A STUDY ON QUEUING SYSTEM WITH MULTIPLE ARRIVALS AND BERNOULLI FEEDBACK SUBJECT TO CATASTROPHICAL EVENTS

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Abstract: The analysis paper influence server takes a vacation of arbitrary distribution whenever remains empty and study multiple vacation policies with batch Poisson arrivals and general service times. Once completion of the service client will instantly be part of the tail of the queue as a feedback client with Bernoulli chance, otherwise, the client might depart forever from the system. The system suffers random Catastrophe that, once they occur, instantly removes all customers from the system. Catastrophe affects the system solely throughout the time the server is busy serving customers. Instantly once the prevalence of a Catastrophe, the server undergoes for a repair with random length. We have a tendency to conjointly contemplate that server takes vacations whenever empties. We have a tendency to investigate two totally different models officiating the holiday policy that is followed, the multiple and therefore the single vacation policy. We have a tendency to analyses this technique exploitation the supplementary variables technique and that we acquire the chance generating a operate of the stationary queue length distribution and therefore the Pierre Simon de Laplace remodel of the busy period's distribution.

Keywords: Queuing System, Bernoulli Feedback, Catastrophe, Probability Generating Function, Multiple Vacations, Laplace Transform and Busy Period.

Introduction: Queuing systems with vacation time have been found to be useful in modeling the systems in which the server has additional tasks. [10] Have studied server vacations based on Bernoulli schedules and a single vacation policy. [5] Have studied a single server queue with two phase of heterogeneous service under Bernoulli schedule and a general vacation time. [2] Have studied a single server M/G/1 feedback queue with two types of service having general distribution. [10] Proposed anqueuing system with restricted admissibility of arriving batches. [9] [3] considered an M/G/1 queue system where disasters occurring at certain random times and studied the model for different disaster rules. We also refer [3] where they studied the effect of catastrophes on the strategic customer behavior and the paper of [7] who studied a M/G/1 system with working breakdowns suffering by disasters. [4] Gave a generalization for the same model. Repair times after disasters occurring for discrete and continuous time models studied respectively by [7]and [6] study single server queues with state dependent feedback. [13] Consider a single server feedback queue with impatient and feedback customers. They study M/M/1 queuing model for queue length at arrival and obtain result for stationary distribution, mean and variance of queue length. The combined effects of balking and reneging in an M/M/1/N queue have been investigated by [1]. [17] Extended this work to study an M/M/C/N queue with balking, reneging and server breakdowns. [12] Obtain the Performance analysis of Queuing System with general conditions Heterogeneous Servers Subject to Catastrophe Intensity. [16] Obtain transient solution of M/M/1 feedback queue with catastrophes using continued fractions. The steady-state solution, moments under steady state and busy period analysis are calculated. Feedback queues play a vital role in the areas of Computer networks, Production systems subject to rework, Hospital management, and Super markets and banking business etc. [11] Studied on Net Profit Associated with Queuing System subject to Catastrophically events.

Then [15] introduced the concept of feedback queues. The queuing systems which include the possibility for a customer to return to the server for additional service are called queues with feedback. [2], [8].
The rest of the paper is organized as follows: In Section 2, the mathematical description of the considered queuing system is given. In Section 3, the probability generating functions of the stationary queue length distribution. In Section 4, steady state probabilities of the system are deduced. In Section 5, we obtained Laplace transform of the busy period's distribution. Finally, in Section 6conclusion is provided.

Description of the Model:
Arrival: customers arrive in batches according to a Poisson process with Parameter λ. Batches are assumed to be independent identical distribution with $X_n\ n=1,2,...\infty$ the nth batch and Probability generating functions $X(z)=\sum_{n=1}^{\infty}P(X_1=n)z^n$.
Service: Service times are assumed to be independent identical distributionrandom variables with common distribution S, Corresponding density $S^{'}$ and hazard rate function $\mu(x)=\frac{S^{'}(x)}{1-S(x)}, x\geq 0$. S denoted by $S^{'}(s)=\int_0^{\infty}e^{-sx}dS(x)$ and mean $m_S=ES$
Feedback: After completion of each service the customer either join at the end of the queue with probability p or leave the system with probability q = 1 – r.
Catastrophes: The catastrophes occur at the server as an independent Poisson process with parameter δ and inactivate the server upon arrival.
Repairs: The repair times of the failed server are independent identical distribution exponential random variable with distribution function R. density function $R^{'}$ and hazard rate function $r(x)=\frac{R^{'}(x)}{1-R(x)}, x\geq 0$.
And Laplace transform $R^{'}(s)=\int_0^{\infty}e^{sx}dR(x)$. And with mean is $m_R=ER<\infty$

Balance Equations:

$$\frac{d}{dx}P_n(x)+(\lambda+\delta+\mu(x))P_n(x)$$
$$=\lambda\sum_{k=1}^{n-1}X_kP_{n-k}(x),\ x>0, n\geq 1 \qquad (3.1)$$
$$\frac{d}{dx}W_0(x)+(\lambda+r(x))W_0(x)=0 \qquad (3.2)$$
$$\frac{d}{dx}W_n(x)+(\lambda+r(x))W_n(x)$$
$$=\lambda\sum_{k=1}^{n-1}X_kW_{n-k}(x),\ x>0, n\geq 1 \qquad (3.3)$$
$$\frac{d}{dx}V_0(x)+(\lambda+u(x))V_0(x)=0 \qquad (3.4)$$
$$\frac{d}{dx}V_0(x)+(\lambda+u(x))V_0(x)$$
$$=\lambda\sum_{k=1}^{n-1}X_kV_{n-k}(x),\ x>0, n\geq 1 \qquad (3.5)$$

The boundary conditions of the differential equations are

$$P_n(0)=(1-r)\int_0^\infty P_{n+1}(x)\mu(x)dx + r\int_0^\infty P_n(x)\mu(x)dx$$

$$+\int_0^\infty V_n(x)u(x)dx + \int_0^\infty W_n(x)r(x)dx \qquad (3.6)$$

$$V_0(0)=(1-r)\int_0^\infty P_1(x)\mu(x)dx + \int_0^\infty V_n(x)u(x)dx$$

$$+\int_0^\infty W_n(x)r(x)dx \qquad (3.7)$$

$$V_0(0)=0, n=1,2,.... \qquad (3.8)$$

$$W_0(0)=\delta\sum_{n=1}^{\infty}\int_0^\infty P_n(x)dx, n\geq 1 \qquad (3.9)$$

With normalization condition

$$\sum_{n=1}^{\infty}\int_0^\infty P_n(x)dx$$

$$+\sum_{n=0}^{\infty}\left(\int_0^\infty W_n(x)dx+\int_0^\infty V_n(x)dx\right)=1 \qquad (3.10)$$

Define the partial probability generating functions

$$P(x;z)=\sum_{n=1}^{\infty} z^n p_n(x),\ W(x;z)$$

$$=\sum_{n=0}^{\infty} z^n W_n(x),\ V(x;z)=\sum_{n=1}^{\infty} z^n V_n(x)$$

The partial probability generating functions of the number of customers in the system in stationary regardless of the value of the supplementary variables is given by

$$p(z)=\int_0^\infty P(x;z)dx, W(z)=$$

$$\int_0^\infty W(x;z)dx, V(z)=\int_0^\infty V(x;z)dx \qquad (3.11)$$

Theorem: 1

The probability generating function for the number of customers in the system when the server is busy under the multiple vacation policy is given by

$$p(z)=P(0;z)\quad \frac{1-\hat{S}\,(\delta+\alpha(z))}{\delta+\alpha(z)} \qquad (3.12)$$

Where $p(0,z)=z\ \dfrac{V(0;z)(1-(\hat{U}\,(\alpha(z))}{\hat{S}\left(\delta+\alpha(z)\right)((1-r(1-z))-z}$ and $\alpha(z)=\lambda(1-X(z))$.

Proof:

Multiplying (1),(2), (3), (4),(5) and (6) by z^n and adding we obtain the linear first order PDEs

$$\frac{\partial P(x;z)}{\partial x}+(\alpha(z)+\delta+\mu(x))\,P(x;z) = 0$$
$$\frac{\partial W(x;z)}{\partial x}+(\alpha(z)+r(x))W(x;z) = 0 \qquad (3.13)$$
$$\frac{\partial V(x;z)}{\partial x}+(\alpha(z)+u(x))V(x;z) = 0$$

And the equation

$$\sum_{n=1}^{\infty} z^n p_n(0)=\int_0^{\infty}\sum_{n=1}^{\infty} z^n V_n(x)u(x)dx+(1-r)$$
$$\int_0^{\infty}\sum_{n=1}^{\infty} z^n p_{n+1}(x)\mu(x)dx + \int_0^{\infty}\sum_{n=1}^{\infty} z^n W_n(x)r(x)dx$$
$$+\int_0^{\infty}\sum_{n=1}^{\infty} z^n P_n(x)\mu(x)dx \qquad (3.14)$$

$$P(0;z)=z^{-1}(1-r)\int_0^{\infty} P(x;z)\mu(x)dx-(1-r)\int_0^{\infty} P_1(x)\mu(x)dx$$
$$+r\int_0^{\infty} P(x;z)\mu(x)dx+\int_0^{\infty} V(x;z)u(x)dx-\int_0^{\infty} V_0(x)u(x)dx$$
$$+\int_0^{\infty} W(x;z)r(x)dx-\int_0^{\infty} W_0(x)r(x)dx \qquad (3.15)$$

Solving (13)we obtain

$$P(x;z)=P(0;z)(1-S(x))e^{-(\delta+\alpha(z))x}$$
$$W(x;z)=W(0;z)(1-R(x))e^{-\alpha(z)x} \qquad (3.16)$$
$$V(x;z)=V(0;z)(1-U(x))e^{-\alpha(z)x}$$

From (16) we obtain

$$\int_0^{\infty} P(x;z)\,\mu(x)dx$$
$$=\int_0^{\infty} P(0;z)(1-S(x))e^{-(\delta+\alpha(z))x}\,\mu(x)dx$$
$$=P(0;z)\,\hat{S}(\delta+\alpha(z)) \qquad (3.17)$$
$$\int_0^{\infty} W(x;z)\,r(x)dx$$
$$=\int_0^{\infty} W(0;z)(1-R(x))e^{-\alpha(z)x}\,r(x)dx$$
$$=W(0;z)\,\hat{R}(\alpha(z)) \qquad (3.18)$$

$$\int_0^\infty V(x;z)u(x)dx =$$

$$\int_0^\infty V(0;z)(1-U(x))e^{-\alpha(z)x}u(x)dx$$

$$= V(0;z)\hat{U}(\alpha(z)) \qquad (3.19)$$

From equation (17)-(18) we obtain

$$P(0;z) = V(0;z)\hat{U}(\alpha(z)) + z^{-1}(1-r)P(0;z)\hat{S}(\delta+\alpha(z))$$

$$+W(0;z)\hat{R}(\alpha(z)) - V_0(0) + rP(0;z)\hat{S}(\delta+\alpha(z))$$

Since no customers are present when vacation periods starts then $V(0;z)=V_0(0)$ and $W(0;z)=W_0(0)$

Then we obtain

$$P(0;z) = z\frac{V_0(0)\left(1-\hat{U}(\alpha(x))\right) - \delta P(1)\hat{R}(\alpha(x))}{\hat{S}(\delta+\alpha(x))(1-r(1-z)) - z} \qquad (3.20)$$

We can prove this via Rouche's Theorem that denominator of P (0; z) has an unique root in the disk $|z|<1$.

The P (0; z) must also vanishes at z_0 So we can establish a relationship for $V_0(0)$ and P(1) as

$$V_0(0)\left(1-\hat{U}(\alpha(x))\right) = \delta P(1)\hat{R}(\alpha(z_0)) \qquad (3.21)$$

Steady State Distributions: We derive the stationary probabilities for the state of the server based on the corresponding marginal customers in the system. In order to simplify the expressions from equations (3.21) we set $\gamma_1 = \dfrac{\hat{R}(\alpha(z_0))}{1-\hat{U}(\alpha(z_0))}$ for multiple models.

When the Server is vacation from the theorem 1 for the multiple models

$$P(z) = z\delta\frac{\gamma_1(1-(\hat{U}(\alpha(z))) - \hat{R}(\alpha(z))}{\hat{S}(\delta+\alpha(z))((1-r(1-z)) - z}$$

$$\frac{1-\hat{S}(\delta+\alpha(z))}{\delta+\alpha(z)}P(1) \qquad (4.1)$$

When the server is under vacation from the equation (3.11), (3.16) and (3.20) we obtain

$$W(z) = \delta\ \frac{1-\hat{S}(\delta+\alpha(z))}{\delta+\alpha(z)}P(1) \qquad (4.2)$$

The probability that the server is under vacation using the Hospital rule is $W(1) = P(1)\delta_{m_R}$. Relation (4.2) is valid for the multiple models using each case the appropriate value of P (1).

When the server is on vacation from the equation (3.11), (3.16) and (3.21) for the multiple vacation then we obtain

$$V(z) = \delta\gamma_1\ \frac{1-\hat{U}(\alpha(z))}{\alpha(z)}P(1) \qquad (4.3)$$

The probability that the server is under vacation using the Hospital rule is $V(1) = \delta P(1)_{mU\gamma_i}$ for $i=1,2$. Respective for multiple models.

The PGF of the number of customers in the system in stationary.
We can now derive the probability generating function of the number of customers in the system in stationary for the multiple models summing the corresponding marginalprobability generating function as given above From the equation (4.1), (4.2), (4.3) then we obtain

$$\phi(z)=\delta P_1\left\{z\frac{\gamma_1(1-(\hat{U}(\alpha(z)))-\hat{R}(\alpha(z))}{\hat{S}(\delta+\alpha(z))(1-r(1-z))}\frac{\hat{S}(\delta+\alpha(z))}{\delta+\alpha(z)}+\gamma_1\frac{1-\hat{U}(\alpha(z)}{\alpha(z)}+\frac{1+\hat{U}(\alpha(z)}{\alpha(z)}\right\} \quad (4.4)$$

Busy Period Analysis: $\psi(t)$ The probability that the typical busy period in stationary starts with n customers present and let $\{t_l\}\ l=0,1,2...$ be the initiation epoch of the busy periods. The N_{t_l} is the number of customers in the system at the initiation epoch of the firstbusy period and $\psi(t)$ the steady state probability that an arbitrary customer finds n customers in the system at a busy period initiation epoch. The probabilities where system begins vacation given respectively by

P [server stopped by catastrophe] = $\dfrac{W_0(0)}{W_0(0)+V_0(0)}$

P [server stopped by Catastrophe]= $\dfrac{V_0(0)}{W_0(0)+V_0(0)}$

Where $W_0(0)$ given by equations (3.20) with appropriate value of P (1) for the model while $V_0(0)$ given equation (3.21) multiple models.
Conditioning on the number of customers which arrive during the vacation we obtain multiple vacation model the probability of $\psi(t)$ as

$$\psi(t)=\frac{W_0(0)}{W_0(0)+V_0(0)}\sum_{k=1}^{n}b_k X_n^{(k)*}$$
$$+\frac{V_0(0)}{W_0(0)+V_0(0)}\sum_{k=1}^{n}\alpha_k X_n^{(k)*}$$

Where $X_n^{(k)*}$ is the k fold convolution of X_n and α_k =P [batch arrives during a vacation time] while $b_k=P[\text{k batches arrive during a vacation time }]$, then we obtain the corresponding probability generating functions for multiple model.

$$\psi(t)=\frac{W_0(0)}{W_0(0)+V_0(0)}\left\{\hat{R}(\alpha(z))-b_0\right\}$$
$$+\frac{V_0(0)}{W_0(0)+V_0(0)}\left\{\hat{U}(\alpha(z))-a_0\right\} \quad (5.1)$$

Conclusion: The research paper deal with study on Queuing System with Multiple arrivals and Bernoulli feedback subject to Catastrophically Events is discussed. A system of differential equations satisfied by the multiple models has been set up. We obtained the probability generating function of the stationary queue length distribution and the Laplace transform of the busy period's distribution through the supplementary variables technique. The pertaining system has moreover been studied, when there are catastrophes, and hence confirmed the respective results obtained by other researchers.

References:

1. Ancker Jr., C. J. and Gafarian, A. V., "Some Queuing Problems with Balking and Reneging". I, Operations Research, (Vol. 11, PP. 88-100), 1963.
2. Thangaraj, V. and Vanitha, S., "On the Analysis of M/M/1 Feedback Queue with Catastrophes using Continued Fractions", International Journal of Pure and Applied Mathematics, (Vol. 53, PP. 131-151), 2009.

3. D' Avignon, G.R. and Disney, R.L., "Single Server Queue with State Dependent Feedback ", INFOR, (Vol. 14, PP. 71-85), 1976.
4. Boudali, O., and Economou, A., (2013). "The Effect of Catastrophes on the Strategic Customer Behavior in Queuing Systems", Naval Research Logistic,(PP. 571-587).
5. D. Narasimha Raju, M. Divya, Hash Blinear Search - A Novel Loom Searching Algorithm; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 263-266
6. O. J. Boxma., D. Perry and W. Stadje. (2001). "Clearing Models for M/G/1 Queues. Queuing Systems"(PP. 287–306).
7. J.Subashini, K.Indirani, A Study On Weakly Ultra Separation Axioms Via (μ)^ B-Kernel Set; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 261-262
8. S.R. Chakravarthy. (2009). "A disaster queue with Markovian arrivals and impatient customers". Applied Mathematics and Computation (PP. 48–59.)
9. R.L. Disney, D.C. McNickle, and B. Simon, "The M/G/1 queue with instantaneous Bernoulli feedback", Nav. Res. Log. Qua(PP. 633-644) ,1980.
10. B.K. Kim., and D.H. Lee, "The M/G/1 queue with disasters and working breakdowns", Applied Mathematics Model (2013).
11. I. Arockiarani, S. Sharmila, Pairwise Intuitionistic Fuzzy Regular Volterra Spaces; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 267-276
12. B. Krishna Kumar, S. Pavai Madheswari., and A.Vijayakumar, "The M/G/1 retrial queue with feedback and starting failures", Applied Mathematical Modeling, 26 (2002), 1057 - 1075.
13. Y.Levi and U.Yechilai, "An M/M/s queue with server vacations", INFOR (Vol.14, PP. 153-163),1976.
14. K.C.Madan and R.F. Anabosi, "A single server queue with two types of service, Bernoulli schedule server vacations and a single vacation policy", Pakistan Journal of Statistics,(Vol.19, PP. 331-442), 2003.
15. Cao, J., Li, K., Stojmenovic, I.: Optimal power allocation and load distribution for multiple heterogeneous multicore server processors across clouds and data centers. IEEE Tran. Comp. 63, 45–57, 2014
16. S. Maria Sylviaa, Dr. S. Vijayarani, Impact Of The Usage Of Dimensionality Reduction Techniques In Medical Datasets: An Analysis; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 277-282
17. Santhakumaran, A. and Thangaraj, V., "A Single Server Queue with Impatient and Feedback Customers", Information and Management Science, (Vol. 11, PP. 71-79), 2000.
18. Sharma S.K. and Kumar R., "A Markovian feedback Queue with Retention of Reneged Customers and Balking", AMO-Advanced Modeling and Optimization, (Vol. 14, PP. 681-688), 2012.
19. Takacs, L., "A Single Server Queue with Feedback", The Bell System Tech. Journal, (Vol. 42, PP. 134-149), 1963.
20. P. Raju, K. Sobhanbabu, D. R. K. Reddy, Conformally Flat Spherically Symmetric Perfect Fluid Distribution In F(R, T) Gravity; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 283-285
21. Thangaraj, V. and Vanitha, S., "On the Analysis of M/M/1 Feedback Queue with Catastrophes using Continued Fractions", International Journal of Pure and Applied Mathematics, (Vol. 53, PP. 131-151), 2009.
22. K-H. Wang and Y-C. Chang.,"Cost analysis of a finite M/M/R queuing system with balking, reneging and server breakdowns", Mathematical Methods of Operations Research, (Vol.56(2),PP 169–180), 2002.
23. Sujith Kanagarajan, Sivakumar Ramakrishnan, Integration Of Internet-Of-Things Facilities And Ubiquitous Learning For Still Smarter Learning Environment; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 5 Issue 2 (2016), Pg 286-289
24. Y. Zhang, D. Yue and W. Yue., "Analysis of an M/M/1/N queue with balking, reneging and server vacations", Proc. of the Fifth International Symposium, (PP.37– 47), 2005.

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