+\frac{h}{24}\left[9\underline{f_r^{+}}\left(t_{k,i+2}\right)+19\underline{f_r^{+}}\left(t_{k,i+3}\right)-5\underline{f_r^{+}}\left(t_{k,i+4}\right)+\underline{f_r^{+}}\left(t_{k,i+5}\right)\right];$$

Step 4: $t_{k,i+1}=t_{k,0}+(i+1)h_k$;

Step 5: let $i=i+1$;

Step 6: if $i \le N$, go to step 3.

Step 7 : The algorithm ends and $\left[\underline{w_r^{+}}\left(t_{k+1}\right),\overline{w_r^{+}}\left(t_{k+1}\right)\right]$ approximate the value of the exact solution $\left[\underline{Y_r^{+}}\left(t_{k+1}\right),\overline{Y_r^{+}}\left(t_{k+1}\right)\right]$.

Similar algorithm can be given to approximate the solution of the fuzzy initial value problem given by the system of equation in eq (8) and (9).

Convergence and Stability: The following lemmas will be applied to show the convergence of the approximate

$\left[\underline{w_r^{+}}\left(t_{k+1}\right),\overline{w_r^{+}}\left(t_{k+1}\right)\right]$ (i.e) $\lim_{h\to 0}\underline{w_r^{+}}\left(t_{k+1}\right)=\underline{Y_r^{+}}\left(t_{k+1}\right)$ and $\lim_{h\to 0}\overline{w_r^{+}}\left(t_{k+1}\right)=\overline{Y_r^{+}}\left(t_{k+1}\right)$

Lemma 5.1: Let the sequence of number $\{W_n\}_{n=0}^{N}$ satisfy $\left|W_{n+1}\right| \le A\left|W_n\right|+B, 0\le n\le N-1$ for some given positive constant A and B , then $\left|W_n\right| \le A^n\left|W_0\right|+B\frac{A^n-1}{A-1}, 0\le n\le N-1$

Proof: see [3]

Lemma 5.2: let the sequence of numbers $\{W_n\}_{n=0}^{N}$, $\{V_n\}_{n=0}^{N}$ satisfy $\left|W_{n+1}\right| \le \left|W_n\right|+\max\{\left|W_n\right|,\left|V_n\right|\}+B$, $\left|V_{n+1}\right| \le \left|V_n\right|+\max\{\left|W_n\right|,\left|V_n\right|\}+B$ for some positive constant A and B , and denote $U_n=\left|W_n\right|+\left|V_n\right|, 0\le n\le N$. Then $U_n \le (1+2A)U_0+2B\frac{\left|1+2A\right|^n-1}{\left|1+2A\right|-1}, 0\le n\le N.$

Proof: see [3]

Theorem 5.1: For arbitrary fixed $r, 0\le r\le 1$ the approximate solution given in equation (12) and (16) converge to the exact solution $\underline{Y_r^{+}}\left(t_{k+1}\right)$ and the approximate solution given in equation (13) and (17) converges to the exact solution $\overline{Y_r^{+}}\left(t_{k+1}\right)$ uniformly in t ,for $\underline{Y_r^{+}}\left(t_{k+1}\right),\overline{Y_r^{+}}\left(t_{k+1}\right)\in C^4\left[t_0,T\right]$

Proof: see [3]

Definition 5.1: an m-step method for solving the initial value problem is one whose difference equation for finding the approximation $y\left(t_{i+1}\right)$ at the mesh point t_{i+1} can be represented by the following equation:

$$y\left(t_{i+1}\right)=a_{m-1}y\left(t_i\right)+a_{m-2}y\left(t_{i-1}\right)+\cdots+a_0y\left(t_{i+1-m}\right)+h\{b_mf\left(t_{i+1},y_{i+1}\right)+b_{m-1}f\left(t_i,y_i\right)+\cdots+b_0f\left(t_{i+1-m},y_{i+1-m}\right)\}$$

for $i=m-1,m,\cdots N-1$ such that $a=t_0\le t_1\le t_2\cdots\le t_N=b$, $h=\frac{(b-a)}{N}$ and $a_0,a_1,\cdots a_{m-1}$, $b_0,b_1\cdots b_m$ are constants with the starting values $y_0=\alpha_0$, $y_1=\alpha$, $y_0=\alpha_0$, $y_{m-1}=\alpha_{m-1}$ when $b_m=0$ the method is known as explicit and when $b_m\ne 0$ the method is known as implicit.

Definition 5.2: Associated with the difference equation

$$y\left(t_{i+1}\right)=a_{m-1}y\left(t_i\right)+a_{m-2}y\left(t_{i-1}\right)+\cdots+a_0y\left(t_{i+1-m}\right)+h\{b_mf\left(t_{i+1},y_{i+1}\right)+b_{m-1}f\left(t_i,y_i\right)+\cdots+b_0f\left(t_{i+1-m},y_{i+1-m}\right)\}$$

the characteristic polynomial of the method is defend by $p(\lambda)=\lambda^m - a_{m-1}\lambda^{m-1} - a_{m-2}\lambda^{m-2}\cdots a_1\lambda - a_0$

If $|\lambda_i| \le 1$ for $i=1,2,3\cdots m$ and all roots with absolute value "1" are simple roots ,then the difference method is said to satisfy the root condition.

Theorem 5.3: The implicit four step method is stable

Proof: for the implicit four step method, there exist only one characteristic polynomial $p(\lambda)=\lambda^4-\lambda^2$, so it satisfies the root condition and therefore, it is a stable method.

Numerical Examples: $y'(t)=-\lambda\Theta y(t), y(t_0)=y_0, t\in I=[t_0,T]$ where $y(t)$ is the number of radio nuclide present in a given radioactive material . λ is the decay constant and y_0 is the initial number of radio nuclide . in the model uncertainty is introduced if we have uncertain information on the initial value y_0 of radio nuclide present in the material . note that the phenomenon of nuclear disintegration is considered a stochastic process uncertainty being introduced by lack of information on the radioactive material under study. However , in some situations, there may be hesitation on the number of radio nuclide present in the radioactive material . in order to take in to account the uncertainty and hesitation : we consider y_0 being a triangular Intuitionistic fuzzy number.

Solution:

Let $\lambda=1$ and $I=[0,1]$, the $\alpha-$cut of the initial value is given by

$y(t_0,\alpha)=y_0(\alpha)=\{[5+2\alpha,9-2\alpha],[3+4\alpha,11-4\alpha]\};$

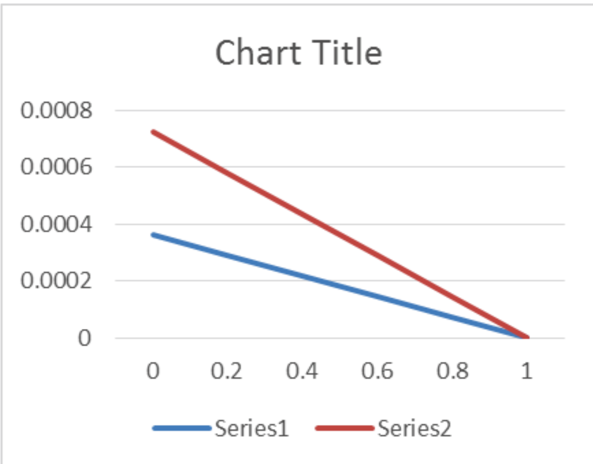
Case 1: (i) – differentiability

The exact solution of equation (20) under (i) – differentiability is given by

$\underline{y}^{+}(t;\alpha)=(2\alpha-2)e^{t}+7e^{-t}$; $\overline{y}^{+}(t;\alpha)=-(2\alpha-2)e^{t}+7e^{-t}$;

$\underline{y}^{-}(t;\alpha)=(4\alpha-4)e^{t}+7e^{-t}$; $\overline{y}^{-}(t;\alpha)=-(4\alpha-4)e^{t}+7e^{-t}$;

Error between exact and approximate solution of membership and non-membership function of eq (20) at t=0.04 with h=0.01 is given in following table



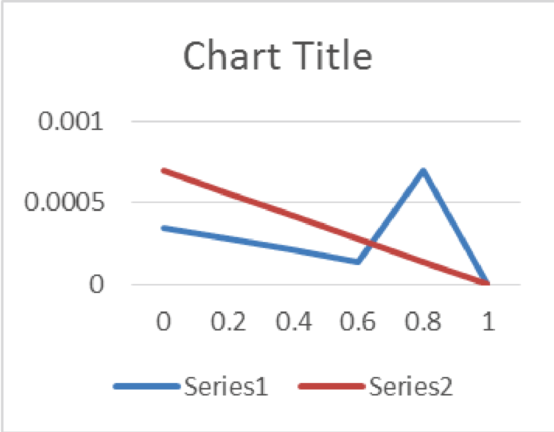
Case 2: (ii) – differentiability

The exact solution of equation (20) under (ii) – differentiability is given by

$\underline{y}^{+}(t;\alpha)=(5+2\alpha)e^{-t}$; $\overline{y}^{+}(t;\alpha)=(9-2\alpha)e^{-t}$;

$\underline{y}^{-}(t;\alpha)=(3+4\alpha)e^{-t}$; $\overline{y}^{-}(t;\alpha)=(11-4\alpha)e^{-t}$;

Error between exact and approximate solution of membership and non-membership function of eq (20) at t=0.04 with h=0.01 is given in the following table



The solution of Eq(10) under (i) differentiability has an increasing length of its support leads us to the conclusion that there is a possibility that the number of radio nuclide increases as increases and even a non zero possibility that it is negative Fortunately the real Situation is different and the number of radio nuclide decreases with time and it cannot be Negative therefore (ii) differentiability is suitable for this type of problem .In both cases the numerical solution of Eq(20) by the Adams's predictor corrector method are closer to the exact solution however the error can be minimised by taking smaller step size h.

Conclusion: We propose a general numerical procedure for treating Intuitionistic fuzzy Cauchy problem The original initial value problem is replaced by four parametric ordinary differential Equation which is then solved numerically using classical algorithms. In this work the Adams's predictor corrector method is used . The method applicability is illustrated by solving first order Intuitionistic fuzzy differential equation This method gives better result than milnes predictor-corrector method.

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