NON EXISTENCE OF SKOLEM MEAN LABELING FOR FOUR STAR GRAPH

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Abstract: In this paper,we prove if $\ell \le m < n$ the four star $G = K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph $|m-n| > 2\ell + 4$ *for* $\ell = 2,3......; m = 2,3.........$

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Introduction: In [2], we proved the following theorems to study the existence of skolem mean graphs. We proved the three star $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m-n| = 4+\ell$ for $1 = 1, 2, 3, K$; $m = 1, 2, 3, K$ and $1 £ m < n$. The three star $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m-n| > 4+\ell$ for $1 = 1, 2, 3, K$; $m = 1, 2, 3, K$ and $1 £ m < n$. The four star $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m-n| = 4+2\ell$ for $1 = 2, 3, K$; $m = 2, 3, K$ and $1 £ m < n$. The four star $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m-n| > 4+2\ell$ for $1 = 2, 3, K$; $m = 2, 3, K$ and $1 £ m < n$. In [3]. The five star $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m-n| = 4+3\ell$ for $1 = 2, 3, K$; $m = 2, 3, K$ and $1 £ m < n$. Further, we prove the four star $K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m-n| = 7$ for $m = 1, 2, 3, K$ and $1 £ m < n$; The four star $K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m-n| > 7$ for $m = 1, 2, 3, K$ and $1 £ m < n$; The five star $K_{1,1} \cup K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m-n| = 8$ for $m = 1, 2, 3, K$ and $1 £ m < n$.

Definition 2.1: The four star is the disjoint union of $K_{1,a} \cup K_{1,b} \cup K_{1,c}$ and $K_{1,d}$ and is denoted by $K_{1,a} \cup K_{1,b} \cup K_{1,c} \cup K_{1,d}$.

Theorem 2.2: The four star $G = K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph $|m-n| > 2\ell + 4$ for $\ell = 2,3....; m = 2,3......; \ell \le m < n$.

Proof: To Prove $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ Lis not Skolem Mean Graph if $|m-n| > 2\ell + 4$.

Consider $|m-n| = 2\ell + 5$.

Let $G = K_{1,1} \cup K_{1,1} \cup K_{1,1} \cup K_{1,8} = 3K_{1,1} \cup K_{1,8}$

Where $V(G) = \left\{ v_{i,j} : 1 \le i \le 3;\ 0 \le j \le 1 \right\} \cup \left\{ v_{4,j} : 0 \le j \le 8 \right\}$

$E(G)=\left\{v_{i,0}\, v_{i,j} : 1 \le i \le 3;\ 1 \le j \le 1\right\} \cup \left\{v_{4,0}\, v_{4,j} : 1 \le j \le 8\right\}.$

Therefore, G has 15 vertices and 11 edges.
The vertex and edge labeling of G is given by

$v(G) \to \{1,2,...15\}$

$E(G) \to \{2,3,....15\}$

First let us consider the case. When $t_{4,0}=15.$

Then the possibilities of $t_{4,j}$: $1 \le j \le 8$ are 1, 2/3, 4/5, 6/7, 8/9, 10/11, 12/13 and 14.

We have two choiceless values (i, e) $t_{4,1}=14$ and $t_{4,8}=1.$

Also $\left\{x_{4,j} : 1 \le j \le 8\right\} = \{8,\ 9,\ 10,\ 11,\ 12,\ 13,\ 14,\ 15\}$

Case (a): Now let us consider the cases of $t_{4,2}$.

$t_{4,2}$ is either 12 and 13.

Without loss of generality we shall first consider $t_{4,2}=12.$

Then 13 should be the value of the remaining vertices (i, e) $t_{i,j}$ for $i=1,\ 2,\ 3$ and $j=0,\ 1.$

Let $t_{1,0}=13.$

Then $t_{1,1}$ should be 1, because if $t_{1,1}>1$ then $x_{1,1}>7.$

But all values greater then 7 are already the edge labels of $K_{1,8}$.

Therefore, $t_{1,0}$ should not be 13.

That is, 13 can not be allocated to any vertices of $t_{i,j}$ for $i=1,\ 2,\ 3$ and $j=0,\ 1.$

Therefore, $t_{4,2}=13.$

Case (b): Now we have, $t_{4,0}=15,\ t_{4,1}=14,\ t_{4,2}=13,\ t_{4,8}=1.$

Also, $x_{4,1}=15,\ t_{4,2}=14$ and $t_{4,8}=8.$

Therefore, $t_{1,0}=12.$

Then $t_{1,1}$ should be ≤ 2 because if $t_{1,1}>2$ then $x_{1,1}>7.$

Therefore, $t_{1,1}=2.$

So the vertex $t_{4,7}$ which should be either 2 or 3 is reduced to choice less vertex.

That is, $t_{4,7}=3.$ Since $t_{1,1}=2.$

Case (c): Now we have $t_{4,0}=15,\ t_{4,1}=14,\ t_{4,2}=13,\ t_{4,8}=1,\ t_{1,1}=2,\ t_{4,7}=3,\ t_{1,0}=12.$

Also, $x_{4,1}=15,\quad x_{4,2}=14,\quad x_{4,8}=8,\quad x_{4,7}=9$ and $x_{1,1}=7.$

Now $t_{4,3}$ should either be 10 or 11.

Let us proceed the proof by considering $t_{4,3}=10.$

Let $t_{2,0}=11.$

Then $t_{2,1}$ should be 1 if $t_{2,1}>1$ then $x_{2,1}>6$ which will be a label of one of $t_{4,j}$ for $1 \le j \le 8.$

But already we have $t_{4,8}=1.$

Therefore, $t_{2,1}$ is left without any choice of label.

Therefore, G is not a skolem mean graph when $t_{4,0} = 15$.

Next we shall consider the case when $t_{4,0} = 14$.

Then the possibilities of $t_{4,j}$ for $1 \le j \le 8$ are 1/2, 3/4, 5/6, 7/8, 9/10, 11/12, 13 and 15.

Now we have $t_{4,0} = 14,\ t_{4,1} = 15,\ t_{4,2} = 13$.

The edge label of $K_{1,8}$.

That is, $\{x_{4,j} : 1 \le j \le 8\} = \{8,\ 9,\ 10,\ 11,\ 12,\ 13,\ 14,\ 15\}$.

Case (d): Now let us consider the cases of $t_{4,3}$.

$t_{4,3}$ should either be 11 or 12.

Supposing if $t_{4,3} = 11$.

Then 12 should be label of a vertex among $t_{i,j}$ for $i = 1,\ 2,\ 3$ and $j = 0,\ 1$.

Let $t_{1,0} = 12$.

Then $t_{1,1}$ should be 2 if $t_{1,1} > 2$ then $x_{1,1} > 7$ which will be a lead to contrary. Let $t_{1,1} = 2$.

Therefore, $t_{1,1} = 7$.

Case (e): Now we have $t_{4,0} = 14,\ t_{4,1} = 15,\ t_{4,2} = 13,\ t_{4,3} = 11,\ t_{1,0} = 12,\ t_{1,1} = 2,\ t_{4,8} = 1$.

Also, $x_{4,1} = 15,\quad x_{4,2} = 14,\quad x_{4,3} = 13,\quad x_{4,8} = 8,\quad x_{1,1} = 7$.

Next, $t_{4,4}$ should either be 9 or 10.

Let $t_{4,4} = 9$.

Then 10 should be a label of one vertex among $t_{i,j}$ for $i = 2,\ 3$ and $j = 0,\ 1$.

If $t_{2,0} = 10$.

Then $t_{2,1}$ should be ≤ 2. Since if $t_{2,1} > 2$ then $x_{2,1} > 7$ which will be a lead to contrary.

Therefore, $t_{2,1} \le 2$.

But $t_{1,1} \le 2$ and $t_{4,8} = 1$. That is, $t_{2,1}$ is left without any suitable label. Which is not possible as f is bijection.

Therefore, G is not a skolem mean graph when $t_{4,0} = 14$.

Similarly we can prove that G is not a skolem mean graph for all the other values of $t_{4,0}$.

$\Rightarrow\ G = 3K_{1,1} \cup K_{1,8}$ is not al skolem mean graph.

Therefore, G is not a skolem mean graph when $|m-n| = 2\ell + 5$ with similar argument we can also prove that G is not a skolem mean graph when $|m-n| = 2\ell + 6$. Hence G is not a skolem mean graph when $|m-n| \ge 2\ell + 5$.

Application of Graph Labeling: The skolem mean labeling is applied on a graph (network), such as bus topology, mesh topology and star topology in order to solve the problems in establishing fastness, efficient communication and various issues in that area, in which the following will be taken into account.

1. A protocol, with secured communication can be achieved, provided the graph (network) is sufficiently connected.
2. To find an efficient way for safer transmissions in areas such as Cellular telephony, Wi-Fi, Security systems and many more.
3. Channel labeling can be used to determine the time at which sensor communicate.

Conclusion: Researchers may get some information related to graph labeling and its applications in communication field and work on some ideas related to their field of research.

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References:

1. J.C. Bermond, Graceful graphs,radio antennae and French windmills, Graph theory and combinatorics, pitman,London,(1979), 13 - 37.
2. V. Balaji, D.S.T. Ramesh,and A. Subramanian, Skolem Mean Labeling, Bulletian of Pure and Applied Sciences, vol. 26E, No. 2, 2007 ,245 - 248.
3. M.Lellis Thivagar, V.Ramesh, M.Arockia Dasan, M.Davamani Christober, Note On Local Functions In Ideal Extended Bitopological Spaces; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 249-252
4. V. Balaji, D.S.T. Ramesh,and A. Subramanian, Some Results On Skolem Mean Graphs, Bulletian of Pure and Applied Sciences, vol. 27E, No. 1, 2008 ,67 - 74.
5. V. Balaji, D.S.T. Ramesh and A. Subramanian, Some Results On Relaxed Skolem Mean Graphs, Bulletian of Kerala Mathematics Association, vol. 5(2), December 2009, 33 - 44.
6. Dr. Samudrala Venkateswarlu, Discrete Group With The Alternative Characterisations Twisted Rapid Decay Property; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 301
7. V. Balaji, D.S.T. Ramesh and A. Subramanian, Relaxed Skolem Mean Labeling, Advances and Application in Discrete Mathematics, vol.5(1), January 2010 ,11 - 22.
8. B. Gayathri, D. Muthuramakrishnan, K-Even Sequential Harmonious Labeling Of Some Special Graphs; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 233-245
9. V. Balaji, Solution of a Conjecture On Skolem Mean Graphs of Stars, $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$, International Journal of Mathematical combinatorics, vol.4(2011), 115 - 117.
10. V. Balaji, D.S.T. Ramesh and S. Ramarao, Skolem Mean Labeling For Four Star, International Research Journal of Pure Algebra, Vol.6 No.1, Jan. - 2016, 221 - 226.
11. V. Balaji, D.S.T. Ramesh and V. Maheswari,Solution of a Conjecture On Skolem Mean Graphs of Stars, $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$, Sacred Heart Journal of Science and Humanities, volume 3, July 2013.
12. G. Mahadevan, V. G. Bhagavathi Ammal, A. Iravithul Basira, Maruthamani, Some More Results On Strong And Weak Triple Connected Dominaiton In Fuzzy Graphs; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 246-248
13. J.A. Gallian, A dynamic survay of graph labeling, The Electronic Journal of Combinatorics, 6(2010), DS6.
14. J. Maria Remona, Dr. K. Julia Rose Mary, Fuzzy Analysis Of Mx/M/1/MWV With Breakdown Queueing Model; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 257-260
15. F.Harary, Graph Theory, Addison - Wesley, Reading, 1969.
16. Kuchi.Padmavathi, Recognition Of General Set And Single Member Descriptions; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 253-256
17. V. Maheswari, D.S.T. Ramesh and V. Balaji, On Skolem Mean Labeling, Bulletin of Kerala Mathematics Association,Vol.10, No.1,2013,89 - 94.
18. G. Vasanthakannan, K. Indirani, Some Results On Contra Nano Rg**B B-Closed And Open Maps In Nanotopological Spaces; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 297-300

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