AN INNOVATIVE METHOD FOR FINDING OPTIMAL SOLUTION TO TRANSPORTATION PROBLEMS

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Abstract: In this paper a new method is proposed for finding an optimal solution for intuitionistic fuzzy transportation problem, in which the supplies and demands are octagonal intuitionistic fuzzy numbers. The procedure is illustrated with a numerical example.

Keywords: Intuitionistic Fuzzy Transportation Problems, Octagonal Intuitionistic Fuzzy Numbers, Ranking Method, Modi Method, Initial Basic Feasible Solution, Optimal Solution.

Introduction: The central concept in the problem is to find the least total transportation cost of commodity. In general, transportation problems are solved with assumptions that the cost, supply and demand are specified in precise manner. However in many cases the decision maker has no precise information about the coefficient belonging to the transportation problem. Intuitionistic fuzzy set is a powerful tool to deal with such vagueness.

The concept of Intuitionistic Fuzzy Sets (IFSs), proposed by Atanassov in [1]and[2], has been found to be highly useful to deal with vagueness. Many authors discussed the solutions of Fuzzy Transportation Problem (FTP) using various techniques.In 2016, Mrs .Kasthuri.B introduced Pentagonal intuitionistic fuzzy. In 2015,A.Thamaraiselvi and R. Santhi [3]introduced Hexagonal Intuitionistic Fuzzy Numbers.In 2015,Thangaraj Beaula – M. Priyadharshini [4] proposedA New Algorithm for Finding a Fuzzy Optimal Solution.K.Prasanna Devi, M. Devi Durga [5] and G.Gokila, Juno Saju [6]introduced Octagonal Fuzzy Number.

A new method is proposed for finding an optimal solution for intuitionistic fuzzy transportation problem, in which the supplies and Demands are octagonal intuitionistic fuzzy numbers. Using Octagonal Intuitionistic fuzzy transportation problem we get best minimum value of optimal solution.

The paper is organized as follows, in section 2, introduction with some basic concepts of Intuitionistic fuzzy numbersin section 3, introduced Octagonal Intuitionistic Fuzzy Definition and proposed algorithm followed by a Numerical example using Modi method and finally the paper is concluded in section 4.

Preliminaries:
Definition (Intuitionistic Fuzzy Set [IFS])[3]:
Let X be a non-emptyset. An Intuitionistic fuzzy set $\bar{A}^I$ of X is defined as
$\bar{A}^I = \{<x, \mu_{\bar{A}^I}(x), \vartheta_{\bar{A}^I}(x) > / x \in X\}$. Where $\mu_{\bar{A}^I}(x)$ and $\vartheta_{\bar{A}^I}(x)$ aremembership and non-membership function. Such that $\mu_{\bar{A}^I}(x), \vartheta_{\bar{A}^I}(x)$: X→ [0, 1] and $0 \leq \mu_{\bar{A}^I}(x) + \vartheta_{\bar{A}^I}(x) \leq 1$ for all $x \in X$.
Definition (Intuitionistic Fuzzy Number [IFN])[3]:
An Intuitionistic Fuzzy Subset $\bar{A}^I = \{< x, \mu_{\bar{A}^I}(x), \vartheta_{\bar{A}^I}(x) > / x \in X\}$ of the real line R is called an Intuitionistic Fuzzy Number, if the following conditions hold,

- There exists m ∈ R suct that $\mu_{\bar{A}^I}(m) = 1$ and $\vartheta_{\bar{A}^I}(m) = 0$.
- $\mu_{\bar{A}^I}$ is a continuous function from R → [0,1] such that
$0 \leq \mu_{\bar{A}^I}(x) + \vartheta_{\bar{A}^I}(x) \leq 1$ for all $x \in X$.
- The membership and non- membership functions of $\bar{A}^I$ are in the following form

$$\mu_{\bar{A}^I}(x) = \begin{cases} 0 & for -\infty < x \le a_1 \\ f(x) & for \;\; a_1 \le x \le a_2 \\ 1 & for \;\; x = a_2 \\ g(x) & for \;\; a_2 \le x \le a_3 \\ 0 & for \;\; a_3 \le x < \infty \end{cases}$$

$$\vartheta_{\bar{A}^I}(x) = \begin{cases} 1 & for -\infty < x \le a_1' \\ f'(x) & for \;\; a_1' \le x \le a_2 \\ 0 & for \;\; x = a_2 \\ g'(x) & for \;\; a_2 \le x \le a_3' \\ 1 & for \;\; a_3' \le x < \infty \end{cases}$$

Wheref, f', g, g' are functions from R → [0,1] . f and g' are strictly increasing functions and g and f' are strictly decreasing functions with the conditions $0 \le f(x) + f'(x) \le 1$ and $0 \le g(x) + g'(x) \le 1$.

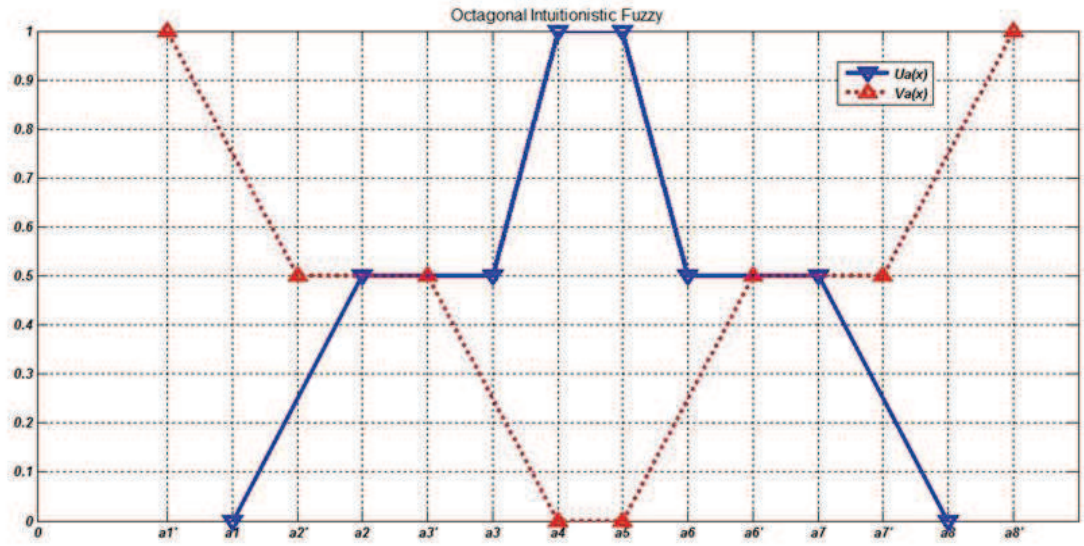
Octagonal Intuitionistic Fuzzy Number:

Definition (Octagonal Intuitionistic Fuzzy Number [OIFN]): An Octagonal Intuitionistic Fuzzy Number is specified by $\bar{A}_{oc}{}^{I}=(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8), (a_1', a_2', a_3', a_4, a_5, a_6', a_7', a_8')$.

Where$a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_1', a_2', a_3', a_6', a_7'$ *and* a_8' and its membership and non-membership functions are given below

$$\mu_{\bar{A}}(x) = \begin{cases} 0 & for \;\; x < a_1 \\ k\left(\dfrac{x - a_1}{a_2 - a_1}\right) & for \; a_1 \le x \le a_2 \\ k & for \; a_2 \le x \le a_3 \\ k + (1-k)\left(\dfrac{x - a_3}{a_4 - a_3}\right) & for \; a_3 \le x \le a_4 \\ 1 & for \; a_4 \le x \le a_5 \\ k + (1-k)\left(\dfrac{a_6 - x}{a_6 - a_5}\right) & for \; a_5 \le x \le a_6 \\ k & for \; a_6 \le x \le a_7 \\ k\left(\dfrac{a_8 - x}{a_8 - a_7}\right) & for \; a_7 \le x \le a_8 \\ 0 & for \;\; x > a_8 \end{cases}$$

$$\vartheta_{\bar{A}^I}(x) = \begin{cases} 1 & for \;\; a_1' < x \\ k + (1-k)\left(\dfrac{a_2' - x}{a_2' - a_1'}\right) & for \; a_1' \le x \le a_2' \\ k & for \; a_2' \le x \le a_3' \\ k\left(\dfrac{a_4 - x}{a_4 - a_3'}\right) & for \; a_3' \le x \le a_4 \\ 0 & for \; a_4 \le x \le a_5 \\ k\left(\dfrac{x - a_5}{a_6' - a_5}\right) & for \; a_5 \le x \le a_6' \\ k & for \; a_6' \le x \le a_7' \\ k + (1-k)\left(\dfrac{x - a_7'}{a_8' - a_7'}\right) & for \; a_7' \le x \le a_8' \\ 1 & for \;\; x > a_8' \end{cases}$$



Graphical Representation of Octagonal Intuitionistic Fuzzy Number

___ Membership Function$\mu_{\bar{A}}$ (x)
---- Non Membership Function$\vartheta_{\bar{A}^I}$(x)

Arithmetic operations on Octagonal Intuitionistic Fuzzy Numbers: Let $\bar{A}_{oc}{}^{I}$ =$(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ $(a_1', a_2', a_3', a_4, a_5, a_6', a_7', a_8')$ and $\bar{B}_{oc}{}^{I}$ = $(b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8)$ $(b_1', b_2', b_3', b_4, b_5, b_6', b_7', b_8')$ be two Octagonal Intuitionistic Fuzzy Numbers , then the arithmetic operations are as follows.

Addition: $\bar{A}_{oc}{}^{I} + \bar{B}_{oc}{}^{I} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7, a_8 + b_8)$
$(a_1' + b_1', a_2' + b_2', a_3' + b_3', a_4 + b_4, a_5 + b_5, a_6' + b_6', a_7' + b_7', a_8' + b_8')$

Subtraction: $\bar{A}_{oc}{}^{I} - \bar{B}_{oc}{}^{I} = (a_1 - b_8, a_2 - b_7, a_3 - b_6, a_4 - b_5, a_5 - b_4, a_6 - b_3, a_7 - b_2, a_8 - b_1)$
$(a_1' - b_8', a_2' - b_7', a_3' - b_6', a_4 - b_5, a_5 - b_4, a_6' - b_3', a_7' - b_2', a_8' - b_1')$

Multiplication: $\bar{A}_{oc}{}^{I} * \bar{B}_{oc}{}^{I} = (a_1 * b_1, a_2 * b_2, a_3 * b_3, a_4 * b_4, a_5 * b_5, a_6 * b_6, a_7 * b_7, a_8 * b_8)$
$(a_1' * b_1', a_2' * b_2', a_3' * b_3', a_4 * b_4, a_5 * b_5, a_6' * b_6', a_7' * b_7', a_8' * b_8')$

Ranking of OIFN based on Accuracy Function: Accuracy function of a Octagonal Intuitionistic Fuzzy Number A^I= $[(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ $(a_1', a_2', a_3', a_4', a_5', a_6', a_7', a_8')]$ is defined as
H(A^I) = $(a_1 + a_1' + a_2 + a_2' + a_3 + a_3' + a_4 + a_4' + a_5 + a_5' + a_6 + a_6' + a_7 + a_7' + a_8 + a_8')/8$

Modi Method: There are many methods to find the basic feasible solution, Modi method is heuristic method. The advantage of this method is that it gives an initial solution which is nearer to an optimal solution. Here in this paper Modi method is suitably modified and used to solving Intuitionistic Fuzzy transportation problem.

Proposed Algorithm:

Step –1: In Octagonal Intuitionistic Fuzzy transportation problem (OIFN) the quantities are reduced into an integer using the ranking method called accuracy function.

Step –2: For an initial basic feasible solution with m + n -1 occupied cells, calculate u_i and v_j for rows and columns. The initial solution can be obtained by any of the three methods discussed earlier.

To start with, any ofu_i's orv_j's is assigned the value zero. It is better to assign zero for a particular u_i or v_j . Where there is maximum number of allocations in a row or column respectively, as it will reduce arithmetic work considerably. Then complete the calculation of u_i's andv_j's for other rows and columns by using the relation.$C_{ij} = u_i + v_J$ for all occupied cells (i,j).

Step – 3: For unoccupied cells, calculate opportunity cost by using the relationship
$d_{ij} = C_{ij}$ - $(u_i + v_J)$ for all i and j.

Step – 4: Examine sign of each d_{ij} .

(i) If d_{ij} >0, then current basic feasible solution is optimal.

(ii) If d_{ij} = 0, then current basic feasible solution will remain unaffected but an alternative solutions exists.

(iii) If one or more d_{ij} <0, then an improved solutions can be obtained by entering unoccupied cell (i,j) in the basis. An unoccupied cell having the largest negative value of d_{ij} is chosen for entering into the solution mix (new transportation schedule).

Step – 5: Construct a closed path (or loop) for the unoccupied cell with largest negative opportunity cost. Start the closed path with the selected unoccupied cell and mark a plus sign (+) in this cell, trace a path along the rows (or columns) to an occupied cell, mark the corner with minus sign (-) and continue down the column (or row) to an occupied cell and mark the corner with plus sign (+) and minus sign (-) alternatively, close the path back to the selected unoccupied cell.

Step – 6: Select the smallest quantity amongst the cells marked with minus sign on the corners of closed loop. Allocate this value to the selected unoccupied cell and add it to other occupied cells marked with plus signs and subtract it from the occupied cells marked with minus signs.

Step – 7: Obtain a new improved solution by allocating units to the unoccupied cell according to step – 6 and calculate the new total transportation cost.

Step – 8: Test the revised solution further for optimality. The procedure terminates when all $d_{ij} \geq 0$, for unoccupied cells.

Numerical Example:

Consider a 3×3 Octagonal Intuitionistic Fuzzy Number.

	D1	D2	D3	Supply
S1	10	12	16	(7,8,9,10,15,16,17,18) (9,10,11,12,13,14,15,16)
S2	11	15	13	(15,16,17,18,20,24,25,27) (0,6,9,10,11,13,14,15)
S3	14	18	12	(16,17,19,20,23,25,27,29) (4,5,7,17,19,24,28,30)
Demand	(2,7,18,25,26,27,29,30) (6,8,10,12,16,19,21,24)	(17,18,21,24,25,26,29,30) (15,16,19,20,22,24,26,28)	(0,3,4,5,7,9,10,11) (2,4,6,9,10,12,13,14)	

ΣDemand = Σ Supply

The problem is a balanced transportation problem. Using the proposed algorithm, the solution of the problem is as follows.

Applying accuracy function on Octagonal Intuitionistic Fuzzy Number (2,7,18,25,26,27,29,30) (6,8,10,12,16,19,21,24)], we have

H(A^I) = (2+7+18+25+26+27+29+30+6+8+10+12+16+19+21+24)/8 = 35

Similarly applying for all the values, we have the following table after ranking

Reduced Table:

	B_1	B_2	B_3	Supply
A_1	10	12	16	25
A_2	11	15	13	30
A_3	14	18	12	40
Demand	35	45	15	

Applying VAM method, Table corresponding to initial basic feasible solution is

Reduced Table:

	B_1	B_2	B_3	Supply
A_1	10	[25] 12	16	25
A_2	[10] 11	[20] 15	13	30
A_3	[25] 14	18	[15] 12	40
Demand	35	45	15	

Since the number of occupied cell m+n-1=5 and are also independent. There exist a non-negative basic feasible solution.

The initial transportation cost is

[(12×25) + (10×11) + (20×15) + (14×25) + (12×15)] = 1240
Applying MODI method, table corresponding to optimal solution is

Reduced Table:

	B_1	B_2	B_3	Supply	u_i
A_1	(2) 10	[25] 12	(10) 16	25	0
A_2	[10] 11	[20] 15	(4) 13	30	3
A_3	[25] 14	(0) 18	[15] 12	40	6
Demand	35	45	15		
v_j	8	12	6		

Since all $d_{ij} \geq 0$ the solution in optimum and unique. The solution is given by x_{12}= 25, $x_{21} = 10$, x_{22}= 20, x_{31}= 25, x_{33}= 15.
The optimal solution is
[(12×25) + (10×11) + (20×15) + (14×25) + (12×15)] = 1240

Conclusion: In this paper, we discussed finding optimal solution for Octagonal Intuitionistic Fuzzy Transportation problem. We have used Accuracy function ranking method and Modi Method to easily understand and to arrive at nearer optimum solution. This methods are using less time and is very easy to understand and apply, so it will be very useful for decision makers. In future research we would propose generalized Octagonal Intuitionistic Fuzzy Numbers to deal problems and handling real life transportation problem having Intuitionistic Fuzzy Numbers.

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